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Validity Domains of Beams Behavioural Models: Efficiency and Reduction with Artificial Neural Networks

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Abstract — In a particular case of behavioural model reduction by ANNs, a validity domain shortening has been found. In mechanics, as in other domains, the notion of validity domain allows the engineer to choose a valid model for a particular analysis or simulation. In the study of mechanical behaviour for a cantilever beam (using linear and non-linear models), Multi-Layer Perceptron (MLP) Backpropagation (BP) networks have been applied as model reduction technique. This reduced model is constructed to be more efficient than the non-reduced model. Within a less extended domain, the ANN reduced model estimates correctly the non-linear response, with a lower computational cost. It has been found that the neural network model is not able to approximate the linear behaviour while it does approximate the non-linear behaviour very well. The details of the case are provided with an example of the cantilever beam behaviour modelling.

Keywords — artificial neural network; validity domain; cantilever beam; non-linear behaviour; model reduction.

I. Introduction

A MODEL is an abstraction of reality and no model represents it perfectly [16]. In mechanical engineering, as in other areas, it is possible to find a number of models to simulate the same phenomenon. These variety is created for many reasons. Mainly, because a model represents only a limited view of reality: some aspects of reality are incorporated, others are leaved out. Also, because disposing of several models permits the engineer to select an more efficient or more adequate model. While that is true, another reason is that the same phenomenon may be modelled differently according the activity where it is to be employed; e.g. within an off-line analysis, models are essentially accurate and precise; while within an on-line simulation, models are primarily fast. Also, *specific* models are usually developed to be more efficient than their *generic* counterparts; but not without a penalty —typically the diminution of the applicability domain. The domain where a model can be applied limits its validity domain, since only there its validity can be assessed. In many situations (as in the reuse of mechanical models, see [40]), a model should always be accompanied of its validity domain in order to be used. Moreover, the validity domain must be verified if the model is modified: during the application of a model reduction to beam behavioural models, a modification of the resulting validity domain was noticed. While using model reduction techniques to create a more efficient model (lower time of response with a negligible loss of accuracy), changes in the validity domain must be expected.

In this paper, the case of Artificial Neural Networks (ANN) employment as model reduction technique for beam behavioural models is considered. The efficiency and validity domain of the reduced model are studied to a means to support decision making in the successful use of models:

- A shortening of validity domain after reduction of the original model is reported.
- An improvement of the efficiency of the model after reduction is reported.

The next section (sect. III) provides a background of the utilisation of ANN in mechanics and engineering, and as model reduction techniques. Section IV presents some behavioural models for beams and discusses their validity domain. Section V illustrates the application of ANN-based model reduction to a cantilever beam case. Section VI discusses the resulting efficiency and validity domain of the cantilever beam case.

II. Problem statement

A recent trend in the creation of virtual prototypes for product design is the inclusion of interactivity. Virtual prototypes are digital representations or simulations of the product concept. Simulation of interactive prototypes (or interactive simulations) can be used to explore and experiment product concepts according to the expertise and intuition of the designer[9] and the future user. Similarly, it has been suggested that the use of interactive simulation shall speed up the findings and reviewing of concept design in the early stages of the development process [5]. Interactivity in the virtual prototype is its capability to simulate the human's interaction with the design. In the past, the effectiveness of an interactive virtual prototype was limited to the following features: realistic visualisation, geometry-related constrains, and realistic simulation of physical behaviour [39]. However, human-product interaction should be included [36] as well as real-time processing and rendering [29], [5] to maintain the

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illusion of realism in the simulation [45]. In fact, as stated by Liu et al. [28], the key problem of virtual prototyping is how to build credible VP models. Today, virtual prototyping for product design must provide interactive simulation that ensures: realism (visual and behavioural), fast processing (computation of models), and integration of the human-object interaction. Also, extensible and reusable models are desired to simulated different design alternatives with a minimal effort. Therefore, the interactive simulation must reflect the following features:

- Accuracy and appropriate speed. Visualisation and simulation of physical behaviour must be accurate to provide a realistic reliable experience to the user [39], and fast enough to maintain the sensation of immersion [45].
- Human integration. Object-object interactions as well as also human-object interaction must be integrated. [45], so that the designer is able to explore and experiment the future user reaction with the design alternatives.
- Extensibility and reusability. Quickly integration of changes in the virtual prototype [39] and easy derivation of virtual prototype variations [13] allow the creation of prototypes for the different design alternatives.

In the current research, exploration of the interactive simulation models is performed. It aims to develop a modelling methodology with the features mentioned above, except for the realistic visualisation.

In this study, the diversity of behavioural models for component simulation is addressed. The efficiency of a model within is validity domain is studied in the search of realistic real-time simulations.

III. Background on ANN in mechanical modelling

Neural networks are a computational approach to build models where complexity or lack of information of the problem make the development of a classical model more difficult. They have found great acceptance in function interpolation and approximation [10], [27], [33]. Moreover, they have been successfully used in engineering [41] in different areas: in mechanics of structures and materials [42], [11], [46]. For modelling: in physics-based modelling [14], in model updating [4].

Many behavioural models are governed by differential equations, as in the case of the cantilever beam. Some general application of neural networks to solve differential equations are found in: linear ordinary differential equations by feedforward neural networks [20], [21], artificial neural networks for solving ordinary and partial differential equations [24]. With the Finite Elements Method: solving partial differential equations in real-time using artificial neural network signal processing as an alternative to finite-element analysis [38], MLP networks for differentiation of finite-element solutions [8], finite element analysis based Hopfield neural network model for solving nonlinear electromagnetic field problems [15], FEM-based neural-network approach to non-linear modeling with application to longitudinal vehicle dynamics control [22], direct solution method for finite element analysis using Hopfield neural network [43], the use of neural networks combined with FEM to optimize the coil geometry and structure of transverse flux induction equipments [44], the use of finite elements and neural networks for the solution of inverse electromagnetic problems [30].

As presented above, neural networks have been used is a vast range of applications related to mechanical engineering in a way or another. But the most important characteristics of neural networks (parsimony, non-linear relations with linearised connections) make them good candidates for reduction of non-linear models.

A. *Beam modelling by neural networks*

Artificial neural networks have been used specifically for modelling beams. ANN were used in [26] for an efficient clamped-clamped microbeam model for the non-linear dynamic response of MEMS, reduced by a neural network method. However, In [1], two neural networks were used to estimated the static response of a large deflection cantilever beam. Its validity domain included the small displacements and the great displacements domains. See Sect. IV for further information about validity domains. Although their proposed model was the fastest compared to four models (linear, elliptic, reversion and numeric), its accuracy was the worst. There was no further information that would lead to understanding the reasons of the lack of accuracy.

Nonetheless, this work supposes that neural networks are a good option to model reduction of beam behavioural models, and it investigates one of the possible reasons of lack of accuracy.

IV. Beams behavioural models and their validity domains

Beam behaviour depends on the geometry behaviour (displacements, deflections), the material behaviour (elasticity, plasticity), and the forces (independent of displacements, follower forces—non-linear functions of displacements).

As a result, many models have been developed and they correspond to different validity domains. The following list presents some beam models grouped by their validity domain (Table IV). For illustration purposes, only geometric and material aspects are included, and no specific values are given.

Geometric domains are related to displacements and deformations. Change in geometry as the structure deforms is taken into account in setting up the strain-displacement and equilibrium equations [12]. Material behaviour depends on current deformation state and possibly past history of the deformation. Other constitutive variables (pre-stress, temperature, time, moisture, electromagnetic fields, etc.) may be involved.

A. *A cantilever beam*

A long thin cantilever beam, statically charged on the free end, is to be modelled for interactive simulation (see Figure 1).

The cantilever beam is considered of uniform rectangular cross section made of a homogeneous and isotropous elastic material, that follows a linear elastic constitutive law. Only

TABLE I
BEAM BEHAVIOURAL MODELS CLASSIFIED BY THEIR VALIDITY DOMAIN.

Domain	Indicator	References
Geometric Domains		
Small displacements and small deformations	$\delta \ll L$; $\epsilon_{eq} \leq 1\%$	[47], [17], [6]
Great displacements and small deformations	$\epsilon_{eq} \leq 1\%$	[32], [35], [31], [25], [6], [1], [7], [19], [3], [34]
Great displacements and great deformations	$\epsilon_{eq} > 1\%$	[23]
Material Domains		
Elasticity	$\sigma_{eq} \leq \sigma_{elast}$	[18]
Elasto-plasticity	$\sigma_{0.2}, \epsilon plast = 0.2\%$,	[25]
Plasticity	$\sigma_{eq} \geq \sigma_{plast}$	[37]

TABLE II
DATA OF THE CANTILEVER BEAM [6].

Description	Value
Length L	300 mm
Width b	30.4 mm
Height h	0.78 mm
Moment of inertia I	1.20×10^{-12} m^4
Young's modulus E	200 GPa
External force F	3.92 N

small deformations are accepted, but large deflections may appear. Since large rotations move away the current configuration ($\mathcal{C}^D$) from the base configuration ($\mathcal{C}^0$), a linear model cannot be used but only for small rotations. A total lagrangian (TL) formulation model is accurate and precise enough; but the computing time exceeds the acceptable threshold for an interactive simulation since it requires an iterative solution process (usually a variant of the Newton-Raphson is used).

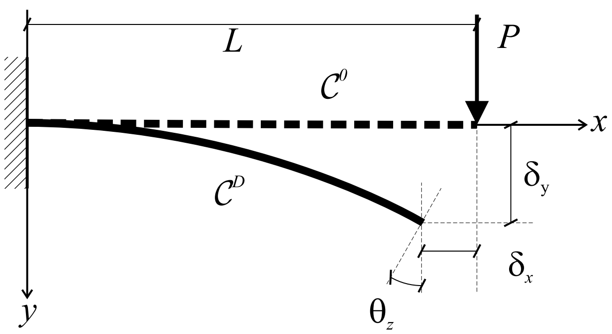

Fig. 1. Long deflection cantilever beam problem for flexible modelling

Table II resumes the data of the cantilever beam used as the test case. It is analogous to the problem experimented in [6]. Their results where validated experimentally. Table III presents the resulting displacements at the free end of the beam (which correspond to the maximal values).

It is important to consider the validity of a model when used in a particular domain. For the beam described above, the non-linear model (TL formulation) is clearly more accurate than the linear model (see Fig. 2). However, the linear model is normally faster than the former.

The models presented above are the base of the creation and comparison of the reduced model. Both models are presented in the following sections.

TABLE III
DISPLACEMENTS AT THE FREE END OF THE CANTILEVER BEAM. NUMERICAL RESULTS OF THE REFERENCE MODEL, VALIDATED EXPERIMENTALLY [6].

Displacements	Response of the reference model
δ_x	31.4 mm
δ_y	121.6 mm
θ_z	36.09°

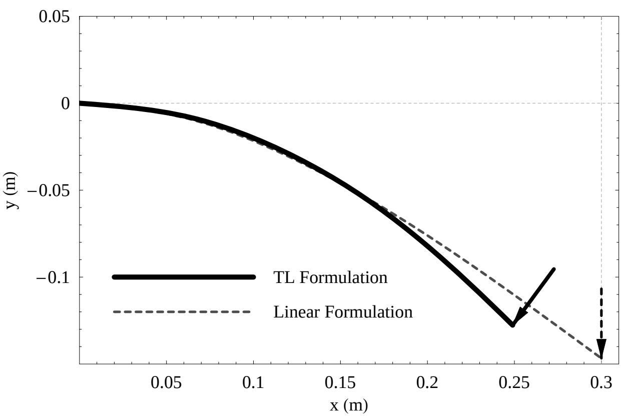

Fig. 2. Comparison of a non-linear model against a linear model in the behaviour of a cantilever beam under great displacements.

B. Selected beam models

The three models are organised in Table IV and in Table V. Their known accuracy, speed and validity domains are included in Table IV; while their definition is presented in table Table V .

The linear model [47] corresponds to the Euler-Bernoulli Beam Theory. The non-linear model [19] only takes geometric non-linearities into account.

V. MODEL REDUCTION OF THE BEHAVIOURAL MODEL OF A CANTILEVER BEAM

In this section, the construction of a reduced behavioural model for a cantilever beam is presented. Artificial neural network is used as model reduction technique. To introduce the concept of reduced model, short definitions of model and model reduction are presented as follows.

Let's define a model and its reduction as follows:

A *model* is a mathematical relation that links the changes of a given response to the changes of one

TABLE IV
BEHAVIOURAL MODELS FOR THE CANTILEVER BEAM. VALIDITY DOMAINS AND REPORTED PERFORMANCE.

Model	Validity Domain	Accuracy	Speed
Linear [47] φ_l	Small displacements	Regular	Very fast
Non-linear [19] φ_{nl}	Small and great disp.	High	Slow

TABLE V
BEHAVIOURAL MODELS FOR THE CANTILEVER BEAM. DEFINITION.

Model	Definition
Linear [47]	$\mathbf{K}\,\mathbf{u} = \mathbf{f}$ $\underbrace{\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI_{zz}}{L^3} & \frac{6EI_{zz}}{L^2} & 0 & -\frac{12EI_{zz}}{L^3} & \frac{6EI_{zz}}{L^2} \\ 0 & \frac{6EI_{zz}}{L^2} & \frac{4EI_{zz}}{L} & 0 & -\frac{6EI_{zz}}{L^2} & \frac{2EI_{zz}}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI_{zz}}{L^3} & -\frac{6EI_{zz}}{L^2} & 0 & \frac{12EI_{zz}}{L^3} & -\frac{6EI_{zz}}{L^2} \\ 0 & \frac{6EI_{zz}}{L^2} & \frac{2EI_{zz}}{L} & 0 & -\frac{6EI_{zz}}{L^2} & \frac{4EI_{zz}}{L} \end{bmatrix}}_{\mathbf{k}_l^e}$ $\mathbf{u} = \varphi_l\left(\mathbf{f}, (p_g, p_m)\right)$
Non-linear [19]	$\mathbf{K}(\mathbf{u})\,\mathbf{u} = \mathbf{f}$ $\mathbf{k}_{nl}^e = \mathbf{k}_l^e + \mathbf{k}_{nlgeo}^e\,;\; N = \frac{EA}{L}\left(L' - L\right)$ $N \underbrace{\begin{bmatrix} 0 & & & & & \\ 0 & \frac{6}{5L} & & & \text{sym.} & \\ 0 & \frac{1}{10} & \frac{2L}{15} & & & \\ 0 & 0 & 0 & 0 & & \\ 0 & -\frac{6}{5L} & -\frac{1}{10} & 0 & \frac{6}{5L} & \\ 0 & \frac{1}{10} & -\frac{L}{30} & 0 & -\frac{1}{10} & \frac{2L}{15} \end{bmatrix}}_{\mathbf{k}_{nlgeo}^e}$ $\mathbf{u} = \varphi_{nl}(\mathbf{f}, \mathbf{u}, (p_g, p_m))$

or many factors.

The *reduction of a model*, or model reduction, is obtaining an equivalent mathematical relation generated from the features of the connections between the changes of a given response and the changes of one or many factors.

As a consequence, it is possible to establish —within this context— that a *reduced model* is another equivalent transformation of the original model.

A. Reduced non-linear beam model

It is assumed that the elastostatic response of a mechanical system can be simulated with more realism by using a reduced model that ensures an appropriate accuracy-speed ratio. In this test case, ANN modelling is used as a technique to replace the non-linear model (see IV-A). The learning capability of neural networks provides an alternative path to obtain the force-displacement connections of the non-linear model. An important feature their capability to approximate non-linear functions by using less variables and linear relations among them.

a) ANN Description.: The architecture to be used is the multilayer perceptron (MPL). This is a network of n_{hl} hidden layers, where every neuron is totally connected with the neurons of the next layer. The first and final layers are dimensioned (i.e. the number of neurons within) accordingly to the size of input and output vectors, respectively.

$$\mathbf{u} = \varphi_{ann}\left(\mathbf{f}, p_g, p_m, c_{ann}\right) \;, \qquad (1)$$

where c_ann is the configuration of the network:

- number of hidden layers,
- dimension of each layer,
- transfer function of each layer.

For the cantilever beam, a possible structure of MLP network is presented in Figure 3.

Backpropagation (BP) learning [2] has been selected as the training algorithm as it is a well known technique used for function approximation. Even if the architecture and learning algorithm are selected, configuring and tuning a neural network and capable to properly response to unknown data is not simple: establishing the appropriate configuration (the number of hidden layers, their dimension and the transfer function) is a complicated task. This is the reason that has fostered the employment of optimisation techniques to find a nearly optimal structure of a neural network to a given problem. Here, the selected technique is based on genetic algorithms (GA).

b) ANN Construction.: In this work, defining the appropriate configuration of a neural network can be seen as an optimisation problem. An optimisation technique is used to automatically define the structure and overall configuration of the ANN to model the cantilever beam. The neural network is built as follows.

Genetic algorithms are a particular class of algorithms that use techniques inspired by evolutionary biology. The

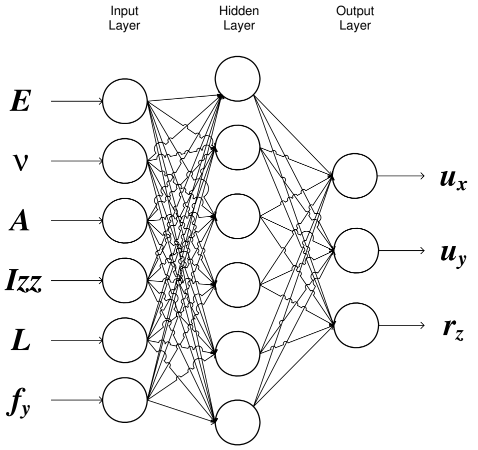

Fig. 3. Non-linear reduced model by means of an artificial neural network. Material and geometric properties as well as the point force are the input to the model; displacements are the output. The configuration of the boundary conditions is fixed.

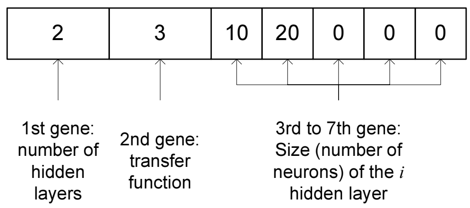

Fig. 4. Abstract representation of a candidate solution (an individual) for the neural network selection. The structure of the chosen network is given by the number of hidden layers (gene 1), their dimensions (genes 3 to 5), and the transfer function (gene 2).

genetic algorithm technique is about searching one of the best individuals of a population of potential solutions; in this case, one of the neural networks for the reduced modelling of static behaviour. Population is composed of the candidate configurations of MLP networks (i.e. hidden layers, neurons by layer, transfer functions). In certain cases, also the training algorithm could be variable in the search of an optimal network. An example of the structure of an individual or candidate solution is shown in Fig. 4. The structure of the chosen network is given by the number of hidden layers (gene 1), their dimensions (genes 3 to 5), and the transfer function (gene 2). The transfer function is indexed from a list of known functions. Some of the functions commonly used are: linear function, logarithmic sigmoid function, radial basis function, tangent sigmoid function, triangular basis function.

The objective of the genetic algorithm is to minimize the residual error in the verification of a candidate neural network.

The neural network selection process for the case presented in section IV-A was executed on a computer for 20 hours (1.81 Ghz processor and 1 Gb RAM). The winning individual was coded as: $(2, 4, 8, 12, 0, 0, 0)$, 2 hidden layers (8 and 12 neurons) with tangent sigmoid transfer function.

c) ANN Specification.: After executing the genetic algorithm, the resulting configuration of the employed neural network is shown in Table VI.

The ANN model is specified in terms of weights and transfer functions (2):

$$\begin{aligned}
\mathbf{u} &= \varphi_{ann}\left(\mathbf{f}\right) \\
\mathbf{u}_j &= \mathrm{Lin}\left(\mathbf{w}_3^{(j)}\ \mathrm{Tansig}\left(\mathbf{w}_2^{(j)}\ \mathrm{Tansig}\left(\mathbf{w}_1^{(j)}\mathbf{f}\right)\right)\right) \\
&\quad \mathrm{Lin}(\mathbf{x}) = \mathbf{x} \\
&\quad \mathrm{Tansig}(\mathbf{x}) = \frac{2}{1+e^{-2\mathbf{x}}} - 1 \\
&\quad j = 1, 2, \ldots, n
\end{aligned} \tag{2}$$

It is important to emphasize that the ANN model (2) provides a representation $\mathbf{u} = \varphi_{ann}\left(\mathbf{f}\right)$ of the non-linear model $\mathbf{u} = \varphi_{nl}\left(\mathbf{f}, \mathbf{u}\right)$ that has been created with different connections between the input $(\mathbf{f})$ and the output $(\mathbf{u})$ without the need of iterations. This is, the dependency of the displacements with themselves and the forces has been reduced to a dependency of the forces only. The non-linear mapping has been captured in the weights of the ANN $(\mathbf{w}_i^{(j)})$.

B. Validity domain of the reduced model

As stated in sect. IV-A, the validity domains considered in this study correspond only to geometric aspect of the beam behaviour, since the material is considered ideally elastic, so the beam stays in the elasticity domain. Also, under the assumption that deformation remains small, the possible domains for the models are:

D_S	small deflections domain
$D_S \cup D_G$	small and great deflections domain
D_G	great deflections domain

The preliminary tests of the reduced model have shown that its validity domain corresponds to D_G; but it is not certain if it covers $D_S \cup D_G$. The results of the application to the cantilever beam are presented in the next section.

VI. Results and Discussion

The behaviour of a cantilever beam has been modelled with three different models: a non-linear, a linear, and a non-linear ANN-reduced model. Their different performance is discussed in the following sections as well as the interest of ANN-reduced model.

A. Validity domain

While the linear model is known to be valid only under small displacements ($\theta_z < 15°$, for this case [6]), the non-linear model is valid under small displacements and great displacements (but small deformation). Sect. V-B Figures 5, 6, and 7 show a detailed view of the zone were the domain switch occur for horizontal displacement, vertical displacement and rotation at the free end of the beam. The reduced model φ_{red} provides not only inaccurate results at the configurations near the initial configuration ($\mathcal{C}^0, F_y = 0$), but also non-logical response in the case of Figure 5. It is stated as non-logical since it provides a negative horizontal displacement while the beam is in equilibrium without the action of external forces.

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TABLE VI
CONFIGURATION OF THE NEURAL NETWORK USED TO MODEL THE NON-LINEAR BEHAVIOUR.

Element	Option
Learning algorithm	backpropagation
Architecture	multi-layer perceptron
Structure	3 layers (8, 12 et 3 neurons), which implies 2 hidden layers
Transfer function	tangent sigmoid (hidden layers) and linear (output layer)
Training epochs	300
Target error	1×10^{-7}

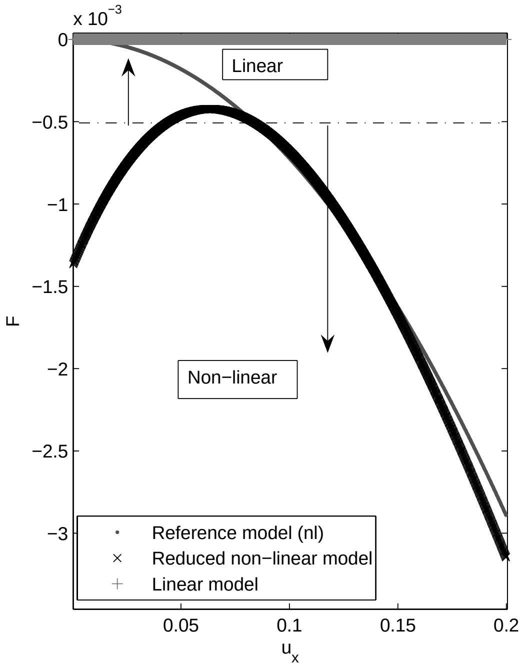

Fig. 5. Comparison of horizontal deflection δ_x estimation at the free end of the beam

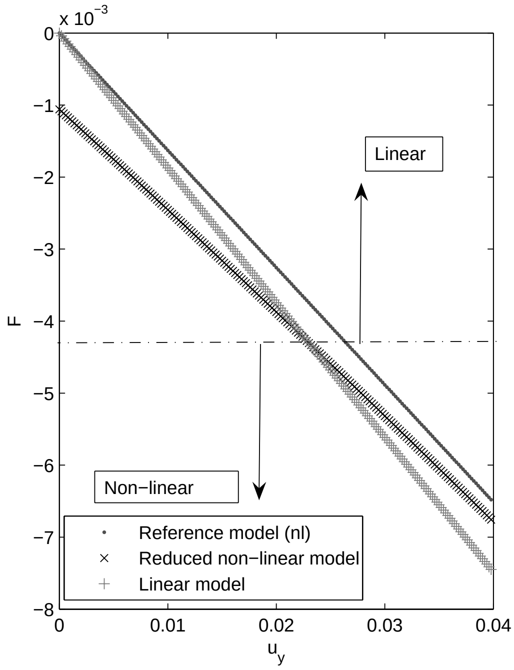

Fig. 6. Comparison of vertical deflection δ_y estimation at the free end of the beam

In that case, the linear model is more "accurate" even if it always estimates the horizontal displacement as zero (see top of figure 5). Also, it is possible to see in Figures 6 and 7 that the linear model approximates better the non-linear response than the reduced model. It is evident that the non-linear reduced model is not capable to estimate the beam behaviour under small displacements.

The resulting validity domains are concentrated in Table VII.

B. *Efficiency*

It has been defined, previously in this section, that the reduced model was only valid in the great displacements domain. In its validity domain, the interest relies in how the model performs compared to the original non-linear model.

The reduced model presents a loss of accuracy compared to the non-linear model. However, the gain in speed is as expected: almost as fast as the linear. Although, the linear model is not valid in this domain ($\theta_z > 15°$, as stated in [6]), it is include for speed comparison. The estimated error and response time t_{calc} for the three models are reported in Table VIII.

As presented above, the behavioural model reduced by ANN has shown that it is a fast alternative to be used in interactive simulations. Its accuracy is not an issue if the ANN-reduced model is to be used only within its validity domain. In fact, the small loss of accuracy compared to the original non-linear model is negligible for an interactive simulation.

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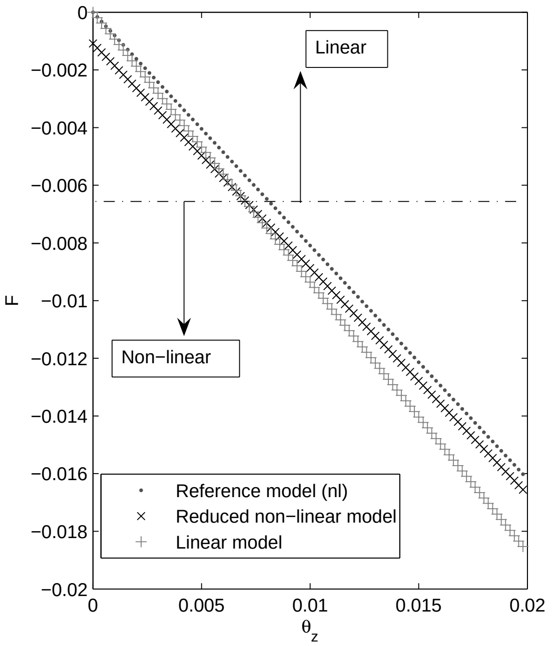

Fig. 7. Comparison of rotation θ_z estimation at the free end of the beam

TABLE VII
VALIDITY DOMAIN OF THE BEAM MODELS. SEE SECT. V-B FOR A DESCRIPTION OF THE DOMAINS.

Model	Validity domain
Linear	Small deflections, D_S
Non-linear	Small and great deflections, $D_S \cup D_G$
Non-linear reduced	Strictly great deflections, D_G

VII. CONCLUSION

A validity domain study is reported in the context of a reduced cantilever beam static behaviour model. The original model is reduced by a multi-layer perceptron network giving an alternative model that is more efficient only in a reduced domain. The validity domain of the reduced model is emphasised as a delicate aspect to verify in the application of ANN as model reduction techniques. In short, for the behavioural modelling of cantilever beam, the application of this reduction technique (based on ANN) has provided a good model (accurate and fast) but with the limitation that it is only valid in a non-linear geometric domain. Thus, for an interactive simulation this model could be used alternatively with the linear model. A dynamic selection strategy is needed to change between those models.

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TABLE VIII
EVALUATION OF VERTICAL DEFLECTION AND ROTATION ERROR AT THE FREE END FOR A POINT FORCE OF 3.92N. ANALYTICAL/EXPERIMENTAL RESULTS OF [6] ARE TAKEN AS REFERENCE.

Model \ Indicator	ε_{δ_x} (%)	ε_{δ_y} (%)	ε_{θ_z} (%)	t_{calc} (s)
Linear (FEM)	100	20.6	16.5	0.0010
Non-linear (TL) [19]	62.4	5.0	0.6	1.050
Reduced (ANN)	67.2	5.12	0.70	0.0015

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APPENDIX

NOMENCLATURE

A	area of the cross section (m^2)
ANN	artificial neural network
MLP	multi-layer perceptron
$\mathcal{B}$	beam definition
$\mathcal{C}^0$	undeformed configuration
c_{ann}	configuration of the neural network
$\mathcal{C}^0$	deformed configuration
δ	displacement (m)
E	Young's modulus (MPa)
ϵ_{eq}	equivalent deformation (%)
ϵ_{plast}	plastic deformation (%)
$\mathbf{f}$	sollicitations (N)
I_{zz}	moment of inertia of the cross section (m^4)
$\mathbf{k}_l^e$	linear element stiffness matrix
$\mathbf{k}_{nl}^e$	non-linear element stiffness matrix
$\mathbf{k}_{nlgeo}^e$	non-linear geometric contribution to the element stiffness matrix
L	length of the beam (m)
L'	length of the deformed beam (m)
ν	Poisson's ratio
p_g	geometric properties
φ	behavioural model
φ_{ann}	ann-based reduced model
φ_{flex}	flexible model
φ_l	linear model
φ_{nl}	non-linear model
φ_{red}	non-linear reduced model
p_m	material properties
$\sigma_{0.2}$	elasto-plastic transition with $\epsilon_{eq} = 0.2\%$ (MPa)
σ_{elast}	elastic limit (MPa)
σ_{eq}	equivalent stress (MPa)
σ_{plast}	plastic limit (MPa)
θ_z	rotation at the free end of the beam (rad)
$\mathbf{u}$	displacements (m)
$\mathbf{w}_i^{(j)}$	weights of the ANN connections