

Weakly generalized closed map

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Abstract: In this paper we introduce a new class of mg-continuous mapping and studied some of its basic properties. We obtain some characterizations of such functions. Moreover we define sub minimal structure and further study certain properties of mg-closed sets.

Keywords: m-structure, mg-continuous mapping, minimal structure, mg T_2 space, sub minimal structure, $T_{\frac{1}{2}}$ space, mg-compact set

1. Introduction

Levine [9] introduced the concept of g -closed sets and studied their properties. A subset A of a space X is g -closed if and only if $cl(A) \subset O$ whenever $A \subset O$ and O is open. Hence every closed set is a g -closed set. The union and intersection of two g -closed set is g -closed set. Regular open sets and stronger regular open sets have been introduced and investigated by Stone[19] and Tang[21] respectively. Complements of regular open sets and strong regular open sets are called regular closed sets and strong regular closed sets. Andrijevic [1], Arya and Nour[2], Bhattacharya and Lahiri[5], Levine[9],[10], Mashour et al[13] and Njastad[17] introduced and investigated semi-preopen sets, generalized semi open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets, generalized open set, semi-open sets pre-open sets and α -open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets respectively. Ganster and Reilly [8] have introduced locally closed sets which are weaker than both open and closed sets. Cameron[6] has introduced regular semi-open sets which are weaker than regular open sets.

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2. Preliminaries

In this section we begin by recalling some definitions and properties.

Let (X, τ) be a topological spaces and A be a subset. The closure of A and interior of A are denoted by $cl(A)$ and $int(A)$ respectively. We recall some generalized open sets.

Definition [9] 2.1: A subset A of a space X is g -closed if and only if $cl(A) \subset G$ whenever $A \subset G$ and G is open.

Definition [20]2.2: A map $f : X \rightarrow Y$ is called g -closed if each closed set F of X , $f(F)$ is g -closed in Y .

Definition[18]2.3: A map $f : X \rightarrow Y$ is called semi-closed if each closed set F of X , $f(F)$ is semiclosed in Y .

Definition [15] 2.4 : A map $f : X \rightarrow Y$ is called α -open if each open set F of X , $f(F)$ is α -set in Y .

Definition [7]2.5 : A map $f : X \rightarrow Y$ is called pre-closed if for each closed map F of X , $f(F)$ is pre-closed in Y .

Definition [12]2.6: A map $f : X \rightarrow Y$ is called regular-closed if for each set F of X , $f(F)$ is regular closed in Y .

Definition (11)2.7: A map $f : X \rightarrow Y$ is said to be strongly continuous if $f^{-1}(V)$ is both open and closed in X for each subset V of Y .

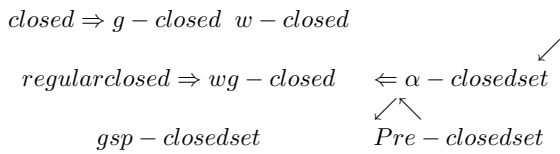
Definition [4] 2.8: A map $f : X \rightarrow Y$ is said to be generalized continuous if $f^{-1}(V)$ is g -open in X for each set V of Y

Definition [15] 2.9 A subset A of a topological space X is said to be weakly generalized closed (wg-closed) set in X if G contains $cl(int(A))$ whenever G

contains A and G is open in X .

Definition[9] 2.10A topological space X is said to be $T1/2$ -space if every g -closed set is closed.

Remark:2.11: The following diagram are well known.



3.Properties of Weakly generalized closed

In this section we studied some of wg -closed sets properties.

Definition 3.1: A map $f : X \rightarrow Y$ is called wg -closed map if for each closed set F of X , $f(F)$ is wg -closed set.

Remark 3.2: Every g -closed map is a wg -closed map and the converse is need not be true from the following example.

Example3.3:Let $X = \{a, b, c\}$ and $\tau_1 = \{\phi, x, \{a\}, \{b\}, \{a, b\}\}, \tau_2 = \{\phi, X, \{a\}, \{a, b\}\}$ be topologies on X . Let $\{a, c\}$ is T_1 -closed but not T_2 -closed.

Theorem 3.4: A map $f : X \rightarrow Y$ is wg -closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a wg -open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$

Proof: Suppose f is wg -closed. Let S be a subset of Y and U is an open set of X such that $f^{-1}(S) \subset U$. Then $V = Y - f^{-1}(X - U)$ is a wg -open set containing S such that $f^{-1}(V) \subset U$.

For the converse suppose that F is a closed set of X . Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F$ is open. By hypothesis there is wg -open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore $F \subset X - f^{-1}(V)$. Hence $Y - V \subset f(F) \subset f(X - f^{-1}(V)) \subset Y - V$ which implies $f(F) = Y - V$. Since $Y - V$ is wg -closed if $f(F)$ is wg -closed and thus f is a wg -closed map.

Theorem 3.5:If $f : X \rightarrow Y$ is continuous and wg -closed and A is a wg -closed set of X then $f(A)$ is

wg -closed.

proof:Let $f(A) \subset O$ where O is an open set of Y . Since f is g -continuous, $f^{-1}(O)$ is an open set containing A . Hence $cl(A) \subset f^{-1}(O)$ is A is wg -closed set. since f is wg -closed, $f(cl(A))$ is a wg -closed set contained in the open set O which implies that $cl(f(Cl(A))) \subset O$ and hence $clf(cl(A)) \subset O$ and hence $cl(f(A)) \subset O$ so f is a wg -closed set.

corollary 3.6: If $f : X \rightarrow Y$ is g -continuous and closed and A is g -closed set of X the $f(A)$ is wg -closed.

Corollary 3.7: If $f : X \rightarrow Y$ is wg -closed and continuous and A is wg -closed set of X then $f_A : A \rightarrow Y$ is continuous and wg -closed set.

Proof Let F be a closed set of A then F is wg -closed set of X . From above theorem 3.5 follows that $f_A(F) = f(F)$ is wg -closed set of Y . Here f_A is wg -closed and continuous.

Theorem 3.8 If a map $f : X \rightarrow Y$ is closed and a map $g : Y \rightarrow Z$ is wg -closed then $f : X \rightarrow Z$ is wg -closed.

Proof Let H be a closed set in X . Then $f(H)$ is closed and $(g \circ f)(H) = g(f(H))$ is wg -closed as g is wg -closed. Thus $g \circ f$ is wg -closed.

Theorem 3.9:If $f : X \rightarrow Y$ is continuous and wg -closed and A is a wg -closed set of X then $f_A : A \rightarrow Y$ is continuous and wg -closed.

Proof:If F is a closed set of A then F is a wg -closed set of X . From Theorem 3.4, It follows that $f_A(F) = f(F)$ is a wg -closed set of Y . Hence f_A is wg -closed. Also f_A is continuous.

Theorem 3.10:If $f : X \rightarrow Y$ is wg -closed and $A = f^{-1}(B)$ for some closed set B of Y then $f_A : A \rightarrow Y$ is wg -closed .

Proof: Let F be a closed set in A . Then there is a closed set H in X such that $F = A \cap H$. Then $f_A(F) = f(A \cap H) = f(H) \cap f(B)$. Since f is wg -closed $f(H)$ is wg -closed in Y . so $f(H) \cap B$ is wg -closed in Y . Since the intersection of a wg -closed and a closed set is a wg -closed set. Hence f_A is wg -closed.

Remark 3.11: If B is not closed in Y then the above theorem is not hold from the following example.

Example 3.12: Take $B = \{a, b\}$. Then $A = f^{-1}(B) = \{a, b\}$ and $\{a\}$ is closed in A but $f_A(\{a\}) = \{a\}$ is not wg-closed in Y . $\{a\}$ is also not wg-closed in B .

4. Normal and Regularity

In this section we introduce the new class of wg-regular and studied some of its properties.

Theorem 4.1: If $f : X \rightarrow Y$ is continuous, wg-closed map from a normal space X onto a space Y then Y is normal.

Proof: Let A, B be disjoint closed sets in Y . Then $f^{-1}(A), f^{-1}(B)$ are disjoint closed sets of X . Since X is normal there are disjoint open sets U, V in X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is wg-closed by theorem 3.4, there are wg-open sets G, H in Y such that $A \subset G, B \subset H$ and $f^{-1}(G) \subset U$ and $f^{-1}(H) \subset V$. Since U, V are disjoint $intG, intH$ are disjoint open sets. Since G is wg-open, A is closed and $A \subset G, A \subset intG$. Similarly $B \subset intH$. Hence Y is normal.

Theorem 4.2: If $f : X \rightarrow Y$ is an open continuous wg-closed surjection, where X is regular then Y is regular.

Proof: Let U be an open set containing a point P in Y . Let X be a point of X such that $f(X) = P$. Since X is regular and f is continuous there is an open set V such that $x \in V \subset cl(V) \subset f^{-1}(U)$. Hence $P \in f(V) \subset f(Cl(V)) \subset U$. Since f is wg-closed $f(Cl(V))$ is wg-closed set contained in the open set U . It follows that $cl(f(Cl(V))) \subset U$ and hence $P \in f(V) \subset cl(f(V)) \subset U$ and $f(V)$ is open. Since f is open. Hence Y is regular.

Remark 4.3: The normality is preserved under regular closed, continuous and surjective.

Example 4.4: In the example 3.12. It is shown that f is wg-closed $\{a, b\}$ is a regular closed set in (X, τ_1) and it is not closed in (X, τ_2) . Hence f is not regular closed.

Example 4.5 Let T_1 be the countable complement topology on the real line R and T_2 be the usual topology on R and $f : (R, T_1) \rightarrow (R, T_2)$ be the identity map. Then f is regular closed by the remark immediately after the above example. But f is not wg-closed. For if $A = \{1/n, n \in N\}$ then A is closed in (R, T_1) and $f(A) = A$ is not wg-closed as $f(A) \subset (0, 2)$ and $(0, 2)$ is open in (R, T_2) . But $clf(A) \subset (0, 2)$.

Theorem 4.6: If A is wg-closed set of a space X then $IndA \leq IndX$

Proof: It suffices to show that if $IndX \leq n$ and A is wg-closed set of X then $IndA \leq n$. We prove this theorem by induction. The result holds trivially for $n=1$. Assume that for every wg-closed set A of X $IndX \leq n-1 \Rightarrow IndA \leq n-1$.

Let X be space with $Ind \leq n$. Let A be a wg-closed set of X . Let E be a closed set of A and G be an open set of A such that $E \subset G$. Then there exist a closed set F of X and an open set H of X such that $E = A \cap F$ and $G = A \cap H$. Since E is closed in A and A is wg-closed. Since $IndX \leq n$, there is an open set V of X such that $clE \subset V \subset H$ and $Indbd(V) \leq n-1$. Then $V \cap A$ is an open set of A such that $E \subset V \cap A \subset G$ and $bd_A(V \cap A) \subset bd(V)$. Now $bd_A(V \cap A)$ is a wg-closed set of $bd(V)$. By induction hypothesis and $Indbd_A(V \cap A) \leq n-1$. Hence $IndA \leq n$.

Theorem 4.7: If A is a wg-closed set of a space X then $dime A \leq dimX$.

Proof If $dimX = 0$ then $dimA \leq 0 = dimX$. Hence $dimA \leq dimX$.

If $dimX \leq 0$ then $dimX = n$, where n is an integer greater than or equal to -1 . If $n = -1$ which implies that $X = \phi$ and hence $A = \phi$ and $dimA = -1 = dimX$ and thus $dimA \leq dimX$.

Next suppose $dimX = n$ where $n \geq -1$ and let A be a wg-closed set of X . Let $\{u_1, u_2, u_3, \dots, u_k\}$ be a finite open cover of A . Then for $i = 1, 2, 3, \dots, k$ there exist open sets V_i of X such that $u_i = A \cap V_i$. Since A is wg-closed and $\bigcup_{i=1}^k V_i$ is an open set containing A , $clA \subset \bigcup_{i=1}^k V_i$. Since $cl(A)$ is a closed set, $dimcl(A) \leq n$ so the finite open cover $\{clA \cap V_i, i = 1, 2, 3, \dots, k\}$ $cl(A)$ has a refinement $cl(A) \cap W_i, i = 1, 2, 3, \dots, k$ or order at most $n+1$, where each W_i is open in X and $clA \cap W_i \subset clA \cap V_i$

for each i . Then $\{A \cap w_i\} : i = 1, 2, \dots\}$ is an open cover of A refining $\{u_i, i = 1, 2, 3, \dots, k\}$ and of order not exceeding $n + 1$. Hence $\dim A \leq n$ which implies that $\dim A \leq \dim X$.

Theorem 4.8: If A is a wg-closed set of a space X then $\dim A \leq \dim X$.

Proof Let X be a space such that $\dim X = n$ and A be a wg-closed set of X . By using the notations of the above theorem, $clA \subset \bigcup V_i$. Since clA is a closed set, $\dim A \leq n$. Hence for every open cover $V_i \cap clA, i = 1, 2, 3, \dots, k$ there is a disjoint family $W_j, j = 1, 2, 3, \dots, k$ of open sets clA refining $V_i \cap clA, i = 1, 2, 3, \dots, k$ and such that $\dim(clA - \bigcup_{j=1}^k W_j) \leq n - 1$. But $A - \bigcup_{j=1}^k W_j \subset clA - \bigcup_{j=1}^k W_j$ and $A - \bigcup_{j=1}^k W_j = A \cap (clA - \bigcup_{j=1}^k W_j)$ is a wg-closed set of clA as the intersection of wg-closed set and closed set is a wg-closed set. By induction hypothesis $\dim(A - \bigcup_{j=1}^k W_j) \leq n - 1$. Also $W_j \cap A, j = 1, 2, 3, \dots, k$ is a disjoint family of open sets of A refining u_1, u_2, \dots, u_k . Thus $\dim A \leq n$ and the theorem is proved.

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