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Vibration Analysis of the Gas Turbine Considering Dependency of Stiffness and Damping on Frequency

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***Abstract*—**In this paper the complete rotor system including elastic shaft with distributed mass, allowing for the effects of oil film in bearings. Also, flexibility of foundation is modeled. As a whole this article is a relatively complete research in modeling and vibration analysis of rotor considering gyroscopic effect, damping, dependency of stiffness and damping coefficients on frequency and solving the vibration equations including these parameters. On the basis of finite element method and utilizing four element types including element of shaft, disk, bearing and foundation and using MATLAB, a computer program is written. So the responses in several cases and considering different effects are obtained. Then the results are compared with each other, with exact solutions and results of other papers.

***Keywords*—**Damping coefficients , Finite element method ,Modeling , Rotor vibration

I. Introduction

CALCULATING natural frequencies and mode shapes and thus predicting and thus predicting critical points in the system is of utmost importance from a design point of view. In the nineteenth century shafts were utilized in industry for the first time. The first research in this case was done by Rankin in 1869. He calculated the critical speed of rotor, considering a simple model [12]. Then in 1894 Dunkerly worked on shafts with several disks and presented an innovative method for predicting natural frequencies [14]. Yet, modeling was very simple and primary including an elastic shaft without mass with a concentrated mass. In 1927 other effects such as rotational inertia and gyroscopic moment were taken into account [15]. In 1972 Finite Element method was utilized and until then models were gradually getting more complete. On the other hand existence of methods such as Transfer Matrix, Finite Element and the use of computer made it possible to analyze more actual shafts with more complexities [2,6,7].

In this study modeling and vibration analysis of complete rotor system considering damping, dependency of stiffness and damping coefficients on frequency, Gyroscopic effect and solving vibration equations including these parameters are taken into account and a relatively complete research is presented. In written program, the way of meshing and the number of elements are free according to the user and thus provide the capability to analyze a variety of rotors with different complexities. On the basis of Finite Element method and utilizing four element types including element of shaft, disk, bearing and suspension using MATLAB, computer program is written. Then complete vibration analysis is done for some examples by the program.The applications of this work are significant in vibration analysis of rotors of compressors and turbines used in jet engines of airplanes and also gas turbines in power plants.

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II. Modeling

To analyze a system, a proper physical model should be made. Then a mathematical model must be presented to solve it and in both steps approximates and errors are inevitable. The important parameter in choosing a model is simplicity and accuracy of the model..

III. Modeling the Complete Rotor System

The main parts in vibration analysis of a rotor are:

Shaft, the assembled parts on shaft, bearings, and foundation;

In this paper, using the finite element method, each part is modeled and then local matrices of each part are obtained. Ultimately to analyze the complete rotor system, the Global matrices of stiffness, damping and mass are assembled and are taken in vibration equation:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{f\} \tag{1}$$

This equation yields frequency responses of rotor system. Four Element types are considered for finite element analysis of rotor which are explained as follows:

IV. Shaft Element

Bar element assuming distributed stiffness and mass are utilized for shaft elements. Using Maximum potential and kinetic energy, stiffness and mass matrices are derived. In this paper, journal bearings with liquid layer are investigated. In these bearings, damping is relatively high so damping of shaft

is negligible and here only stiffness and mass matrices are considered for shaft elementsunits for each quantity in an equation.In bar element used for shaft, each node has two degrees of freedom in xy plane (y,θ) and two degrees of freedom in xz plane (z,φ), Assuming that X axis coincides to the shaft axis. Each node has four degrees of freedom and thus each element has eight degrees of freedom.Finally using energy method stiffness and mass matrices are derived as follow:

$$[K_r]_e = EI\begin{bmatrix} 12/L^3 & 6/L^2 & 0 & 0 & -12/L^3 & 6/L^2 & 0 & 0 \\ 6/L^2 & 4/L & 0 & 0 & -6/L^2 & 2/L & 0 & 0 \\ 0 & 0 & 12/L^3 & 6/L^2 & 0 & 0 & -12/L^3 & 6/L^2 \\ 0 & 0 & 6/L^2 & 4L & 0 & 0 & -6/L^2 & 2/L \\ -12/L^3 & -6/L^2 & 0 & 0 & 12/L^3 & -6/L^2 & 0 & 0 \\ 6/L^2 & 2/L & 0 & 0 & -6/L^2 & 4/L & 0 & 0 \\ 0 & 0 & -12/L^3 & -6/L^2 & 0 & 0 & 12/L^3 & -6/L^2 \\ 0 & 0 & 6/L^2 & 2/L & 0 & 0 & -6/L^2 & 4/L \end{bmatrix} \quad (2)$$

$$[M_r]_e = \frac{\mu}{420}\begin{bmatrix} 156L & 22L^2 & 0 & 0 & 54L & -13L^2 & 0 & 0 \\ 22L^2 & 4L^3 & 0 & 0 & 13L^2 & -3L^3 & 0 & 0 \\ 0 & 0 & 156L & 22L^2 & 0 & 0 & 54L & -13L^2 \\ 0 & 0 & 22L^2 & 4L^3 & 0 & 0 & 13L^2 & -3L^3 \\ 54L & 13L^2 & 0 & 0 & 156L & -22L^2 & 0 & 0 \\ -13L^2 & -3L^3 & 0 & 0 & -22L^2 & 4L^3 & 0 & 0 \\ 0 & 0 & 54L & 13L^2 & 0 & 0 & 156L & -22L^2 \\ 0 & 0 & -13L^2 & -3L^3 & 0 & 0 & -22L^2 & 4L^3 \end{bmatrix} \quad (3)$$

Where L is the length of shaft element, I is the moment of inertia of cross section of element, E is the module of elasticity, μ is mass per unit length of element and e represents element.

V. Disk Element

Meshing is made in such a way that each disk is positioned on one node. Assuming that disk is rigid, there is no stiffness matrix for disk. Since each disk is located on one node, there are four degrees of freedom and thus mass and damping matrices of disk are as follows:

$$[M_d]_e = \begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I_T & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & I_T \end{bmatrix} \quad (4)$$

$$[C_d]_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega I_p \\ 0 & 0 & 0 & 0 \\ 0 & -\omega I_p & 0 & 0 \end{bmatrix} \quad (5)$$

VI. Bearing Element

In the case of bearings, meshing is in such a way that one node of the bearing element is positioned on shaft node and another node on the other side of bearing element is connected to foundation element. Each node has four degrees of freedom, therefore as a whole the element has eight degrees of freedom. Considering short bearing assumption, stiffness and damping matrices are as follows:

$$[K_b]_e = \begin{bmatrix} k_{xx} & 0 & k_{xy} & 0 & -k_{xx} & 0 & -k_{xy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{yx} & 0 & k_{yy} & 0 & k_{yx} & 0 & -k_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_{xx} & 0 & -k_{xy} & 0 & k_{xx} & 0 & k_{xy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k_{yx} & 0 & -k_{yy} & 0 & -k_{yx} & 0 & k_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6)$$

$$[C_b]_e = \begin{bmatrix} c_{xx} & 0 & c_{xy} & 0 & -c_{xx} & 0 & -c_{xy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{yx} & 0 & c_{yy} & 0 & -c_{yx} & 0 & -c_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{xx} & 0 & -c_{xy} & 0 & c_{xx} & 0 & c_{xy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{yx} & 0 & -c_{yy} & 0 & +c_{yx} & 0 & c_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

where Kxx, Kxy,… and Cxx, Cxy… are stiffness and damping coefficients of oil film. The first indices represent the direction of forces and second indices represent the direction of displacements.

VII. Foundation Element

Each node of foundation element has two degrees of freedom and thus foundation element has four degrees of freedom. Then stiffness and damping matrices are as follows:

$$[K_s]_e = \begin{bmatrix} k_x & 0 & -k_x & 0 \\ 0 & k_y & 0 & -k_y \\ -k_x & 0 & k_x & 0 \\ 0 & -k_y & 0 & k_y \end{bmatrix} \quad (8)$$

$$[C_s]_e = \begin{bmatrix} c_x & 0 & -c_x & 0 \\ 0 & c_y & 0 & -c_y \\ -c_x & 0 & c_x & 0 \\ 0 & -c_y & 0 & c_y \end{bmatrix} \quad (9)$$

Where kx, ky are the stiffness of foundation in x, y direction and cx, cy are damping coefficients of foundation.

VIII. Gyroscopic Effect

When the rotation is around x axis and at the same time another rotation around y axis occurs, then gyroscopic

moment will be around z axis, In fact damping matrix of disk is the gyroscopic effect of disk.

IX. Modeling the Bearing

The model of displacement constraint:

In this model shaft displacement in special directions at bearing position is assumed to be zero. It should be noted that this model has the required accuracy when flexibility of bearing is low in comparison with flexibility of shaft.

X. 10-Model of Spring and Damper

In order to obtain more accurate model, flexibility and damping can be considered for bearing. So bearing can be modeled as spring and damper. Equations of applied forces are as follows:

$$Q = Ku + C\dot{u} \tag{10}$$

$$\begin{Bmatrix} Q_y \\ Q_z \end{Bmatrix} = \begin{bmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{bmatrix} \begin{Bmatrix} \upsilon \\ w \end{Bmatrix} + \begin{bmatrix} C_{yy} & C_{yz} \\ C_{zy} & C_{zz} \end{bmatrix} \begin{Bmatrix} \dot{\upsilon} \\ \dot{w} \end{Bmatrix} \tag{11}$$

Stiffness and damping coefficients are basically due to the liquid under pressure in journal bearings. These coefficients are derived by solving Navier-Stokes Equations. Applying simplified assumptions, Navier-Stokes Equations convert to Reynolds Equations and here the required coefficients for bearing are on the basis of Reynolds Equations.

XI. Considering damping terms in solving vibration equations

Considering damping terms, solving the equations do not easily lead to theory of eigenvalues and eigenvectors so it involves using more complex and inventive methods.

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = 0 \tag{12}$$

In order to find an algorithm for programming and calculating natural frequencies and mode shapes by an innovative method, one can convert the above equation into the following equation:

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \ddot{X} \end{Bmatrix} + \begin{bmatrix} 0 & -M \\ K & C \end{bmatrix} \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{13}$$

Now following terms are defined:

$$\{\dot{y}\} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} \quad , \quad \{y\} = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

$$[A] = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \quad , \quad [B] = \begin{bmatrix} 0 & -M \\ K & C \end{bmatrix} \tag{14}$$

Substituting Eq.(14) into Eq. (13), we have:

$$[A]\{\dot{y}\} + [B]\{y\} = \{0\} \tag{15}$$

As it can be seen, Eq.(15) is the state equation. Assuming an answer as $\{y\}=\{S\}e^{i\omega t}$ and substituting it in Eq.(15) yields:

$$(i\omega)[A]\{S\}e^{i\omega t} + [B]\{S\}e^{i\omega t} = \{0\} \tag{16}$$

And thus:

$$[B]\{S\} = (i\omega)(-[A])\{S\} \tag{17}$$

Resulting that natural frequencies and mode shapes are derived from the eigenvalues and eigenvectors of matrix [B] with respect to matrix [A]. In fact the imaginary part of eigenvalues represent natural frequencies.

XII. Considering the dependency of stiffness and damping coefficients on frequency

Another new work which is presented here is that in addition to analysis of vibration equations with constant stiffness and damping coefficients, the effects of frequency on stiffness and damping coefficients are considered and thus vibration equations with variable coefficients dependent on frequency are obtained. In order to solve the equations, the rotational speed Ω of rotor is divided into arbitrary division from zero to ultimate speed and in each division the speed is assumed to be constant. By this procedure, the arrays of stiffness and damping matrices can be calculated as constant in each division. So in each division, vibration equations with constant coefficients consisting damping terms will be produced. The way of solving such equations with damping terms has already explained.

$$M\ddot{X} + C_f)_{\Omega=\Omega_0}.\dot{X} + K_f)_{\Omega=\Omega_0}.X = f \tag{18}$$

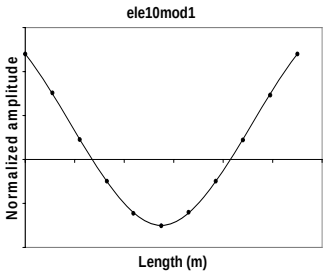
XIII. The results of computer program by solving some examples

First of all in order to examine the accuracy of program, a simple example which has analytical solution is solved by program. Then some examples including the complete rotor system and considering different positions of disk on shaft are analyzed.

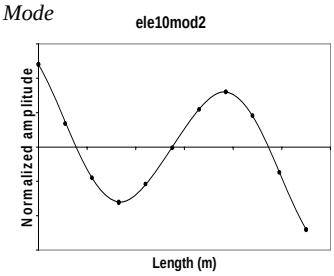
Example 1:

Calculating mode shapes of a free-free beam. The beam has following parameters:

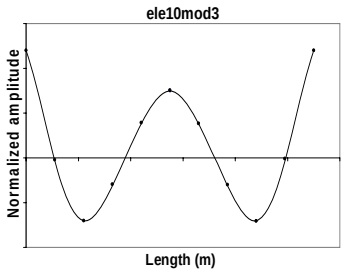
L = 1.5 m, D = 0.025 m, EI = 4032, ρ=7800 kg/m^3. Mode shapes are shown in Fig. 1.



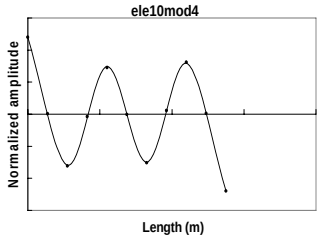
a) First Mode



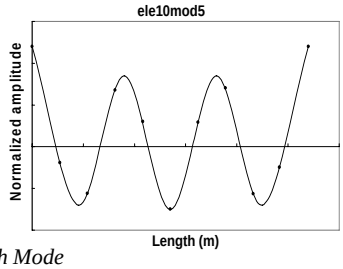
b) Second Mode



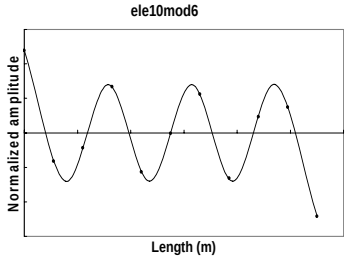
c) Third Mode



d) Forth Mode



e) Fifth Mode



f) Sixth Mode

Fig. 1 Natural Modes of free-free beam with 10 elements

Example 2:

A complete rotor system (shaft with its components) is considered. The rotor system includes shaft, three disks and two linear bearings and foundation. Dimensions of shaft, disks and positions of bearings and disks are shown in Fig. 2

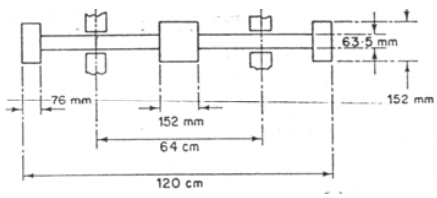


Fig. 2 Rotor with all disks

The parameters of different parts of rotor are as follows:

Shaft parameters:

L = 1.2 m, M = 135 kg, I = 0.798E-6, E = 0.210E+12.

Disks parameters:

m = 8.25 kg mass of end disks

m' = 16.5 kg mass of middle disk

J = 0.024 mass moment of inertia of end disk

J' = 0.048 mass moment of inertia of middle disk

I = 0.012, area moment of inertia of end disk

I' = 0.024 area moment of inertia of middle disk.

Here linear analysis is executed. Also the dependency of stiffness and damping coefficients on rotational speed of rotor is considered Fig3.The result of analysis executed by program is presented as Unbalance Response shown in Fig. 4.

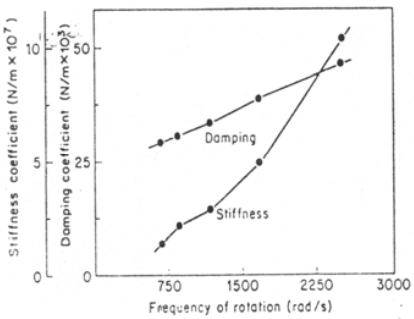


Fig. 3 Dependency of stiffness and damping on frequency of rotation

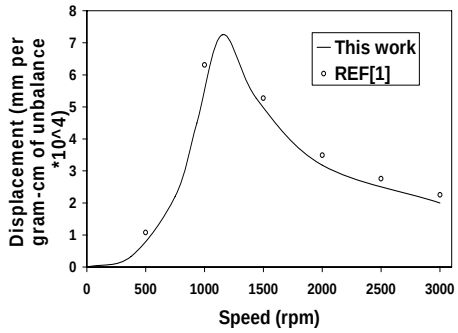


Fig. 4 Unbalance Response of the complete rotor system with linear analysis considering dependency of stiffness and damping coefficients on rotor speed (middle disk)

In this example, just the middle disk is considered and other disks are eliminated. Then in example 3, the three disks are considered in different positions

Example 3:

Again the complete rotor system (including shaft, disks, bearings and foundation) shown in Fig. 5 is considered.

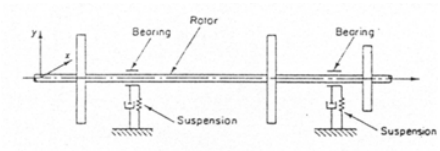


Fig. 5 A rotor with its components

In this example the three disks are considered in their positions respectively and Unbalance Response in three positions; end, center and all disks (Figs.2,6,7) are obtained by the program shown in Fig. 8.

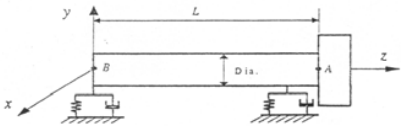


Fig. 6 Rotor with end disk position

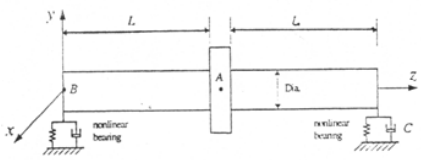
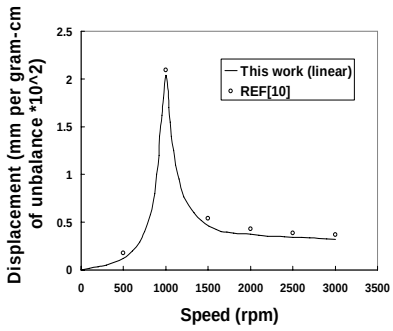
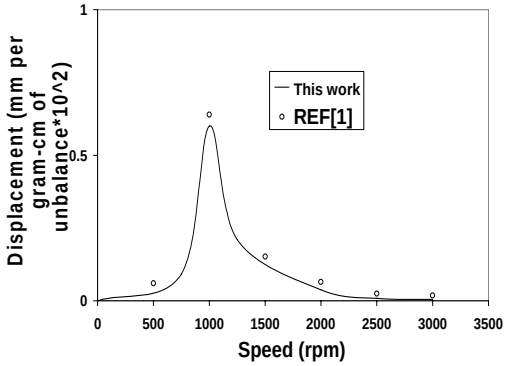


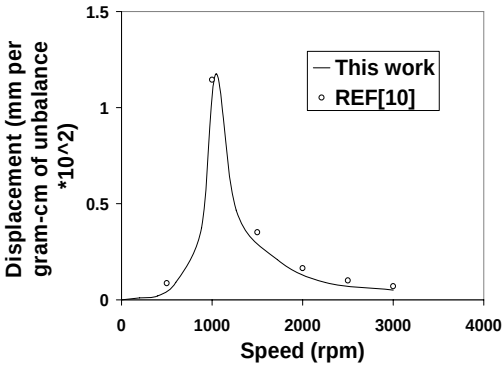
Fig. 7 Rotor with middle disk position



a) Disk in middle position



b) Disk in end position



c) All three disks in their positions

Fig. 8 Unbalance Response of the complete rotor system with linear analysisWith three disk positions

XIV. Conclusions

In this paper the complete rotor system including elastic shaft with distributed mass considering the effects of oil film in bearing and also flexibility of suspension is modeled. As a whole this article is a relatively complete research in modeling and vibration analysis of rotor considering gyroscopic effect, damping, dependency of stiffness and damping coefficients on frequency. On the basis of linear theory by finite element

method and utilizing four element types including element of shaft, disk, bearing and suspension using MATLAB, a computer program is written. At first the accuracy of the written program is checked by comparing the analytical solution and the results of execution of program for a simple example. Then complete vibration analysis is executed for some complete rotor systems and ultimately conclusions are made.

XV. Conclusions

Gyroscopic effect is considered in vibration analysis and thus accuracy of results is increased.

Damping effect is also taken into account yielding vibration equations which can not be easily solved. In this paper an innovative method is utilized in order to solve such equations using state space equations. Fortunately the results accurately match those of Ref [10].

The dependency of stiffness and damping coefficients of bearings on rotational speed of rotor are applied. Since these coefficients are of high order and the range of the coefficients is limited in normal speeds e.g. from 0 to 3000 RPM, so the effect on ultimate responses is negligible. In written program, the way of meshing and the number of elements are free according to the user and thus provide the capability to analyze variety of rotors with different complexities.

On the whole by considering gyroscopic effect, damping effect and dependency of stiffness and damping coefficients on rotor speed, the unbalance response is too close to that of Transfer Matrix method in Ref [10].

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