International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:10, No:7, 2016

Vibration Control of a Functionally Graded Carbon Nanotube-Reinforced Composites Beam Resting on Elastic Foundation

Gholamhosein Khosravi, Mohammad Azadi, Hamidreza Ghezavati

***Abstract*—**In this paper, vibration of a nonlinear composite beam is analyzed and then an active controller is used to control the vibrations of the system. The beam is resting on a Winkler-Pasternak elastic foundation. The composite beam is reinforced by single walled carbon nanotubes. Using the rule of mixture, the material properties of functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) are determined. The beam is cantilever and the free end of the beam is under follower force. Piezoelectric layers are attached to the both sides of the beam to control vibrations as sensors and actuators. The governing equations of the FG-CNTRC beam are derived based on Euler-Bernoulli beam theory Lagrange-Rayleigh-Ritz method. The simulation results are presented and the effects of some parameters on stability of the beam are analyzed.

***Keywords*—**Carbon nanotubes, vibration control, piezoelectric layers, elastic foundation.

I. Introduction

THE carbon nanotubes (CNTs) have been used to reinforce structures [1]. Unlike the carbon fiber-reinforced composites, carbon nanotube-reinforced composites (CNTRCs) can only contain a low percentage of CTNs (2–5% by weight) [2]. The concept of the functionally graded (FG) has been applied to CNTRCs by Shen [3]. Ke et al. studied free vibration of FG-CNTRC Timoshenko beam [4]. They found that linear frequencies of FG-CNTRC beams with symmetric CNT distribution are bigger than asymmetric ones. This research then was extended by Rafiee et al. to FG-CNTRC Euler-Bernoulli beams with piezo electric layers [5]. Yas and Heshmati presented free vibrations and buckling analysis of CNT-reinforced composite Timoshenko beams on elastic foundation [6]. Nowadays, because of destroying effect of vibration on structures, vibration control of structures is one of favorite subjects for researchers. Using finite element method, Mahieddine and Ouali formulated a beam with attached piezoelectric layers [7]. They used Kirchhoff first order theory for formulation and calculation of lateral strains. Li et al. [8] analyzed free vibration of functionally graded material beams with surface-bonded piezoelectric layers in thermal environment. Vibration analysis of nanotube-reinforced composite beams resting on elastic foundations was done by Shen and Xiang [9]. They studied the impact of thermal environment on the vibration characteristic of the beam. Shen and Xiang [10] extended their studies to work on the impact of thermal environment on cylindrical panels which are resting on elastic foundation. The forced vibration behavior of nanocomposite beams reinforced by single-walled carbon nanotubes (SWCNTs) based on the Timoshenko beam theory along with von- Kármán geometric nonlinearity has been studied by Ansari et al. [11]. Azadi et al. studied the active control of a FGM beam under follower force with piezoelectric sensors/actuators [12]. Yildirim et al. used an active piezoelectric vibration controller for a Timoshenko beam [13].

In this paper, the nonlinear forced vibration of FG-CNTRC beam resting on elastic foundation is investigated based on Euler–Bernoulli beam theory. And then, it has been controlled by an active controller. To do this, better piezoelectric layers as sensors/actuators are attached to the beam. The cantilever beam is under a follower force from its free end. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through the rule of mixture. Using Lagrange-Rayleigh-Ritz method and considering Winkler-Pasternak Elastic foundation, the governing equations of the system is derived. Finally, the effect of elastic foundation stiffness, follower force and the vibration controller on stability of the FG-CNTRC beam is investigated.

II. Governing Equations of System

As it is shown in Fig. 1, Winkler-Pasternak elastic foundation is considered. This beam is under follower force from its free end. Piezoelectric layers are attached to both sides of the beam and work as sensor/actuator.

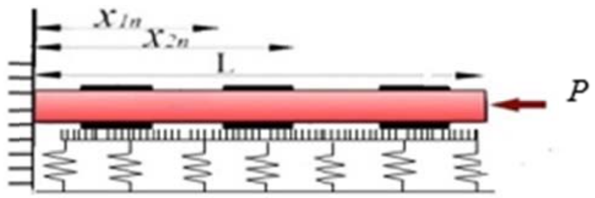

Fig. 1 FG-CNTRC beam resting on elastic foundation

It is assumed that the CNTRC is made from a mixture of SWCNT and an isotropic matrix. Effective Young module and shear module are calculated as:

M. A. is with the Department of Mechanical Engineering Marvdasht Branch, Islamic Azad University, Marvdasht, I.R. Iran (e-mail: mazadi@shirazu.ac.ir).
G. K. and H. G. are with the Department of Mechanical Engineering Arsanjan Branch, Islamic Azad University, Arsanjan, I.R. Iran.

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_m E^m$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_m}{E^m} \tag{1}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_m}{G^m}$$

where E_{11}^{cnt} , E_{22}^{cnt} and G_{12}^{cnt} are Young's modulus and shear modulus respectively, of CNT, E^m and G^m are the corresponding properties for the isotropic matrix; η_j (*j = 1, 2, 3)* is the CNT efficiency parameter accounting for the scale-dependent material properties. V_{cnt} and V_m are the volume fractions for CNT and matrix and are related by:

$$V_{cnt} + V_m = 1 \tag{2}$$

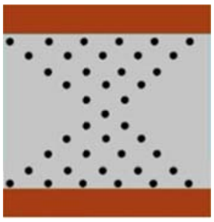

Fig. 2 Geometry of CNTs' distribution in the thickness direction of X-form beam

The volume fraction of CNT V_{cnt} varies linearly along the thickness. Relation for V_{cnt} and its related geometry (Fig. 2) are shown below:

$$X - Beam \quad : V_{cnt} = 4\frac{|z|}{h} V_{cnt}^{*} \tag{3}$$

$$V_{cnt}^{*} = \frac{\gamma_{cnt}}{\gamma_{cnt}} + \left(\rho^{cnt} / \rho^m\right) - \left(\rho^{cnt} / \rho^m\right)\gamma_{cnt} \tag{4}$$

where γ_{cnt} is the mass fraction of CNT, and ρ^{cnt} and ρ^m are the densities of CNTs and matrix, respectively. Similarly, Poisson's ratio ν and mass density ρ can be calculated by:

$$\begin{aligned} \nu &= V_{cnt}\nu^{cnt} + V_m \nu^m \\ \rho &= V_{cnt}\rho^{cnt} + V_m \rho^m \end{aligned} \tag{5}$$

where ν^{cnt} and ν^m are Poisson's ratios of CNT and matrix, respectively.

A nonlinear nanotube reinforced composite Euler-Bernoulli beam of thickness h, length L and width b is shown in Fig. 1. Piezoelectric layers are attached to top and down side of the beam and act as sensors/actuators where their thickness is h_p, length L_p, width b_p, density ρ_p, elasticity module E_p and piezoelectric fitness coefficient is e_{31}. Kinetic energy of the system is calculated as:

$$T = T_b + T_p \tag{6}$$

$$\begin{aligned} T_b &= \frac{1}{2}\int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} b\,\rho(z)\left(\dot{u}^2 + \dot{w}^2\right) dz\, dx \\ T_p &= \frac{1}{2}\sum_{n=1}^{Y} \int_0^L \iint_{A_n} \rho_{p_n} \dot{w}^2 \\ &\left[H\left(x - x_{1_n}\right) - H\left(x - x_{2_n}\right)\right] dA_n dx \end{aligned} \tag{7}$$

$u(x,t)$ is longitudinal displacement in x direction and $w(x,t)$ is transverse displacement in z direction. Also b, p and f indices represent beam, piezoelectric layers and foundation respectively. Y is number of piezoelectric layers, ρ_{p_n} equals density of n_{th} layer, x_{1_n} and x_{2_n} are coordinates of first and last point of piezoelectric layer, $(\cdot)$ represents derivative of parameters to time and $H(x)$ is Heaviside function. And so the potential energy of the system (O):

$$O = O_b + O_p \tag{8}$$

$$\begin{aligned} O_b &= \frac{b}{2}\int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}\left(-z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)^2 dx\, dz \\ &+ \frac{1}{2}\int_0^L P\frac{\partial w}{\partial x} dx \end{aligned} \tag{9}$$

$$\begin{aligned} O_p &= \frac{1}{2}\sum_{n=1}^{Y} \int_{PZT_n} E_{p_n} z^2 \left(\frac{\partial^2 w_{p_n}}{\partial x^2}\right)^2 dv \\ &+ \sum_{n=1}^{Y} \int_{PZT_n} z e_{31} E_{z_n} \frac{\partial^2 w_{p_n}}{\partial x^2} dv + \frac{1}{2}\sum_{n=1}^{Y} \int_{PZT_n} E_{z_n} d_n\, dv \end{aligned} \tag{10}$$

$$Q_{11}(z) = \frac{E_{11}^{cnt}(z)}{1 - \nu^2(z)} \tag{11}$$

where P is follower force and d_n represents electrical displacement of couple of piezoelectric layers and calculated as:

$$d_n = \varepsilon_{pn}\frac{v_n}{h_{pn}} \tag{12}$$

ε_{pn} is dielectric constant of piezoelectric materials that is dependent on n_{th} layer. According to Lagrange theory, governing equations of system is as:

$$\frac{d}{dt}\left(\frac{\partial \Gamma}{\partial \dot{\boldsymbol{q}}}\right)-\frac{\partial \Gamma}{\partial \boldsymbol{q}}=Q \tag{13}$$

where Γ is Lagrangian, q is vector in generalized coordinate and Q is the generalized force. Non-conservative work of system equals to:

$$W = P\frac{\partial w(L,t)}{\partial x}w(L,t) \tag{14}$$

To calculate the generalized force:

$$\delta W = Q^T \delta q$$

Based on Rayleigh-Ritz method to solve the governing equations of system:

$$w(x,t)=\Sigma \varnothing_i q_i = \boldsymbol{\Phi}^T \boldsymbol{q} \tag{15}$$

where ϕ_i , q_i , Φ and q are shape function, a time dependent function in generalized coordinate, mode shape and generalized coordinate respectively. By replacing (15) into (6)-(14):

$$\begin{gathered}[\mathbf{M}]\ddot{\boldsymbol{q}}+[\mathbf{K}]\boldsymbol{q}=\boldsymbol{F}\\ \boldsymbol{v}_s = K_{pelect}^{-1}K_{pelastelect_s}^{-1}\boldsymbol{q}\end{gathered} \tag{16}$$

$$\begin{gathered}F=F_f+F_p\\ F_p=-K_{pelastelect\,a}\boldsymbol{v}_a\\ F_f=-k_L\sum_{i=1}^{n}q_i\varphi_i+k_s\sum_{i=1}^{n}\frac{\partial^2\varphi_i}{\partial x^2}q_i\\ -k_{NL}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}q_iq_jq_k\varphi_i\varphi_j\varphi_k\end{gathered} \tag{17}$$

where F , $[\mathbf{M}]$, $[\mathbf{K}]$, $\boldsymbol{v}_s$ and v_a represent force, mass matrix, stiffness matrix and piezoelectric sensor/actuator voltages. F involves piezoelectric actuator force (F_p) and the force that elastic foundation enters on the beam (F_f). And also k_L , k_{NL} and k_s are linear stiffness of Winkler foundation, nonlinear stiffness of Winkler foundation and shear stiffness of Pasternak foundation. And matrix of mass equals to:

$$\mathbf{M}=\mathbf{M}_b+\mathbf{M}_p \tag{18}$$

$$\mathbf{M}_b=\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}b\,\rho(z)\begin{bmatrix}z^2\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial\varnothing_i(x)}{\partial x}\\ \frac{\partial\varnothing_j(x)}{\partial x}\\ +\sum_{i=1}^{N}\sum_{j=1}^{N}\varnothing_i(x)\\ \varnothing_j(x)\end{bmatrix}dzdx$$

$$\mathbf{M}_p=\sum_{n=1}^{Y}\int_{x_{1_n}}^{x_{2_n}}\rho_{pn}b_{pn}h_{pn}\sum_{i=1}^{N}\sum_{j=1}^{N}\varnothing_i(x)\varnothing_j(x)dx$$

And the stiffness matrix:

$$\mathrm{K}=\mathrm{K}_b+\mathrm{K}_p+\mathrm{K}_w \tag{19}$$

$$\begin{gathered}\mathbf{K}_b=\int_0^L P\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial\varnothing_i(x)}{\partial x}\frac{\partial\varnothing_j(x)}{\partial x}dx\\ +\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}bz^2Q_{11}(z)\left[\sum_{i=1}^{n}\sum_{k=1}^{n}z^2q_k\frac{\partial^2\varphi_i}{\partial x^2}\frac{\partial^2\varphi_k}{\partial x^2}\right.\\ \left.+\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial\varphi_i}{\partial x}\frac{\partial\varphi_j}{\partial x}\frac{\partial\varphi_k}{\partial x}\frac{\partial\varphi_l}{\partial x}\right]dzdx\right.\end{gathered} \tag{20}$$

$$\mathbf{K}_p=\sum_{n=1}^{Y}\iiint_{PZT_n}\begin{bmatrix}E_{pn}z^2\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial^2\varnothing_i(x)}{\partial x^2}\\ \frac{\partial^2\varnothing_j(x)}{\partial x^2}\end{bmatrix}d\mathrm{v} \tag{21}$$

$$\mathbf{K}_w=-P\sum_{i=1}^{N}\sum_{j=1}^{N}\varnothing_i(x)\frac{\partial\varnothing_j(x)}{\partial x}\bigg|_{x=L} \tag{22}$$

which $K_{P_{elatelect_a}}$ and $K_{P_{elatelect_s}}$ show piezoelectric actuator and sensor layers elastic-electric effect matrixes.

$$\begin{gathered}K_{peen}\Big|_{actuator}=\frac{e_{31n}b_{pn}}{h_{pn}}J_{an}\int_{x_{1_n}}^{x_{2_n}}\frac{\partial^2\boldsymbol{\Phi}}{\partial x^2}dx\\ K_{peen}\Big|_{sensor}=\frac{e_{31n}b_{pn}}{h_{pn}}J_{sn}\int_{x_{1_n}}^{x_{2_n}}\frac{\partial^2\boldsymbol{\Phi}}{\partial x^2}dx\end{gathered} \tag{23}$$

According to similarity of piezoelectric layers' properties, the amount of J_{a_n} and $J_{s_n}$is equal to J_{a_n} and J_{s_n} and it is considered for actuators and sensors respectively.

$$J_{an} = \int_{\frac{h}{2}}^{\frac{h}{2}+h_{pn}} z\,dz\,dx\,, J_{sn} = \int_{-\frac{h}{2}-h_{pn}}^{-\frac{h}{2}} z\,dz \tag{24}$$

$$K_{P_{elatelect_s}} = \frac{J_s}{J_a} K_{P_{elatelect_a}} \tag{25}$$

$K_{P_{elatelect_a}}$ $K_{P_{elatelect_s}}$ are matrix of piezoelectric patches diagonal capacity:

$$K_{pelect} = \sum_{n=1}^{Y} \int_{PZT_n} \varepsilon_{pn} p_n p_n^T \, dv \tag{26}$$

where $Y \times 1$ vector, p_n for all inputs unless n which is $1/h_{p_n}$ equals to zero:

$$K_{P_{elect}} = \varepsilon_p L_p b_p I_{Y \times Y} = \frac{1}{\mu} I_{Y \times Y} \tag{27}$$

where $I_{Y \times Y}$ is a $Y \times Y$ unit matrix, Lyapunov controller is employed to control vibrations of the beam and make it stable. This is amount of applied voltage to actuators:

$$\mathbf{v}_a = -K_d \dot{\mathbf{v}}_s - K_p \mathbf{v}_s \tag{28}$$

where K_d and K_p are certain positive matrixes. To prove stability of the controller, Lyapunov parameter

$$V_{Lyapunov} = \frac{1}{2}\dot{\mathbf{q}}^T \mathbf{M}\dot{\mathbf{q}} + \frac{1}{2}\mathbf{q}^T \mathbf{K}\mathbf{q} + \frac{1}{2} K_q \mathbf{q}^T K_{p_{elastelect_a}} K^T_{p_{elastelect_a}} \mathbf{q} \tag{29}$$

K_q is certain positive matrix. After deriving Lyapunov function and replacing M$\ddot{\text{q}}$ in it and by using (29) and $(\mu \frac{|J_s|}{|J_a|} K_P) = K_q$ the following relation would be achieved:

$$\dot{V}_{Lyapunov} = -\mu \frac{|J_s|}{|J_a|} K_d \dot{\mathbf{q}}^T K_{p_{elastelect_a}} K^T_{p_{elastelect_a}} \dot{\mathbf{q}} \tag{30}$$

$\mu \frac{|J_s|}{|J_a|} K_d$ is a certain positive matrix. So, it is proof that derivative of Lyapunov function has negative sign and stability of system to control vibration using of LaSalle constant collection theory is guaranteed.

III. Simulation and Discussion

It is difficult to find an equation that can consider all geometrical and natural conditions of system. So, some shape functions are defined instead of displacement parameter to estimate that parameter [12]. Poly Millet Methacrylate (PMMA) and armchair (10, 10) CNTs are used as matrix and reinforcement respectively. Material properties of this materials and piezoelectric layers are the same as the mentioned paper [12]. The parameters that are used to derive natural frequencies are:

$$\bar{P} = \frac{P}{P_{cr}}\,,\; I_{10} = \int_{-h/2}^{h/2} \rho_m dz\,,\; A_{110} = \int_{-h/2}^{h/2} E_m dz\,,$$
$$\omega = \Omega L \sqrt{I_{10}/A_{110}}$$

$P_{cr} = \pi^2 E_m I_m / L^2$ is critical buckling force for the composite beam without carbon- nanotubes. I_{10}, A_{110} *and* I_m represent Moment of inertia of that composite beam and ω is the natural frequency of homogenous beam. In Table I, dimensionless frequency of studied FG-CNTRC beam is compared with the presented results of Yas and Samadi [6].

TABLE I
VALIDATION OF FIRST DIMENSIONLESS FREQUENCY

$V^*_{cnt} = 12\%$	FG-X	
	present	[6]
First Frequency (ω_1)	1.7177	1.6000

In the worse condition maximum of error, comparing with reference is 7%.

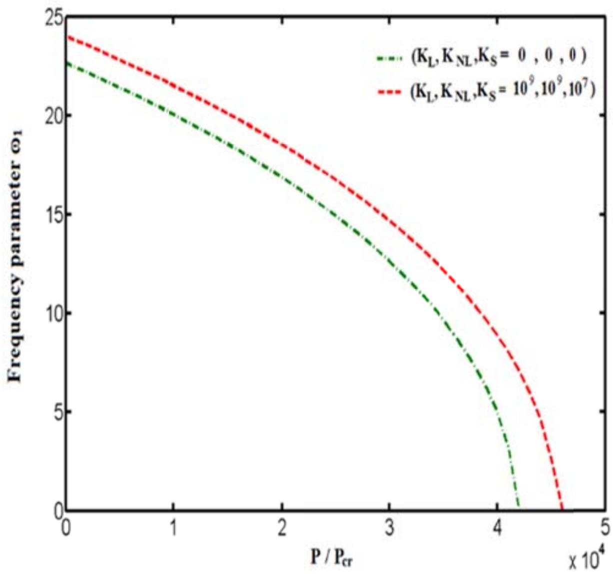

Fig. 3 Impact of follower force on dimensionless first frequency

Looking at Fig. 3, increasing the amount of follower force decreases the flutter vibrations capacity and increases the vibration amplitude. Increasing the coefficient of linear and nonlinear elastic foundation helps the system to show more resistance against external loads and the result is less vibration while increasing the shear vibration stiffness coefficient of elastic foundation induces a reserve trend to the system and even causes the system lead to buckling phenomenon faster than the condition that there is no elastic foundation. In the process of this experiment, the volume fraction in the x-form distribution was considered 12%.

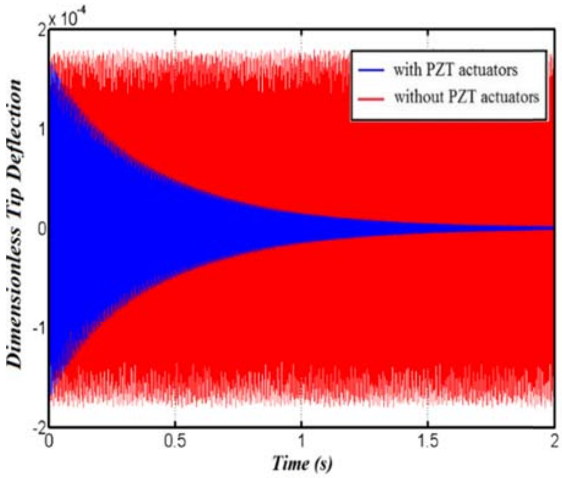

Fig. 4 Control of FG-CNTRC beam under follower force

In this experiment, the distribution of CNTs in thickness direction of the composite beam in the X -form, the volume fraction is considered 17% and the value of dimensionless follower force is 10_4. Length of the piezoelectric layers with a thickness of 1 mm assumed to be 8 cm. as the chat shows vibrations of the beam are shown by red and the controlled vibrations are blue. Here the controller system could converge the beam vibrations to zero. The control system that is designed for this model is Unstable.

TABLE II
DIMENSIONLESS FIRST FREQUENCIES OF FG-CNTRC BEAM FOR DIFFERENT STIFFNESSES

$V_{cnt}^{*}=12\%$	K_L=0	K_{NL}=0	K_S=0
	ω_1	ω_2	ω_3
X	13.7633	55.8300	108.1748
$V_{cnt}^{*}=12\%$	K_L=10^9	K_{NL}=10^9	K_S=0
	ω_1	ω_2	ω_3
X	15.7531	56.3262	108.4277
$V_{cnt}^{*}=12\%$	K_L=10^9	K_{NL}=10^9	K_S=10^7
	ω_1	ω_2	ω_3
X	15.6489	56.1002	108.1178

In Table II, the effect of elastic foundation coefficients when volume fraction of CNTs is 12%, on dimensionless parameters, the first three frequencies of system $\omega = \Omega L\sqrt{I_{10}/A_{110}}$ for one mode of distribution of CNTs (X-From) in the thickness direction of the beam is displayed. Three different cases for elastic foundation coefficients were raised to be shown. As the numbers show existence of linear and nonlinear elastic stiffness coefficients, increases systematic frequencies while adding the shear stiffness coefficient decreases systematic frequencies. It shows that linear and nonlinear elastic stiffness coefficients increase the system vibration capacity.

IV. CONCLUSION

In this paper vibration characteristic of a FG-CNTRC beam resting on elastic foundation has been analyzed and then controlled by an active controller. The effect of elastic foundation stiffness, follower force and the vibration controller on stability of the FG-CNTRC beam is investigated. The results of this study show that increasing of follower force increases the stresses of the system and decreases frequencies of the system. The other parameter that has effect on the system vibration is elastic foundation. By adding elastic foundation to the system increases resistance of the system against external loads totally and frequencies will increase too. The controlling system works well and converged vibration amplitudes to zero.

REFERENCES

[1] A.M.K. Esawi, M.M. Farag, “Carbon nanotube reinforced composites: potential and current challenges”, Mater Des, vol.28, pp.2394–401, 2007.
[2] Y. Han, J. Elliott, “Molecular dynamics simulations of the elastic properties of Polymer/carbon nanotube composites”, Compute Mater Sci, vol.39, pp.315–23, 2007.
[3] H.S. Shen, “Nonlinear bending of functionally graded carbon nanotube-reinforced composite plates in thermal environments”, Compos Struct, vol.91, pp.9–19, 2009.
[4] Ke L-L, Yang J, Kitipornchai S, “Nonlinear free vibration of functionally graded carbon nanotube-reinforced composit beams”, Compos Struct, vol.92, pp.676–83, 2010.
[5] M. Rafiee, J. Yang, S. Kitipornchai, “Large amplitude vibration of carbon nanotube reinforced functionally graded composite beams with piezoelectric layers”, Composite Structures, vol.96, pp.716-725, 2013.
[6] M.H. Yas, N. Samadi, ”Free vibrations and buckling analysis of carbon nanotube-reinforced composite Timoshenko beams on elastic foundation”, Int J Pres Ves Pip, vol.98, pp.119–28, 2012.
[7] A. Mahieddine and M. Ouali, “Finite element formulation of a beam with piezoelectric patch”, J. Eng. Appl. Sci, vol.3, pp.803–807, 2008.
[8] S. Li, H. Su and C. Cheng, “Free vibration of functionally graded material beams with surface-bonded piezoelectric layers in thermal environment”, Appl. Math. Mech., Vol.30, pp.969–982, 2009.
[9] H. Shen and Y. Xiang, “Nonlinear analysis of nanotube-reinforced composite beams resting on elastic foundations in thermal environments”, Engineering Structures, vol.56, pp.698–708, 2013.
[10] H. Shen and Y. Xiang,” Nonlinear vibration of nanotube-reinforced composite cylindrical panels resting on elastic foundations in thermal environments”, Composite Structures, vol.111, pp.291–300, 2013.
[11] R. Ansari, M. FaghihShojaei, V. Mohammadi, R. Gholami, F. Sadeghi, “Nonlinear forced vibration analysis of functionally graded carbon nanotube-reinforced composite Timoshenko beams”, Compos Struct, vol.113, pp.316-327, 2014.
[12] V. Azadi, M. Azadi, S.A. Fazelzadeh, E. Azadi, “Active control of a FGM beam under follower force with piezoelectric sensors/actuators, IJSSD, vol.3, 1350063 (19 pages), 2014.
[13] K. Yildirim, I. Kucuk, “Active piezoelectric vibration control for a Timoshenko beam”, Journal of the Franklin Institute, vol.353, pp.95-107, 2016.