Verification Process of Cylindrical Contact Force Models for Internal Contact Modeling

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***Abstract*—**In the numerical solution of the forward dynamics of a multibody system, the positions and velocities of the bodies in the system are obtained first. With the information of the system state variables at each time step, the internal and external forces acting on the system are obtained by appropriate contact force models if the continuous contact method is used instead of a discrete contact method. The local deformation of the bodies in contact, represented by penetration, is used to compute the contact force. The ability and suitability with current cylindrical contact force models to describe the contact between bodies with cylindrical geometries with particular focus on internal contacting geometries involving low clearances and high loads simultaneously is discussed in this paper. A comparative assessment of the performance of each model under analysis for different contact conditions, in particular for very different penetration and clearance values, is presented. It is demonstrated that some models represent a rough approximation to describe the conformal contact between cylindrical geometries because contact forces are underestimated.

***Keywords*—**Clearance joints, Contact mechanics, Contact dynamics, Internal cylindrical contact, Multibody dynamics.

I. Introduction

IMPACT is a noticeable phenomenon in most mechanical systems, such as mechanisms with intermittent motion, kinematic discontinuities and clearance joints [1]-[15]. Impact can be mainly due to: i) the collision between two or more bodies that can be unconstrained or may belong to a multibody mechanical system, or ii) joint clearances in the system. In both cases, the impact phenomenon is characterized by contact forces generated during impact applied and removed in short periods of time [16]-[23]. When impact occurs, there is an abrupt change in the state of the system, most visible in the discontinuities of velocities and accelerations due to changes in the kinetic energies of the impacting masses and in the reaction forces or impulses of the kinematic joints [7]-[30]. The large impulsive and short lived contact forces combined with kinematic constraints result in a strong nonlinear response of the mechanical system. The knowledge of the peak forces developed in the impact process and their transmission throughout the constrained system is very important for the dynamic analysis of multibody mechanical systems and has consequences in the engineering design process [7]-[38].

In the continuous formulation for contact dynamics analysis, the occurrence of penetration is used as the basis to evaluate the local deformation of the bodies in contact [16], [17], [21]-[26], [28], [35], [37-44]. Given that the penetration is known at every time step from the state variables, i.e., positions and velocities of the bodies in the system, the contact forces are evaluated using appropriate contact models. Therefore, knowledge on the relation contact force versus penetration during the contact period is required. Knowing the variation of the contact force during the contact period, the dynamic response of the system is obtained at each time step by simply incorporating updated forces into the equations of motion [16], [17], [21-26], [28]. With the continuous contact method it is possible to account for the changes in the system configuration and its velocities during the contact period since the equations of motion are integrated over the period in which contact takes place, thus allowing for a comprehensive description of the real behavior of multibody systems, including friction [36], [45], [46]. But to efficiently evaluate the contact forces between the bodies that integrate a multibody system, proper modeling of contact forces is required. The contact force model must be computed through suitable constitutive laws that include information about the geometric characteristics, material properties and impact velocities of the contacting surfaces [22], [28], [36], [46], [47]-[50]. In addition, special attention must be given to the modeling of joints with very low clearance values because firstly, the contact forces are nonlinear functions of the joint's relative motions and secondly, because they also depend on the joint internal geometry and material properties [51].

The goal of this paper is to identify and discuss the applicability domain of actual cylindrical contact force models, which is done through a comparative assessment of the performance of each model under analysis for different contact conditions, in particular for very different penetration and clearance values.

II. Analytical Models for Cylindrical Contact

Based on the Hertz pressure distribution, several analytical cylindrical contact force models describing the relation between the penetration experienced by contacting cylindrical bodies and the applied contact force, are available in the literature [52]-[58]. These models are all nonlinear since the penetration is represented as an implicit function of the

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contact force. A comparative assessment of the performance of each current cylindrical model available in the literature to describe internal cylindrical has been performed by the authors calculating the relative difference of each one in relation to Johnson's model. From this study it was concluded that, when compared with other cylindrical and spherical models, the cylindrical contact model presented by Johnson [52] is the one that best describes the contact involving colliding cylinders in most practical applications [51]. In the Johnson model, the total penetration of two deformable contacting cylinders of radius R_i and R_j made of materials with similar elastic modulus and Poisson ratios denoted by E_i, υ_i and E_j, υ_j, respectively, submitted to the action of a compressive normal load, P, is given by (1) [52].

$$\delta = \frac{P}{\pi E^*}\left\{\ln\left(\frac{4\pi E^* \Delta R}{P}\right) - 1\right\} \qquad (1)$$

In (1) the compressive load P is expressed per unit of the axial length of the cylinder. Furthermore, the penetration, which accounts for the contribution of both cylinders, is assumed to be measured at a point distant enough from the contact point. E* represents the composite modulus of the two colliding cylinders and is evaluated as defined by (2).

$$\frac{1}{E^*} = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_j^2}{E_j} \qquad (2)$$

However, if the contacting cylinders are characterized by similar elastic properties, Poisson's coefficient and Young's modulus, (2) takes the form of (3).

$$E^* = \frac{E}{2\left(1-\nu^2\right)} \qquad (3)$$

Depending on what parameter ΔR represents, (1) can be applied to internal and external contacts. When ΔR represents the sum of the cylinders' radii, (R_i+R_j), an external contact geometry is considered. Otherwise, for internal contact, ΔR is quantified by the difference between the cylinders' radii, (R_i-R_j), corresponding to the clearance between the two cylindrical bodies.

In the numerical solution of the forward dynamics of a mechanical system, the positions and velocities of the bodies in the system are obtained first. The internal and external forces acting on the system are calculated for the state variables of the system at each integration time step. Therefore, for each given penetration, (1) has to be solved iteratively to evaluate the contact force that fulfills it. When used in the framework of forward dynamic analysis, this procedure is not only computationally costly but also represents a numerical difficulty for the performance of a computational program especially if a greater number of contacting bodies are involved [13], [15]-[17], [23]-[26], [28], [40], [45], [46]. Nevertheless, the nonlinearity of Johnson model is not the only drawback associated with this model. It was proposed as purely elastic models and therefore it is unable to explain the energy dissipation during the impact process [39]-[41], [47]-[50]. Moreover, it includes a logarithmic function, which imposes mathematical and physical limitations on its application, particularly for conformal contact conditions with lower clearance values, which means that it has a validity domain that depends on the clearance value and material properties. These same drawbacks are associated to most current cylindrical contact models [51]. To avoid these shortcomings, a new Enhanced cylindrical contact force model, without domain validity problems, has been recently proposed by the authors [59], where the contact force is defined as an explicit function of the known penetration as given by (4).

$$\delta = \left(\frac{P\Delta R}{\left(a\,\Delta R + b\right)E^*}\right)^{1/n} \qquad (4)$$

In (4), for internal cylindrical contact, a=0.49, b=0.10, $n=Y\Delta R^{-0.005}$ and $\Delta R=R_i-R_j$, in which $Y=1.56[\ln(1000\Delta R)]^{-0.192}$ if $\Delta R=[0.005,\ 0.750[$ or $Y=0.0028\Delta R+1.083$ if $\Delta R=[0.750,\ 10.0[$ mm. The remaining quantities in (4) have the same meaning described for (1). As in the new enhanced model, conversely to Johnson model, the pseudo-stiffness is defined as not dependent on the contact force, it is possible to add the term that accounts for energy dissipation in the form suggested by Lankarani and Nikravesh as represented by (5) [39]-[41]. In (5) c_e is the restitution coefficient, $\dot{\delta}^{(-)}$ is the relative impact velocity and $\dot{\delta}$ is the actual penetration velocity.

$$P = \frac{\left(a\ \Delta R + b\right)E^*}{\Delta R}\delta^n\left[1 + \frac{3(1-c_e^2)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \qquad (5)$$

Relative analytical differences lower than 10% are obtained from the comparison between the results presented by the enhanced model with respect to the Johnson model for all the contact parameter variations - clearance value, elastic modulus and Poisson ratios - tested [59].

C.-S. Liu et al. [38] adopted the combined Lagrange–Penalty method, where the penetrations are controlled directly by the penetration tolerance and not indirectly by the penalty factor, to establish a comparison between FEM results and those obtained using the Johnson model, defined by Equation (1), and the Persson theory for describing contact in cylindrical joints with clearance. Based on the works developed by Ciavarella and Decuzzi [60], [61], and considering contacting bodies with identical elastic material properties, C.-S. Liu et al. present the Persson model as described by (6), where $b=\tan(\varepsilon/2)$ and the semi-angle of contact, $\varepsilon=\arccos(\Delta R/(\Delta R+\delta))$.

The numerical results obtained by these authors show that both Johnson and Persson models have limitations in their application. To overcome the drawbacks associated with

Johnson and Persson models a simple and straightforward cylindrical conformal contact model, where the contact force is directly calculated from a given penetration, has been proposed by Liu and co-workers [38]. This approximate model is based on Winkler elastic foundation with the penetration depth related to the normal force as defined by (7).

$$P = \frac{E^{*}\Delta R\pi\left(b^{2}+1\right)b^{2}}{2-\left[\log\left(b^{2}+1\right)+2b^{4}\right]} \tag{6}$$

$$P = \frac{1}{2}\pi\delta E^{*}\left[\frac{\delta}{2\left(\Delta R+\delta\right)}\right]^{1/2} \tag{7}$$

When compared with FEM results the model proposed by C.-S. Liu et al. gives, for the contact conditions considered, a more effective solution than those achieved with Johnson and Persson models. However, since the pseudo-stiffness is not defined as constant, the C.-S. Liu et al. model is unable to account for the energy dissipation process that characterizes impact mechanisms [47]-[50], at least in the form suggested by Lankarani and Nikravesh [39]-[41], [47]. In addition, although a detailed finite element analysis is used, the validation of the model proposed by C.-S. Liu et al., and of other models, was done for a range of contact conditions, dimensions of contacting cylinders, clearances and penetration depths, specific for the applications foreseen in each one of them [37], [38], [48]. Clearances in the range of 0.1 to 5 mm, penetration depths from 0.02 to 0.2 mm and an external cylinder with a radius of 100 mm are the contact conditions used by C.-S. Liu et al [38]. For these contact conditions, the authors reported that when compared with the FEM results, the model proposed by Johnson is effective only in the condition that the clearance is large enough and the normal load is very small, i.e. for non-conformal contact conditions, while the Persson model can be applied only in the case that there is a small clearance and the contact semi-angle is large enough. For very different contact conditions than those used by Liu and co-workers, in particular what concerns clearance and penetration depth values as well as contacting bodies' dimension, it is demonstrated by C. Pereira et al. [62] that the Johnson cylindrical model is appropriate and suitable to describe the cylindrical conformal contact, which does, however, contradict the results obtained by Liu and co-authors [38]. In fact, for clearance values between 0.005 and 1.5 mm, penetration depths from 0.00024 mm to 0.00254 mm, corresponding to an external cylinder radius of 2.245 mm, relative differences lower than 10% are found between FEM results and the results obtained using the Johnson model, regardless of the load value applied [62]. Moreover, it has been experimentally demonstrated that maximum divergence of less than 15% separate experimental results from those obtained by the Johnson model, and that when the enhanced model results are compared with Johnson results, divergences lower than 10% are verified [63].Therefore, and in order to identify and discuss the applicability domain of the cylindrical contact force models proposed by Johnson, Persson, C.-S. Liu et al. and the new enhanced model, a comparative assessment of the different models for different contact conditions, in particular for very different penetration and clearance values, is here performed. This issue is presented and discussed in the following section.

III. Comparative Assessment of Different Cylindrical Contact Force Models

The behavior of the different models to describe the contact for cylindrical clearance joints is presented by Fig. 1a) through 1l) for low penetrations and for high penetrations by Fig. 2a) through 2l). The results are grouped according to penetration depth values considered, and so, two sets can be identified: i) low penetrations, for penetration depth values in the range of 0.001 to 10 μm; and ii) high penetrations, for penetration depths values in the range of 5 to 200 μm. The high penetration set is here considered in order to establish a comparison with the results presented by C.-S. Liu and co-authors [38]. For the same reason a composite modulus of 205 GPa for the contacting cylinders is considered. Since, in the Johnson model the penetration is an implicit function of contact force, the contact forces calculated from the Person model are used to obtain the normal load versus penetrations curves for Johnson model.

Radial clearance values between 5 μm and 2 mm are considered. This range of clearances is selected based on the standard ISO system information [64], [65]. The system ISO implements 20 grades of accuracy and defines 28 classes of basic deviations for holes and shafts. It is principally impossible to produce machine parts with absolute dimensional accuracy. In fact, it is not necessary or useful. For normal ordinary engineering purposes, only a limited range of tolerance zones is used. For fits in precision and general engineering the individual tolerances of the system ISO is in the range of IT5 to IT12 ensuring the correct functioning of engineering products. Fits may, however, differ depending on the type and field of production, local standards, constructional and technological views, economic reasons and so on. The desired clearances and interferences in the fit are achieved by combinations of various shaft tolerance zones with the hole tolerance zone "H". For some of these combinations and for standard dimensions of common examples of mechanical engineering practice in which internal contacting cylinders are involved, Table I summarizes the maximum permissible clearances using the hole-basis system. The maximum clearance values presented in Table I are calculated as the difference between the upper limit of size of the hole and the lower limit of size of the shaft.

From Table I it can be concluded that a maximum radial clearance of 0.570 mm, corresponding to the running fit H11/d11 and to a maximum nominal dimension of 630 mm, is allowable. It should be noted that for general engineering applications the running fits more used are identified as b) and c) in Table I where a) is used less for economic reasons. For b) and c) running fits and for the maximum nominal dimension considered, a maximum radial clearance of 0.163 mm, corresponding to H7/e8 fit, is acceptable. This means that for the nominal dimension of 200 mm, corresponding to a radius of 100 mm, considered by C.-S. Liu et al [38], a maximum radial clearance of 0.109 mm is tolerable if the running fit with great clearances without any special requirements for fit accuracy or for accuracy of guiding shafts is neglected. Except for the first clearance value considered by C.-S. Liu et al [38], all the others correspond to typical clearance of worn equipments. The same is applied, in the range of clearance values under analysis in this work, to the higher clearances considered.

to 1l). In fact, for clearances higher than 0.02 mm, that correspond to contact semi-angles lower that 48º, the differences between Persson and C.-S. Liu et al. models increase substantially, because the basic assumption of Persson theory is violated, as demonstrated by C.-S. Liu et al. [38].

In what concerns the behavior presented by Johnson and the Enhanced models, it can be concluded that a good agreement is obtained between both models for clearances higher than 0.05 mm. This agreement decreases with the increase of penetration depth values, probably due to the fact that the Johnson's penetrations are obtained using the normal forces

TABLE I
MAXIMUM CLEARANCE VALUES, IN MILLIMETERS, FOR RUNNING FITS OF THE HOLE BASIS SYSTEM WITH: A) VERY SMALL CLEARANCES FOR PRECISE GUIDING AND CENTERING OF PARTS; B) VERY SMALL CLEARANCES FOR ACCURATE GUIDING OF SHAFTS; C) SMALL CLEARANCES WITH GENERAL REQUIREMENTS FOR FIT ACCURACY OR FOR ACCURATE GUIDING OF SHAFTS; D) GREAT CLEARANCES WITHOUT ANY SPECIAL REQUIREMENTS FOR FIT ACCURACY OR FOR ACCURACY OF GUIDING SHAFTS

	A)		B)			C)					D)	
Nominal Size [mm]	H6/h5	H6/g5	H6/f6	H7/h6	H7/g6	H7/f7	H7/e8	H8/h7	H8/h8	H8/f8	H10/d10	H11/d11
6-10	0.015	0.020	0.031	0.024	0.029	0.043	0.062	0.037	0.044	0.057	0.156	0.220
10-18	0.019	0.025	0.038	0.029	0.035	0.052	0.077	0.045	0.054	0.070	0.190	0.270
18-30	0.022	0.029	0.043	0.034	0.041	0.063	0.094	0.054	0.066	0.086	0.233	0.325
30-50	0.027	0.036	0.057	0.041	0.050	0.075	0.114	0.064	0.078	0.103	0.280	0.400
50-80	0.032	0.042	0.068	0.049	0.059	0.090	0.136	0.076	0.092	0.122	0.340	0.480
80-120	0.037	0.049	0.080	0.057	0.069	0.106	0.161	0.089	0.108	0.144	0.400	0.560
120-180	0.043	0.057	0.093	0.065	0.079	0.123	0.188	0.103	0.126	0.169	0.465	0.645
180-250	0.049	0.064	0.108	0.075	0.090	0.142	0.218	0.118	0.144	0.194	0.540	0.730
250-315	0.055	0.072	0.120	0.084	0.101	0.160	0.243	0.133	0.162	0.218	0.600	0.830
315-400	0.061	0.079	0.134	0.093	0.111	0.176	0.271	0.146	0.178	0.240	0.670	0.930
400-500	0.067	0.087	0.148	0.103	0.123	0.194	0.295	0.160	0.194	0.262	0.730	1.030
500-630	0.076	0.098	0.164	0.114	0.136	0.216	0.325	0.180	0.220	0.296	0.820	1.140

A. Low Penetration Values

For penetrations in the range of 0.001 to 10 μm the curves produced by Persson model agree well with those obtained using the model suggested by C.-S. Liu et al. for very low clearances, i.e., for clearances lower than 0.02 mm, as shown by Fig. 1a) to 1d). For clearances higher than 0.02 mm, Fig. 1e) to 1l) demonstrate that the deviation between these two models increases with the increasing of clearance values. The C.-S. Liu et al. model always leads to small values of contact force for the same penetration than those obtained using Persson model, and the difference between models is greater the higher the clearance is. This means that with Persson theory the contact force values are overestimated, in particular for clearances higher than 0.02 mm. Although C.-S. Liu and co-workers have reported that for high penetration values, when compared with FEM results, the Persson model can only be applied in the case that there is a small clearance and the contact semi-angle is large enough. This same behavior is here observed for the contact conditions considered. Maximum contact semi-angle values around 70º, 64º, 60º, 48º, 33º, 28º, 24º, 17º, 11º, 9º, 8º and 5º are obtained for the range of clearances under analysis, respectively, represented in Fig. 1a) achieved by Persson model. However, for the three lowest clearances under analysis, the difference between these models
cannot be quantified for all the range of penetrations tested. The Johnson model leads to a much more stiff contact, increasing this stiffness with the decreasing of clearance, i.e. with the increase of conformity/conformal contact conditions. In fact, the increase of penetration is not proportional to the increase in load. Beyond certain load value, which depends on the clearance value, the increase of penetration is minimum [51]. The model proposed by Johnson represents the penetration as a function of contact force in a logarithmic form. From a physical point of view, the function expressed in (1) should exhibit a continuous monotonically increasing behavior, i.e. the values obtained for the indentation must always be positive and must increase with the increase in load. This trend is not observed for loads beyond certain values, and for very low clearances the penetration value decreases with increasing load, which is physically inconsistent. To guarantee that the mathematical and physical requirements are satisfied, the logarithmic function of

Johnson's contact model equation must be equal to or greater than 2. This leads to a load limit value for each clearance value given by (8), where all parameters have been defined before.

$$\ln\left(\frac{4\pi E^{*}\Delta R}{P}\right) \geq 2 \;\rightarrow\; P_{lim} \leq \frac{4\pi E^{*}\Delta R}{e^{2}} \tag{8}$$

From (8) it can be concluded that the Johnson model has a specific validity domain, which depends on the clearance and material properties. The validity domain decreases with the decrease in clearance and/or with the increase of contacting elastic properties materials. Thus, special attention is required in the application of this model, particularly for contacting conditions with low clearance values and high loads, since its validity domain depends on the value assumed by the logarithmic function.

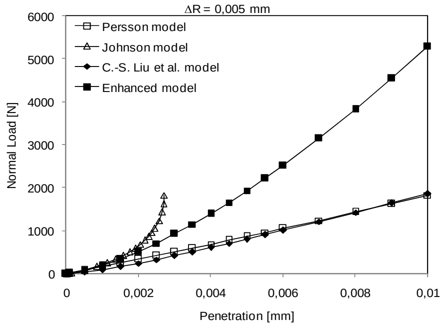

Fig. 1 (a) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.005 mm

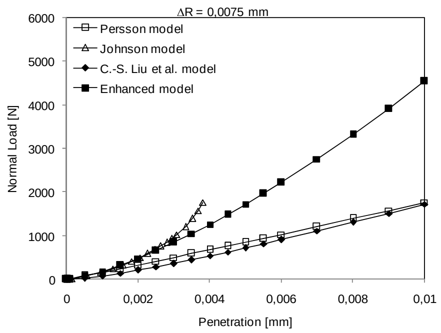

Fig. 1 (b) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.0075 mm

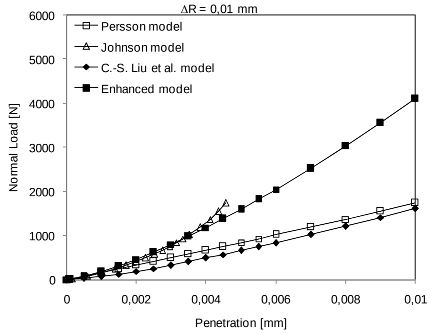

Fig. 1 (c) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.01 mm

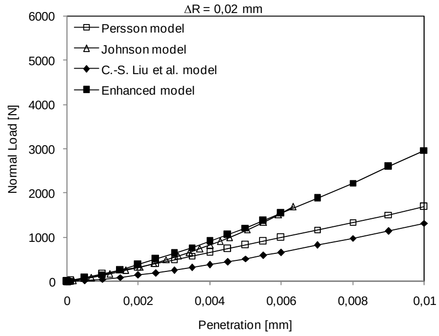

Fig. 1 (d) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.02 mm

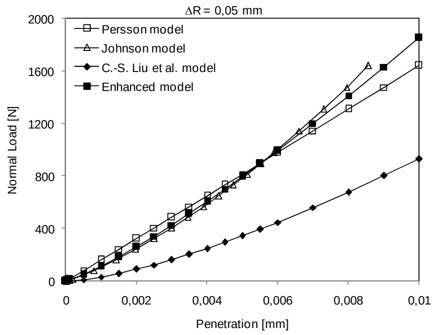

Fig. 1 (e) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.05 mm

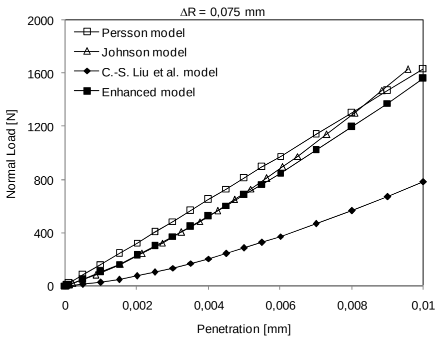

Fig. 1 (f) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.075 mm

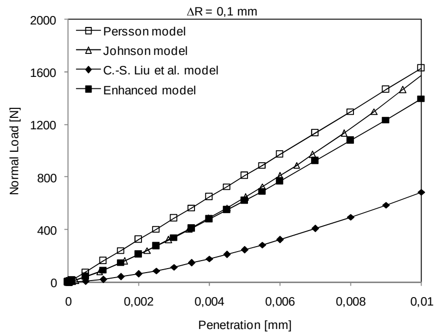

Fig. 1 (g) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.1 mm

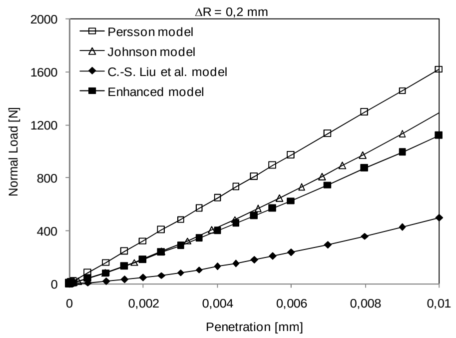

Fig. 1 (h) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.2 mm

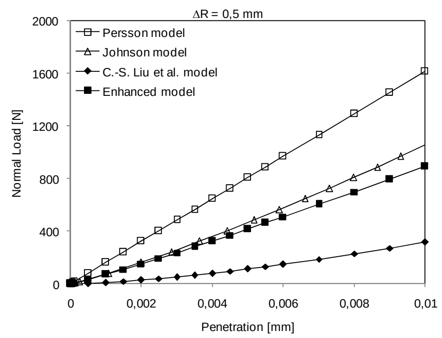

Fig. 1 (i) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.5 mm

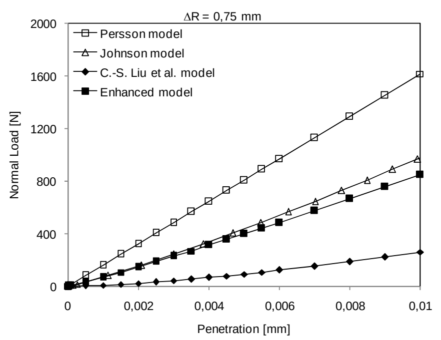

Fig. 1 (j) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 0.75 mm

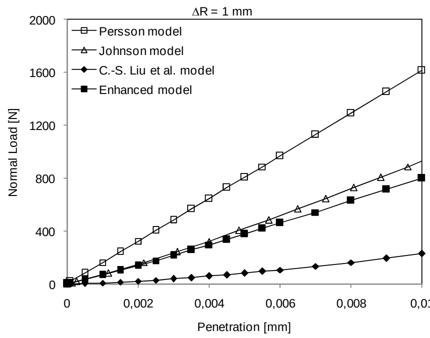

Fig. 1 (k) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 1 mm

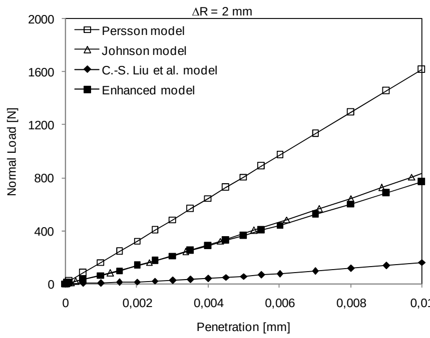

Fig. 1 (l) Comparison of the different contact force models for the set of low penetrations and for a clearance value of 2 mm

Even though based on Johnson model, the Enhanced model is defined without domain validity problems since it does not include any logarithmic function and is free of mathematical and physical limitations. Regardless of clearance values, when compared with Persson and mainly with the C.-S. Liu et al. model, the behavior presented by the Enhanced model confirms the high stiffness of cylindrical contact, in particular as low as the clearance is.

From Fig. 1 (a) to (1l), it can be concluded that, depending on the clearance and contact semi-angle values, Persson model presents a very different behavior. Clearances higher than 20 μm it lead to very high normal loads when compared to those achieved with the other models under analysis, in particular by the model suggested by Liu and co-workers, where normal loads are largely underestimated leading to a softer contact. For clearances lower than 20 μm a good agreement between Persson and C.-S. Liu et al. models is found, leading the Persson model to lower normal loads than those obtained by the Johnson and the Enhanced models. For the same penetration value high loads are always obtained using these models, which means, once again, that the contact is stiffer than that evaluated by Persson and C.-S. Liu et al. models.

B. High Penetration Values

Fig. 2a) through 2l) show, for penetrations in the range of 5 to 200 μm, the relations between the normal load and the penetration depths obtained from the different models.

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:7, No:5, 2013

Although not represented, for clearances lower than 20 μm, the Persson model leads to negative normal loads. The load increases exponentially reaching a maximum, which corresponds to very high contact semi-angle value, assuming negative values beyond this value. This means that, similarly to the Johnson model, the Persson model also has a specify validity domain, which depends on the contact semi-angle value, i.e. on clearance and penetration values. For clearances lower than 20 μm, and for penetrations between 5 and 200 μm, the inconsistence of Persson theory appears, regardless of clearance values, for contact semi-angle, ε, higher than 87º. This contact semi-angle value occurs with the maximum penetration depths of 0.08; 0.12 and 0.16 corresponding to the clearances of 5; 7.5 and 10 μm, respectively. Thus, the validity domain of Persson model is assured if the condition defined by (9) is verified. All the parameters in (9) have been defined before.

$$\left(\log\left(b^2+1\right)+2b^4\right)<2 \;\rightarrow\; b<\sqrt{10^{\left(2-2b^4\right)}-1} \qquad (9)$$

For clearances higher than 5 μm and lower than 200 μm, the curves obtained by the Persson model present a good agreement with those achieved using the model proposed by C.-S. Liu et al.. For clearances higher than 100 μm, however, the results produced by Persson model are largely different from the C.-S. Liu et al. results, the greater this difference is, the higher the clearance is. As referred before, to avoid the violation of the basic assumption of Persson theory for the set of low penetrations, the contact semi-angle value must be large enough [38], which occurs only for clearances lower than 200 μm. For a clearance of 200 μm a maximum contact semi-angle value around of 60º is obtained, decreasing continually with the increases of clearance until a value of 24º, which corresponds to the highest clearance under analysis.

In what refers to the behavior presented by the Johnson model and for clearances lower than 100 μm, its validity domain is restricted to maximum loads of 1743; 2614; 3486; 6972; 17431 and 26147 N/mm. For clearances higher than 75 μm the Johnson model is valid for all pairs penetration depths/clearances tested. Similar to the observed set of low penetrations, and for the same penetration depth, high value of normal loads are obtained using the Johnson model or the Enhanced model when compared with those achieved by the Persson model, within its validity domain, or the C.-S. Liu et al.. This means that also for the high penetration depths using Johnson or the Enhanced models a stiffer contact is obtained. The differences observed between these two models are justified by the fact that to produce the curves normal load versus penetrations corresponding to the Johnson model, the normal loads obtained by the Persson model are used. If the Enhanced model is used instead of Persson model the agreement with Johnson model is largely improved, mainly for higher clearance values, due to Persson model validity domain.

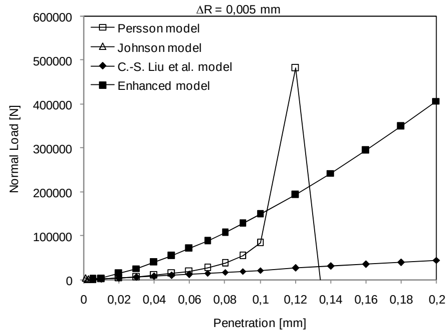

Fig. 2 (a) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.005 mm

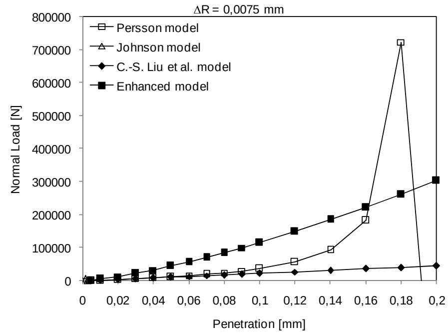

Fig. 2 (b) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.0075 mm

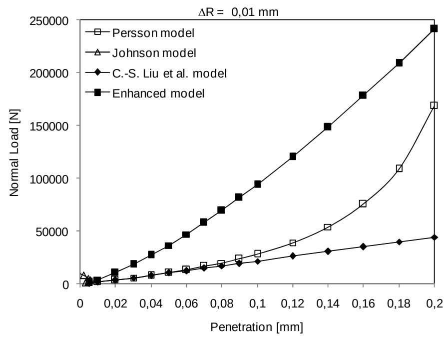

Fig. 2 (c) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.01 mm

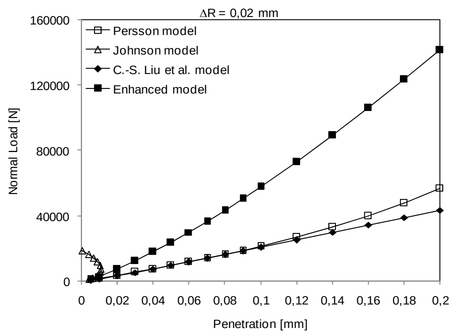

Fig. 2 (d) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.02 mm

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:7, No:5, 2013

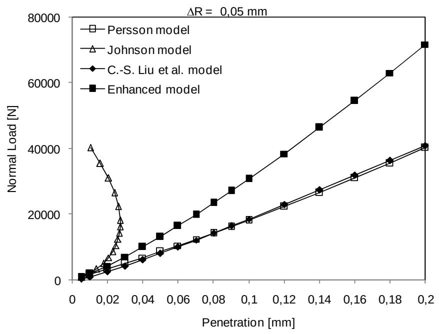

Fig. 2 (e) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.05 mm

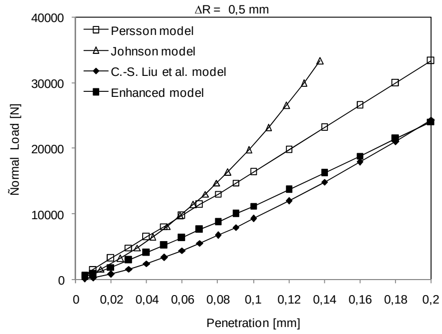

Fig. 2 (i) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.5 mm

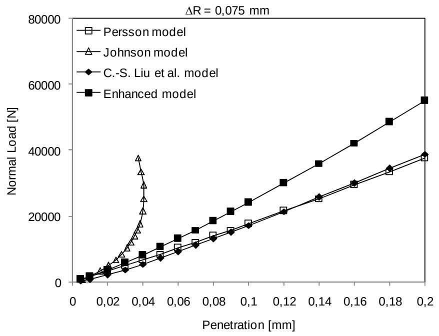

Fig. 2 (f) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.075 mm

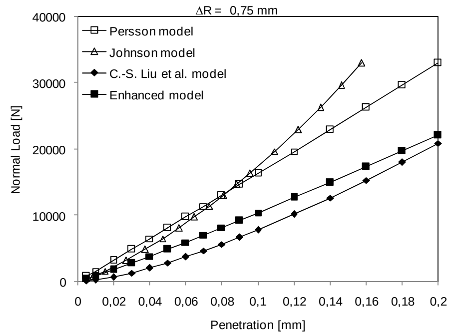

Fig. 2 (j) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.75 mm

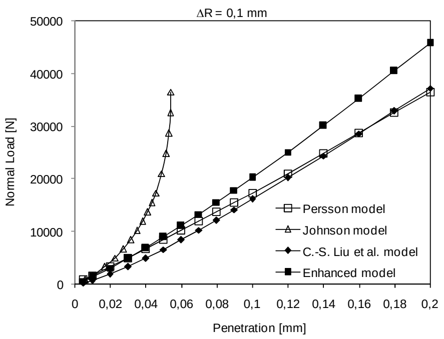

Fig. 2 (g) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.1 mm

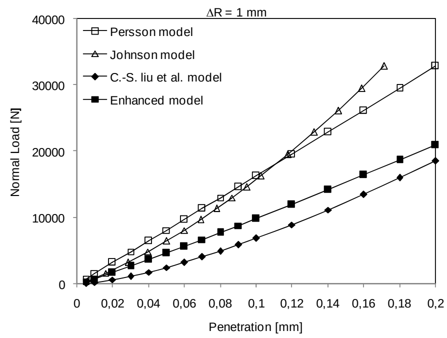

Fig. 2 (k) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 1 mm

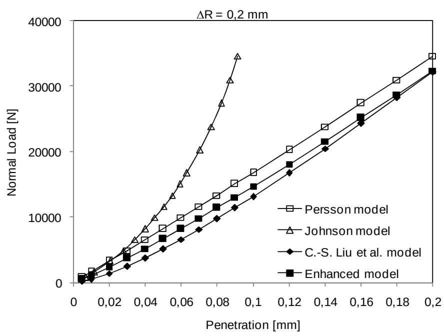

Fig. 2 (h) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 0.2 mm

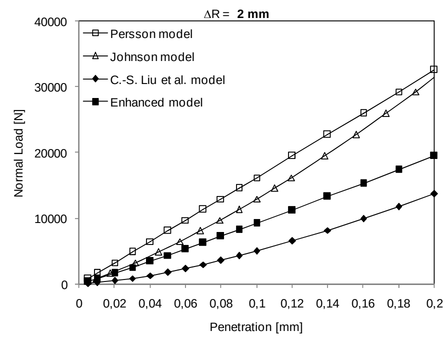

Fig. 2 (l) Comparison of the different contact force models for the set of high penetrations and for a clearance value of 2 mm

For the set of high penetration depths and for low clearance values it can be concluded that the model proposed by Johnson presents a very limited validity domain due to the physical and mathematical limitations associated with the logarithmic function that describe it. Related to the contact semi-angle value, the model derived by Persson also has restrictions on its application, in particular for the cases of very low clearances associated with high penetration values and also for high clearances. In what C.-S. Liu et al. model concerns and regardless of clearance values, the same penetration always leads to greater penetrations that those achieved with other models, in particular with the Enhanced model which produced a stiffer contact.

Regardless of the results presented here, and from a practical standpoint, the set of high penetration depths should be evaluated if contact conditions such as the combination low clearances/high loads and high clearances/low loads make physical sense in common engineering applications. In fact, for the first combination extremely high normal contact loads are required to produce high penetrations, while for the second the limit of plastic deformation is quickly reached for very low loads. But for common engineering practice will these usually be contact conditions? Perhaps not.

IV. Conclusion

A comparative assessment of the behavior presented of some actual cylindrical contact force models is done in this work, for different contact conditions, i.e., for very different penetration depths and clearance values. It can be concluded that the application of both the Johnson and the Persson models require special attention due to their validity domain. It is demonstrated that the validity domain of the Johnson model depends on the clearance and material properties values and that decreases with the decreasing of clearance and/or with the increasing of contacting elastic materials properties. Thus, special attention is required in the application of this model, particularly for contacting conditions with low clearance values and high penetration depths, since its validity domain depends on the value assumed by the logarithmic function that defines the model. Otherwise, the Enhanced cylindrical contact force model proposed by the authors does not have validity domain problems and since the pseudo-stiffness is defined as a constant, the term that accounts for energy dissipation, in the form suggested by Lankarani and Nikravesh, can be easily added to the model. It is additionally demonstrated that it is an appropriate and suitable model for modeling the cylindrical conformal contact becomes a good alternative to the Johnson model for modeling the contact between cylindrical geometries, mainly for its implementation in a computational code for the dynamic analysis of multibody systems.

In what the Persson model concerns, it is verified that its validity domain depends on the contact semi-angle value, i.e. on clearance and penetration values. For penetrations between of 0.001 to 10 μm, it should only be applied in the case that there is a small clearance and the contact semi-angle is large enough. But for penetrations between 5 and 200 μm and for very low clearances, which corresponds to very high contact semi-angle value, the normal load increases exponentially reaching a maximum, assuming negative values beyond this value, which is physically inconsistent. Thus, related to the contact semi-angle value, the model derived by Persson also has restrictions on its application, in particular for the cases of very low clearances associated with high penetration values and for high clearances.

Concerning the C.-S. Liu et al. model it has been demonstrated that, for the same penetration depth, smaller normal loads than those reached with the Johnson and the Enhanced models are always obtained. This means that using the C.-S. Liu et al. model the contact stiffness and, as a result, the contact forces are underestimated. The Johnson and the Enhanced models lead to a stiffer contact, increasing this stiffness with the decreasing of clearance, i.e. with the increasing of contact conformity. The C.-S. Liu et al. model presents, however, an opposite tendency, the contact stiffness decreases with the increasing of contact conformity. For this reason, this model represents rough approximation to describe the contact between cylindrical geometries, in particular for conditions of conformal contact.

Acknowledgment

This research is sponsored by FEDER funds through the program COMPETE – Programa Operacional Factores de Competitividade – and by national funds through FCT – Fundação para a Ciência e a Tecnologia –, under the project PEst-C/EME/UI0285/2011.

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