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Wavelet Compression of ECG Signals Using SPIHT Algorithm

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***Abstract*—** In this paper we present a novel approach for wavelet compression of electrocardiogram (ECG) signals based on the set partitioning in hierarchical trees (SPIHT) coding algorithm. SPIHT algorithm has achieved prominent success in image compression. Here we use a modified version of SPIHT for one dimensional signals. We applied wavelet transform with SPIHT coding algorithm on different records of MIT-BIH database. The results show the high efficiency of this method in ECG compression.

***Keywords*—** ECG compression, wavelet, SPIHT.

I. Introduction

ELECTROCARDIOGRAM (ECG) signal is a very useful source of information for physicians in diagnosing heart abnormalities. With the increasing use of ECG in heart diagnosis, such as 24 hour monitoring or in ambulatory monitoring systems, the volume of ECG data that should be stored or transmitted, has greatly increased. For example, a 3 channel, 24 hour ambulatory ECG, typically has storage requirement of over 50 MB. Therefore we need to reduce the data volume to decrease storage cost or make ECG signal suitable and ready for transmission through common communication channels such as phone line or mobile channel. So, we need an effective data compression method.

The main goal of any compression technique is to achieve maximum data reduction while preserving the significant signal morphology features upon reconstruction. Data compression methods have been mainly divided into two major categories: 1) direct methods, in which actual signal samples are analyzed (time domain), 2) transformational methods, in which first apply a transform to the signal and do spectral and energy distribution analysis of signals.

Examples of direct methods are: differential pulse code modulation (DPCM), amplitude zone time epoch coding (AZTEC), turning point, coordinate reduction time encoding system (CORTES), Fan algorithm, ASEC. [1] is a good review of some direct compression methods used in ECG compression.

Some of the transformations used in transformational compression methods are Fourier transform, discrete cosine transform (DCT), Walsh transform, Karhunen-Loeve transform (KLT), and wavelet transform.

The main idea in using transformation is to compact the energy of signal in much less samples than in time domain, so we can discard small transform coefficients (set them to zero). Wavelet transform has a good localization property in time and frequency domain and is exactly in the direction of transform compression idea. Here, we use wavelet transform with SPIHT coding algorithm, modified for 1-D signals, for coding the wavelet coefficients.

II. Wavelet Transform

A. Introduction

In wavelet transform, we use wavelets as transform basis. Wavelet functions are functions generated from one single function ψ by scaling and translation:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \qquad (1)$$

The mother wavelet $\psi(t)$ has to be zero integral, $\int \psi(t)dt = 0$. From (1) we see that high frequency wavelets correspond to $a < 1$ or narrow width, while low frequency wavelets correspond to $a > 1$ or wider width.

The basic idea of wavelet transform is to represent any function f as a linear superposition of wavelets. Any such superposition decomposes f to different scale levels, where each level can be then further decomposed with a resolution adapted to that level. One general way to do this is writing f as the sum of wavelets $\psi_{m,n}(t)$ over m and n. This leads to discrete wavelet transform:

$$f(t) = \sum c_{m,n}\psi_{m,n}(t) \qquad (2)$$

By introducing the multi-resolution analysis (MRA) idea by Mallat [3], in discrete wavelet transform we really use two functions: wavelet function $\psi(t)$ and scaling function $\varphi(t)$. If we have a scaling function $\varphi(t) \in L^2(\mathbb{R})$, then the sequence of subspaces spanned by its scalings and translations $\varphi_{j,k}(t) = 2^{j/2}\varphi\left(2^j t - k\right)$, i.e.:

$$V_j = span\left\{\varphi_{j,k}(t),\ j,k \in \mathbb{Z}\right\} \qquad (3)$$

constitute a MRA for $L^2(\mathbb{R})$.

$\varphi(t)$ must satisfy the MRA condition:

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$$\varphi(t) = \sqrt{2}\sum_{n} h(n)\varphi(2t - n) \qquad (4)$$

for $n \in \mathbb{Z}$. In this manner, we can span the difference between spaces V_j by wavelet functions produced from mother wavelet: $\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k)$. Then we have:

$$\psi(t) = \sqrt{2}\sum_{n} g(n)\varphi(2t - n) \qquad (5)$$

For orthogonal basis we have:

$$g(n) = (-1)^n h(-n + 1) \qquad (6)$$

If we want to find the projection of a function $f(t) \in L^2(\mathbb{R})$ on this set of subspaces, we must express it in each subspace as a linear combination of expansion functions of that subspace [4]:

$$f(t) = \sum_{k=-\infty}^{\infty} c(k)\varphi_k(t) + \sum_{j=0}^{\infty}\sum_{k=-\infty}^{\infty} d(j,k)\psi_{j,k}(t) \qquad (7)$$

where $\varphi_k(t)$ corresponds to the space V_0 and $\psi_{j,k}(t)$ corresponds to wavelet spaces.

By using the idea of MRA, implementation of wavelet decomposition can be performed using filter bank constructed by a pyramidal structure of lowpass filters $h(n)$ and highpass filters $g(n)$ [3, 4].

B. Biorthogonal Wavelet Basis

Many signals we use are mostly smooth (except for sharp slopes). For example images have regions of low gray level difference, 1-D signals have smooth parts between some peaks. So, it seems appropriate that an exact reconstruction subband coding scheme for signal compression should correspond to an orthonormal basis with reasonably smooth mother wavelet. For having fast computation, the length of filter must be short, but short filter leads to less smoothness and we must do a tradeoff between them. On the other hand, it is desired that FIR filter to be linear phase, since such filters can easily be cascaded in pyramidal filter bank structure without need for phase compensation. As there are no nontrivial orthonormal linear phase FIR filter with exact reconstruction property, we can relax the orthonormal property by using biorthogonal filters.

If the basis of a wavelet expansion is not orthogonal, we can find another set of basis functions that is a dual for the first function set and satisfies the orthogonality relation:

$$\langle \varphi_k(t), \tilde{\varphi}_l(t) \rangle = \int \varphi_k(t)\tilde{\varphi}_l(t)\,dt = \delta(k - l) \qquad (8)$$

We have similar dual functions for wavelet functions ($\tilde{\psi}(t)$). In reconstruction using filter bank algorithm, we must use dual filters. In order to have exact reconstruction, we impose:

$$\tilde{g}(n) = (-1)^n h(-n + 1) \text{ and } g(n) = (-1)^n \tilde{h}(-n + 1) \qquad (9)$$

In [2] some biorthogonal wavelet bases are derived. Here we use the spline filters with symmetric filters $h(n)$ with length 9 and $g(n)$ with length 7. This wavelet basis is commonly said "biorthogonal 9/7 tap filters". It has been shown that this wavelet has the best performance for wavelet ECG compression [8]. The filter coefficients are given in Table I.

Table I. Coefficients of the Biorthogonal 9/7 Tap Filters

n	0	±1	±2	±3	±4
$h(n)$	0.852699	0.377403	-0.11062	-0.023849	0.037829
$g(n)$	0.788485	0.418092	-0.04069	-0.064539	

III. SPIHT Coding Algorithm

A. Overview of SPIHT Algorithm

SPIHT is an embedded coding technique. In an embedded coding algorithm, all encodings of the same signal at lower bit rates are embedded at the beginning of the bit stream for the target bit rate. Effectively, bits are ordered in importance. This type of coding is especially useful for progressive transmission and transmission over a noisy channel. Using an embedded code, an encoder can terminate the encoding process at any point, thereby allowing a target rate or distortion parameter to be met exactly. Typically, some target parameters, such as bit count, is monitored in the encoding process and when the target is met, the encoding simply stops. Similarly, given a bit stream, the decoder can cease decoding at any point and can produce reconstruction corresponding to all lower-rate encodings.

Embedded coding is similar in spirit to binary finite precision representations of real numbers. All real numbers can be represented by a string of binary digits. For each digit added to the right, more precision is added. Yet, encoding can cease at any time and provide the best representation of the real number achievable within the framework of the binary digit representation. Similarly, the embedded coder can cease at any time and provide the best representation of the signal achievable within its framework.

EZW, introduced by J. M. Shapiro [5] is an embedded coding algorithm for image compression. It works on discrete wavelet transform coefficients of an image. It is very effective and computationally simple technique for image compression. SPIHT algorithm introduced for image compression in [6] is a refinement to EZW and uses its principles of operation. These principles are partial ordering of transform coefficients by magnitude with a set partitioning sorting algorithm, ordered bit plane transmission and exploitation of self-similarity across different scales of an image wavelet transform. The partial ordering is done by comparing the transform coefficients magnitudes with a set of octavely decreasing thresholds. In this algorithm, a transmission priority is assigned to each coefficient to be transmitted. Using these rules, the encoder always transmits the most significant bit to the decoder. SPIHT has even better performance than EZW in image compression. In [7], SPIHT algorithm is modified for 1-D signals and used for ECG compression.

B. *Proposed Compression Method*

For faster computations, firstly we divide ECG signal to contiguous non-overlapping frames of 1024 samples and we use each frame for encoding separately. We apply wavelet transform to the frames of ECG signal to 6 levels of decomposition. The wavelet used is biorthogonal 9/7 (Table I). We can assume that each wavelet coefficient is represented by a fixed-point binary format, so we treat it as an integer, because SPIHT algorithm works on integer values. Therefore we apply SPIHT algorithm to these integers (produced from wavelet coefficients) for encoding them. The termination of encoding algorithm is specified by a threshold value determined in the program; changing this threshold, gives different CRs. The output of the algorithm is a bit stream (0 and 1). This bit stream is used for reconstructing signal after compression. From it and by going through inverse of SPIHT algorithm, we compute a vector of 1024 wavelet coefficients and using inverse wavelet transform, we make the reconstructed 1024 sample frame of ECG signal.

IV. Results And Discussion

A. *Simulation Results*

The ECG signals used in the simulation are from MIT-BIH arrhythmia database. This database includes different shapes of ECG signals. The records used are 100, 101, 102, 103, 104, 105, 106, 107, 118, 119, 200, 201, 202, 203, 205, 207, 208, 209, 210, 212, 213, 214, 215, 217 and 219 (25 records). The distortion between original signal and reconstructed signal is measured by percent root mean square difference (PRD):

$$\text{PRD} = \sqrt{\frac{\sum_{i=1}^{N}[x(i)-\hat{x}(i)]^2}{\sum_{i=1}^{N}[x(i)]^2}} \times 100\% \qquad (10)$$

where $x(i)$ is the ith sample of the original signal, $\hat{x}(i)$ is the ith sample of reconstructed signal and N is the number of samples of signal. Although PRD does not account for differences between morphology of two signals and may not report shape distortions, is used widely in signal compression literature as a standard measurement, because it's easy to compute and compare. Compression ratio (CR) is computed from the ratio of original file size (in bits) to the length of output bit stream. The CR-PRD diagram for 25 different records is plotted in Fig. 1. In Fig. 2, the average PRD vs. average CR for all tested records is plotted. The standard deviation of PRDs is also shown on the Fig. 2. The original signal, reconstructed signal and error between them for three records with the corresponding CR and PRD values are shown in Fig. 3, 4 and 5. In Fig. 6, the result of compressing record 117 with three different CRs is shown.

B. *Discussion*

Fig. 1 shows that PRD slightly increases by increasing CR. It also reveals that results for all tested records are in an acceptable range. This means the usefulness of the compression method for different ECG records. Fig. 2 depicts the close results for all records and efficiency of the method for all shapes of ECG, though by increasing the CR, the PRDs change in a wider range. From plots of reconstructed signals, we see that for a CR around 20, almost all of important details and features of the shape of the signal are preserved. Other simulations showed that by increasing the CR over 20, the shape of the reconstructed signal begins to distort unacceptably. For example, comparison for record 117 in Fig. 6, shows that for CR about 40, the reconstructed signal is composed of some flat and some sharp parts, and many details of signal, clearly is lost.

V. Summary And Conclusion

In this paper we applied wavelet transform to the ECG signal and encoded the wavelet coefficients with SPIHT

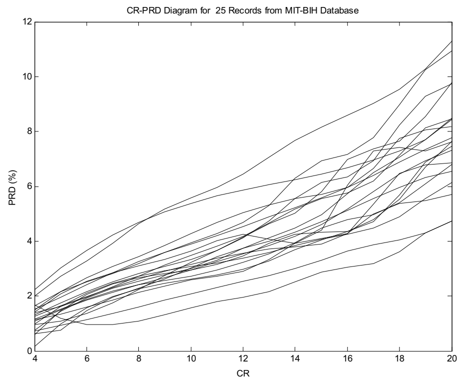

Fig. 1. CR-PRD results for 25 records from MIT-BIH database.

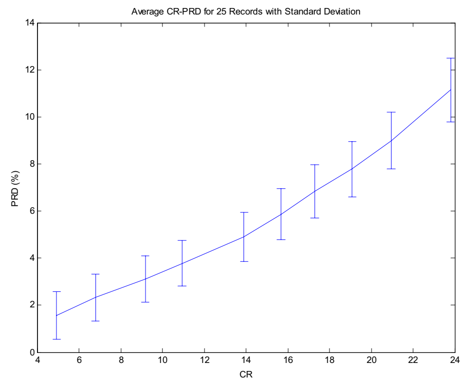

Fig. 2. Average result for tested records. Standard deviation of PRD values is also plotted.

Table II. Compression performance for record 117 from MIT-BIH database with different compression methods

Compression method	CR	PRD (%)
AZTEC [1]	6.8	10.0
Djohan [9]	8	3.9
Hilton [8]	8	2.6
LPC [10]	11.6	5.3
Proposed method	21.4	3.1

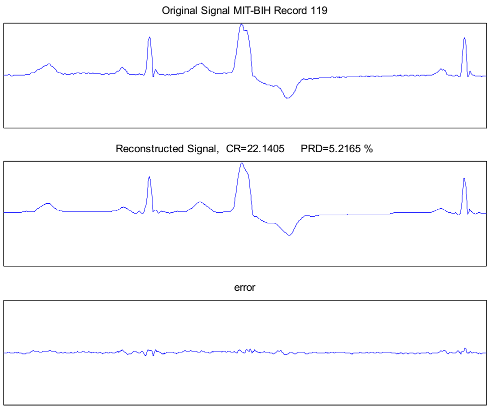

Fig. 3. ECG compression using bi 9/7 wavelet and SPIHT algorithm for record 119 from MIT-BIH database. Top figure is original signal, the middle is reconstructed signal and bottom signal is error. CR=22.1, PRD=5.2%

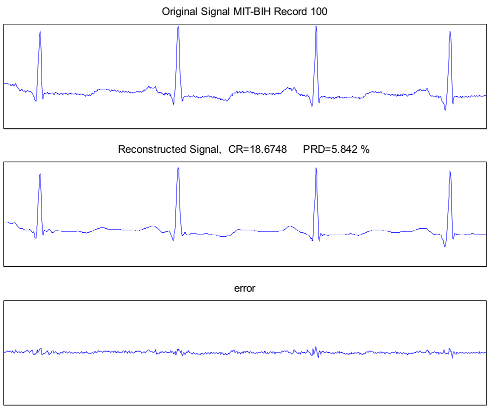

Fig. 4. ECG compression using bi 9/7 wavelet and SPIHT algorithm for record 100 from MIT-BIH database. Top figure is original signal, the middle is reconstructed signal and bottom signal is error. CR=18.6, PRD=5.8%.

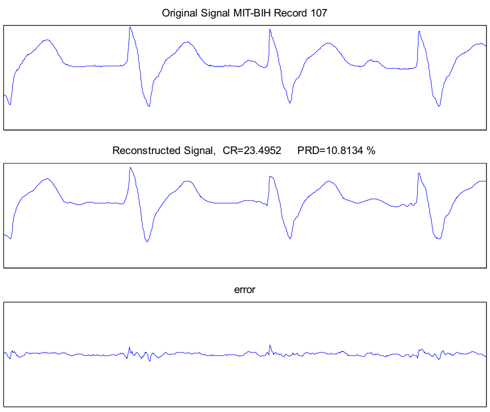

Fig. 5. ECG compression using bi 9/7 wavelet and SPIHT algorithm for record 107 from MIT-BIH database. Top figure is original signal, the middle is reconstructed signal and bottom signal is error. CR=23.5, PRD=10.8%.

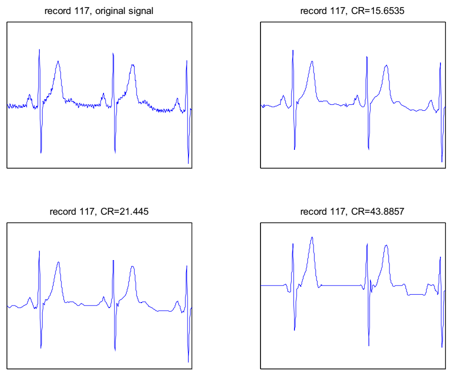

Fig. 6. Results of reconstructing 1024 samples of record 117 with three different CRs.

algorithm. The results show the high efficiency of this method for ECG compression. By this method, we achieved the CR about 20 with a very good reconstruction quality. In Table II is given the CR and PRD values for some other compression methods. It shows that SPIHT method has very better results. SPIHT is a very computationally simple algorithm and is easy to implement, in comparison with many complex coding methods. It's also an embedded coding algorithm that makes it useful for transmission purposes. Although we can achieve higher CRs by utilizing some lossless arithmetic coding (such as run-length coding which increases CR by about 5%), but we lose the important feature of embedded coding; applying lossless coding can only helps in storage applications.

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