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Wind Power Forecast Error Simulation Model

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Abstract—One of the major difficulties introduced with wind power penetration is the inherent uncertainty in production originating from uncertain wind conditions. This uncertainty impacts many different aspects of power system operation, especially the balancing power requirements. For this reason, in power system development planing, it is necessary to evaluate the potential uncertainty in future wind power generation. For this purpose, simulation models are required, reproducing the performance of wind power forecasts.

This paper presents a wind power forecast error simulation models which are based on the stochastic process simulation. Proposed models capture the most important statistical parameters recognized in wind power forecast error time series. Furthermore, two distinct models are presented based on data availability. First model uses wind speed measurements on potential or existing wind power plant locations, while the seconds model uses statistical distribution of wind speeds.

Keywords—Wind power, Uncertainty, Stochastic process, Monte Carlo simulation.

I. Introduction

PRESENCE of renewable generation in power systems worldwide is constantly increasing, following the obligations taken from the Kyoto protocol. Although the dynamics of these changes are highly dependent on ambitions and opportunities in each country, it is hard to find any country that did not have some experience with this type of generation. As a continuation of consecutive aims, some countries integrated significant levels of renewable generation, higher than required by international obligations. However, these countries are rare examples, combining both national strategy and domestic industrial development in achieving such results. Increase in renewable generation presence followed an extreme fall in costs of renewable technology. Altogether, both wind and PV sources exhibit favourable trends in costs, and general acceptance and further development and increase in their presence can be expected in the following decades.

Previously introduced trends in power system development are not performed without difficulties. Namely, although highly beneficial from the environment's perspective, renewable generation introduces several difficulties in power system operation. In this sense, one of these specificities presents the main focus of this work, i.e. the uncertainty in wind power production. Power system uncertainty results from the necessity of power production and consumption planning—which is a typical procedure in power system operation planning—aiming at matching generation and consumption as close as possible. Consequently, wind production requires planning, typically based on weather forecasts. Due to the inherent uncertainty in these plans, wind power output is unreliable.

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In many power system development studies it is necessary to analyse impact of this uncertainty on different aspects of the power system operation, for example, analysis of potential changes in balancing power requirements with the introduction of a new wind generation facility. In such studies, it is necessary to simulate performance of wind power forecasts, accounting for different factors that are influencing them. This paper presents two simulation models, depending on data availability, which can be further modified depending on desired level of accuracy.

Compared to existing models proposed in relevant publications, models presented in this work account for several important factors and provide generality in terms of wind power plant allocation. Majority of models proposed in these publications consider wind power forecast error time series and attempt to reproduce its statistical parameters. Typical drawback in such models is a poor generality. Namely, since the typical wind power forecasting is based on weather models, statistical parameters depend on wind power plant disposition and capacity. Models presented here relay on investigations considering weather based forecasting but compared to similar models provide simpler Monte Carlo based wind power simulation model combined with autoregressive wind speed forecast error model reproducing autocorrelation and correlation.

In the following sections, a general overview of stochastic processes modeling is given. These models are used in later sections, accounting for specificities in wind uncertainty. Finally, proposed model is applied on a case study.

II. Stochastic Processes Simulation

Stochastic process is a collection of random values used for describing the evolution of some random variable over time. This evolution can take several or even infinitely many outcomes. In this sense, forecast error evolution over time is a typical example of a stochastic process. Therefore, the existing knowledge on stochastic process modeling can be used for the forecast error simulation.

Each stochastic process is described with different mathematical parameters and a basic task set for a model used for stochastic process generation is to provide which parameters are preserved and to which extent. Based on these requirements, in following paragraphs, a typical parameters requiring preservation are examined and modeling techniques enabling their preservation are provided.

Common approach adopted for stochastic process modeling is the application of one of the autoregressive models. In [1], [2] an autoregressive moving average model is proposed,

while [3] adopts a native autoregressive model for the forecast error modeling. General mathematical expression describing autoregressive model in vector form (i.e. VAR model) is:

$$y_t = a + \sum_{i=1}^{p} A_i y_{t-i} + \epsilon_t \tag{1}$$

Variables and parameters in these models are:

y_t	Vector of time series variables at time t
a	Vector of offsets
A_i	Autoregressive matrices
ϵ_t	Multivariate normal random vectors

Basic characteristic of such models is their ability to capture certain interdependencies among variables at different time instances (autocorrelation) and, in case of vector forms, interdependencies among variables in different vectors (correlation).

Choosing appropriate values for parameters of autoregressive process allows for exact autocorrelation function to be preserved. Estimation of these parameters requires historical knowledge on time series of modeled process which are often unavailable. Furthermore, preservation of these interdependencies is often not required to be exact since it is usually not deterministic in the first place.

Second important parameter is the correlation between variables of several stochastic processes. More precisely, correlation between each vector in VAR model. In a simple form, this requirement is met by inclusion of desired covariance structure in Gaussian noise of VAR model. This covariance structure is later transferred through the complete autoregressive process. For this purpose, it is necessary to apply multivariate random number generation—a procedure for generation of random vectors from multivariate distribution given its mean vector μ and covariance matrix Σ. Probability density function of the multivariate normal distribution is given with:

$$y = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} e^{\frac{-1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)} \tag{2}$$

where, together with previously introduced variables, d is the dimension of vectors. Procedure for generating multivariate normal random vectors with desired mean vector and covariance matrix is described in Appendix A.

Described procedure enables simulation of stochastic process with desired covariance matrix, mean vector and autocorrelation function within each vector. However, model is limited by means of statistical distribution of each vector in stochastic process. Namely, only normal stochastic processes are captured with previous model and further improvements are required in order to capture other distributions as well. Furthermore, another important requirement, characteristic for forecast errors, is the functional dependence of standard deviation over forecasting horizon. Namely, if a 24 hours ahead forecasting is considered, then each hour's forecast has different standard deviation. If such requirement is included in multivariate Gaussian noise then the autoregressive evolution distorts this dependence and resulting process fails in capturing this characteristic. Since this requirement is far more important than the autocorrelation and correlation, an improvement of previous model is necessary. The same improvement resolves both of the mentioned issues. Following paragraphs give an overview of the procedure used for this purpose.

Stochastic process can be transformed from normal distribution to any desired distribution with specific mean and standard deviation. First, normal distribution is transformed to uniform, accounting for its mean and standard deviation, via cumulative distribution function

$$x^{unif} = F_{\bar{x}_1\sigma_1}(x^{\bar{x}_1\sigma_1}).$$

This uniform process can then be transformed to any desired process if an inverse cumulative distribution function is known for this process

$$x^{\bar{x}_2\sigma_2} = F^{-1}_{\bar{x}_2\sigma_2}(x^{unif}).$$

This transformation can be used for the generation of a stochastic process with desired distribution, mean and standard deviation. However, in case of a correlated and autocorrelated process, this transformation introduces deviations from the original parameters. In the case of statistically similar distributions, these deviations are negligible. Furthermore, it was stated previously that the most important parameter in this case is the dependence of standard deviation on the time horizon. Accounting for this requirement, losing accuracy in correlation and autocorrelation could be seen as an acceptable compromise.

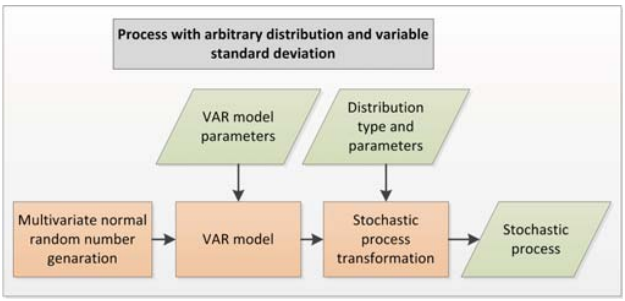

Fig. 1 Models for stochastic process simulation

Fig. 1 displays a principal depiction of the procedure outlined above. First component in procedure is the Gaussian noise simulation preserving the desired mean vector and a covariance structure. The second component is the autoregressive model which generates autocorrelated series. Finally, obtained series are transformed to a series with a desired distribution and standard deviation dependence on horizon. Main disadvantage of the proposed model is the distortion of the original autocorrelation and correlation through the transformation process. Further improvements can be incorporated in order to eliminate this issue; however, since these distortions are not significant and the importance of autocorrelation and correlation accuracy is not of interest here, these improvements will not be considered. It should be pointed out that in the case where autocorrelation is not of importance, a VAR model can be excluded from the proposed procedure.

III. Wind Power Uncertainty

Wind power uncertainty has been seen as the greatest challenge in early years of wind power integration. During this period large number of publications covered this topic. In these first works a wide terminology was used for describing this particular issue. Terms like variability and uncertainty were often misunderstood and other terms such as intermittency were used inappropriately. However, years of research resulted in wide knowledge on wind power uncertainty.

In order to understand uncertainty in wind power it is necessary to examine its origins. Uncertainty in wind power is a direct consequence of wind power production forecasts preformed on different time horizons and with different models. Following paragraphs distinguish several forecasting techniques recognised in existing publications [4]:

The simplest technique is of course the persistence model. In this approach, the forecast value is equal to a previous realisation. This model also serves as the base model, and the validation of forecasting methods is performed through the comparisons with this model. Application of this model is appropriate for a single or few steps ahead forecast. Furthermore, the major disadvantage of this model is that it can not capture extreme events.

Second modeling approach includes time-series analysis [5]. Based on this knowledge, autoregressive models are developed. These models perform better than persistence model. However, the issue of forecasting extreme events remains unresolved in this approach as well.

The best approach for capturing extreme events is the incorporation of the weather models [6]. Advanced weather forecasting techniques allow for precise wind conditions forecasting and later transformation to power output. This model has proven to be the best solution for mid- and long-term forecasting.

In this work, a weather model forecasting is assumed and forecast error simulation model is based on specificities of such a model. Following paragraphs give an overview of the application of previously introduced general stochastic process simulation models for this purpose.

As it was previously stated, basic requirement for a forecast error simulation model is the preservation of several important parameters. Similarly, certain characteristics of forecast error time series have been considered in models developed here.

First, the autocorrelation function within forecasting error evolution over considered period was considered. Preservation of this parameter is in direct relation with forecasting horizon and later usage of the process. For example, if a 24 hour ahead forecasts are performed then the autocorrelation should be preserved on this temporal level. Such requirement introduces even further complications, since it is difficult to maintain certain autocorrelation for such a low lag samples. Furthermore, if a simulated process is later used for analyses, where each hour is considered separately, then this requirement is pointless. In [3] an autocorrelation of wind power forecast error series was investigated and an exponentially decreasing dependence of autocorrelation on a time lag was reported. Except for this publication, this issue was not given a significant importance—as far as the authors are informed. In the mentioned publication it was concluded that this function is highly variable, depending on sites, but certain autocorrelation must be included in the forecast error models. Following this premise, a VAR model used here accounts for this dependence.

On the other hand, a matter of forecast error correlation is vary important due to the effect of error reduction. In [7], [8] this issue has been investigated and it was concluded that the wind power forecast errors are correlated, and expression combining correlation coefficient with distance among the plants has been proposed. It can be assumed that the similar correlation is exhibited among wind speed forecast errors as well. This dependence was included in stochastic process simulation model presented in this work.

Finally, the most important parameter that needs to match real data is the standard deviation of the forecast error. Since the 24 hour ahead forecast error is modeled here, a standard deviation varies with the forecast horizon. In [9] a standard deviation of forecast error for different horizons was examined. Fig. 2 displays this dependence.

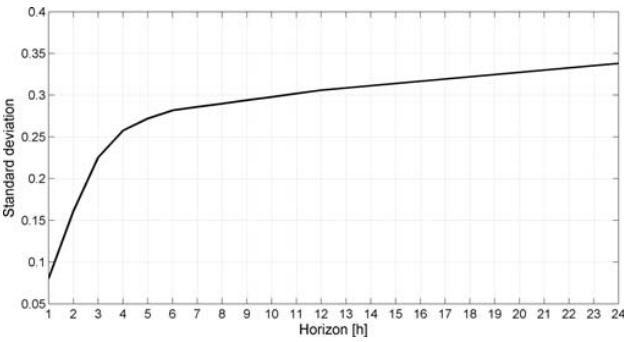

Fig. 2 Dependence of standard deviation of wind power forecast error on forecasting horizon

Knowing all of these characteristics that require preservation, it is possible to apply models presented in the previous section for the wind speed forecast error simulation. In [9] it was stated that the wind speed forecast error is normally distributed around the forecast value. Furthermore, it was concluded that the parameters for this distribution are dependent on the forecast value, but this dependence is not significant and it was neglected in further examinations in the mentioned publication. The same assumption is introduced here, since there are no publications reporting on this dependence in detail, as far as the authors are informed. As a final remark on the mentioned publication—which serves as the basic background for this work—wind speed to wind power error transformation has been investigated. Following remarks relay on the examinations presented in this publication.

Forecasting error transformation from wind speed to wind power domain is performed via the wind power plant P/w curve. High non-linearity of this curve is the main reason for the big disputes surrounding the distribution fit of wind power forecast error discussed in several publications [10], [11]. Fig. 3 depicts this transformation graphically. It can be

seen that the same distribution of forecast error in wind domain can result in very different distributions in the power domain, depending on the mean wind speed.

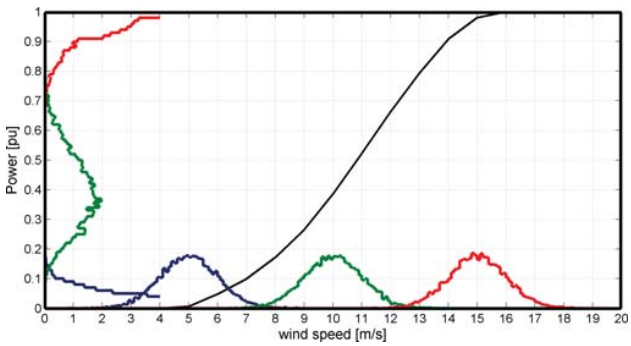

Fig. 3 Wind uncertainty transformation to power

Incorporating specific parameters of wind speed forecast error mentioned in the previous paragraphs, a model for the yearly simulation of forecast error can be developed. As an input for this model, a certain wind power plant (WPP) disposition and yearly wind speed data (or Weibull distribution parameters) for each location are used. In accordance, two models are developed, based on different requirements and input data. In its core, each of these models is a Monte Carlo simulation with a rather complex interdependencies among random variables.

First model is based on the measured hourly wind speed data which can be seen as prefect day-ahead wind speed forecasts for each location. Simulated forecast error can be superposed on these prefect forecasts and day-ahead wind speed scenarios can be obtained for each location and for each day of the year. Such wind speed scenarios can then be transformed into the power domain, thus obtaining the wind power scenarios. Summation of these scenarios allows for the error reduction caused by WPP dispersion. This model accounts for the autocorrelation function, but if this parameter was not of interest for a specific applications, proposed model can be further simplified by the simple elimination of the VAR model component. A principal depiction of this model is given in Fig. 4.

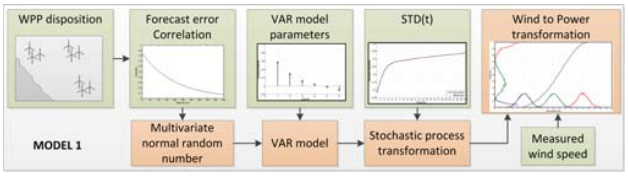

Fig. 4 Principal depiction of Wind power yearly forecast error simulation model 1

Second model is developed based on the unavailability of wind speed measurements time series, but with a knowledge of the Weibull statistical distribution parameters of wind speeds at each location. In such circumstances it is possible to simulate wind speed data in a similar manner, like in the case of forecast error simulation. However, autocorrelation function in the case of wind speeds is more important than in the case of forecast errors. Since the 24 hour simulation can not capture this function, only the variant in which autocorrelation is not required is considered here. In such model, wind speed correlation based on WPP disposition is introduced in multivariate normal random number generator. In the next step, this process is transformed into Weibull distribution with desired parameters. A principal depiction of the second model is given in Fig. 5. The issue of the wind speed autocorrelation can be resolved by a yearly wind speed data simulation; however, such approach is not considered here since it is not a primary concern of this work.

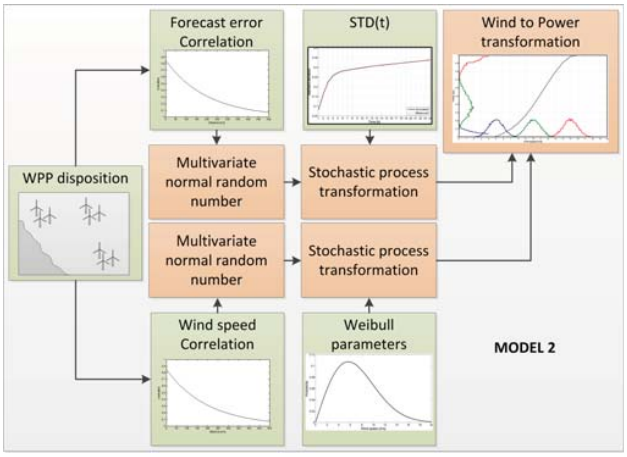

Fig. 5 Principal depiction of Wind power yearly forecast error simulation model 2

Both of these models are applicable for power system development analysis. Knowing the possible locations of WPP it is possible to estimate increase in balancing power and reserve capacity requirements caused by the introduction of the new WPPs.

IV. Case Study

For a first case study, a single WPP was considered with a known wind speed 24 hours ahead forecast. Previously described Model 1 was used for the simulation of wind speed and wind power forecast error scenarios. Fig. 6 depicts forecasts of wind speed and wind power, together with percentile plot of forecast error scenarios for both wind speed and wind power. In this example, a linear increase in wind speed was chosen as forecast in order to examine error transformation at all points in a power / wind curve. Previously mentioned non-linearity of transformation is even more pronounced in this example. It is obvious from this depiction that, in simulation of wind power forecasting performance, it is important to account for the actual forecasting model it aims to simulate. Weather based forecasting models introduce certain specificities which can not be reproduced with simple models, and neglecting those causes major simplification which can result in an unrealistic results.

For a further validation, a second case study is used, considering yearly wind power forecast error distribution in

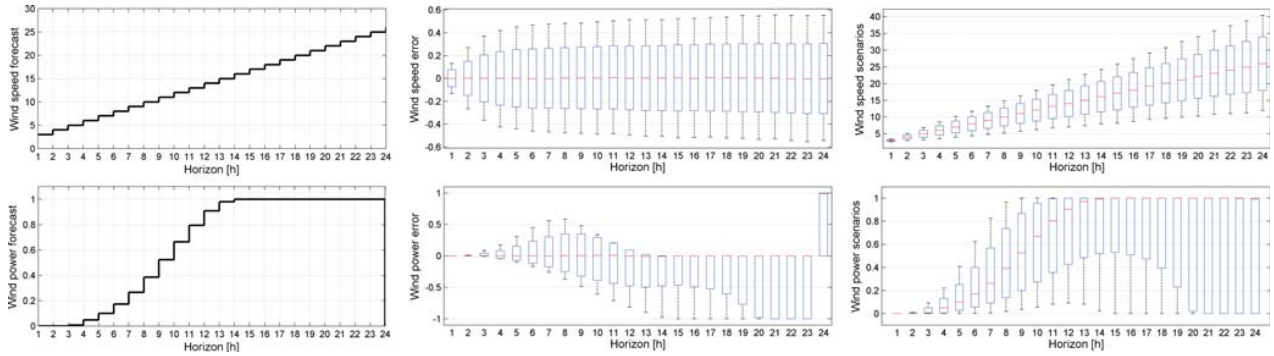

Fig. 6 Percentile depiction of wind power forecast scenarios

a system consisting of four WPPs, each having installed power of 100 MW and the same equivalent power / wind curve. Characteristic power / wind curve used in this example is a typical conversion curve in wind power plants with a cut in wind speed at 3 m/s, nominal power output starting at 14 m/s and with a cut out wind speed at 24 m/s. It should be pointed out that no modification of wind turbine power / wind characteristic is required when representing whole plant except for the necessary scaling on this temporal level. Fig. 7 graphically depicts physical disposition of these four WPPs.

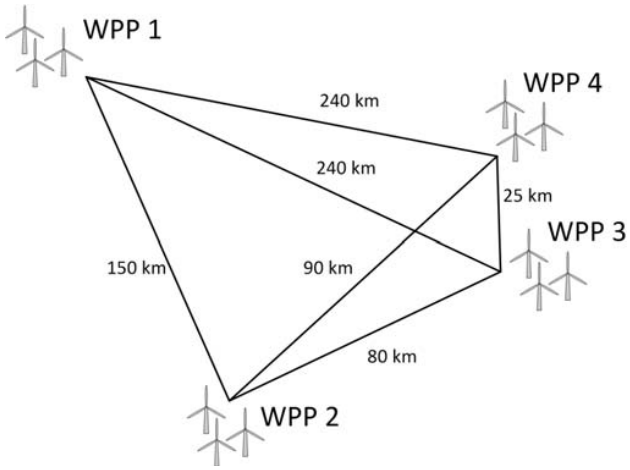

Fig. 7 Case study used for comparison of wind forecast error simulation models

Year long measurements are available for these locations serving as an input for Model 1. In case of Model 2, a Weibull fit for each of these location's wind speed data is calculated and used as an input. Fig. 8 depicts forecast error distribution for both of these models. First, it is necessary to point out the resemblance in yearly error distribution with results in [9], contributing to general validity of the proposed models. In further matter, a significant resemblance can be noticed in results for both models, allowing for validation of Model 2. However, it is necessary to point out that accuracy of Model 2 is highly dependent on accuracy of the Weibull fit for each location's data. Accounting for this precondition, Model 2 can be used as a simple, but accurate, model for wind power forecast error simulation.

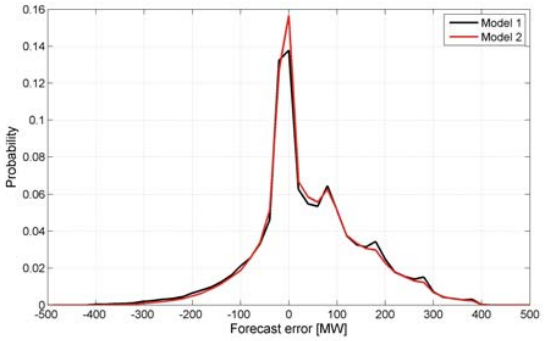

Fig. 8 Wind power forecast error distribution for models 1 and 2

V. Conclusions

In this paper, a novel approach for wind power forecast error simulation is proposed. Model is based on stochastic process simulation which is used for wind speed error simulation. Together with known disposition of wind power plants and their equivalent power / wind curves, together with measured or simulated wind speed data on these locations it is possible to simulate wind power forecast error data. In this sense, two distinct models are proposed in this paper depending on data availability. First model uses wind speed measurements, while the second model uses Weibull parameters of wind speed data.

Both models can be used for wind power forecasting performance simulation. The basic advantages in proposed models can be summed up in following. Proposed models account for the forecast error correlation and therefore reproduce important effect, the forecast error spatial smoothing. Model 1 accounts for the autocorrelation in wind power forecast error. Although often neglected, this parameter can play important role in certain analyses considering longer temporal instances. Finally, proposed models simulated performance of weather based forecasting techniques, enabling reproduction of non-linear transformation of wind to power

forecast error. Accounting for all of these advantages, proposed models can find application in many different studies involving wind power uncertainty.

APPENDIX A
GENERATING MULTIVARIATE NORMAL RANDOM VECTORS

Generating multivariate normal random vectors with desired mean vector and covariance matrix is based on several theorems in statistics. The most important of these is the *Mean and Variance of Linear Transformation* theorem [12]. Namely, if X has mean $E[X]$ and covariance Σ_X, then

$$Y = AX + b$$

has mean

$$E[Y] = b + AE[X]$$

and covariance matrix

$$\Sigma_Y = A\Sigma_X A^T.$$

Based on these considerations it is obvious that by careful choice of parameters A and b it is possible to transform process with arbitrary mean and covariance matrix to a process having desired parameters.

Firstly, it is necessary to choose mean and covariance matrix for X. For convenience, let $E[x] = 0$ and $\Sigma_X = I$. Such process is easy to simulate and later linear transformations are simplified. Now, it is necessary to chose b and A in order for resulting process Y to have desired mean $E[Y] = \mu$ and covariance matrix $\Sigma_Y = \Sigma_X$. Obviously,

$$E[Y] = b + AE[X] = b,$$

therefore choosing $b = \mu$ results in desired mean of Y. On the other hand, choice for A is less straightforward. In the case of previously introduced process X, A should satisfy $\Sigma_Y = AIA^T$. It is out of the scope of this work to derive a complete mathematical background of this problem and a final conclusions will be provided only. Namely, if the matrix A is chosen as

$$A = \Lambda^{\frac{1}{2}}\Phi,$$

where Φ is a matrix containing normalized eigenvectors of desired Σ_X as columns and Λ is diagonal matrix with eigenvalues of Σ_X as diagonal elements, then the transformed process Y has covariance matrix $\Sigma_Y = \Sigma_X$ [13].

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