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Using Interval Constrained Petri Nets for the Fuzzy regulation of quality: case of assembly process mechanics

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Abstract**—**The indistinctness of the manufacturing processes makes that a parts cannot be realized in an absolutely exact way towards the specifications on the dimensions. It is thus necessary to assume that the effectively realized product has to belong in a very strict way to compatible intervals with a correct functioning of the parts. In this paper we present an approach based on mixing tow different characteristics theories, the fuzzy system and Petri net system. This tool has been proposed to model and control the quality in an assembly system. A robust command of a mechanical assembly process is presented as an application. This command will then have to maintain the specifications interval of parts in front of the variations. It also illustrates how the technique reacts when the product quality is high, medium, or low.

Keywords**—**Petri Nets, Production rate, Performance evaluation, Tolerant system, Fuzzy sets.

I. Introduction

Studies on design or scheduling of time constrained workshops focus most of the times on the optimization of the production rate. Processes of these workshops are usually subjected to temporal constraints. [1], [2] Most of these studies are based on Petri Nets models, which are well fitted for the study of robustness properties, where the staying duration of the products has to be guaranteed even when there are disturbances.

However, in many processes, the specified constraint is not time. For instance, the 'conformity intervals' per the geometry of parts to be produced in the assembly mechanical systems, has to strictly belong to a well defined validity interval. Several critical parameters in this process impact the quality of the product.

The components of a mechanical system are with difficulty separable and their mutual influence is principally important. There are several different causes of failure of a mechanical system. The one which is considered here are: Failure due to the manufacturing: not respected tolerances, degraded state of surface, etc.

We look for a robust command to manage the variations of the process by associating, in its representative model, a mechanism of follow-up and control of the specifications over the dimensional and geometrical tolerances of parts to be assembled.

We modify Interval Constrained Petri Nets as a tool for fuzzy modelling. Basic concepts and relations between Fuzzy Petri Nets and Fuzzy IF-THEN rules will be described and algorithm for decomposition of fuzzy Petri net into set of linguistic descriptions will be presented and its implementation. At first, we present the Intervals Constrained Petri Nets (ICPN). This tool presents a complement to the P-temporal Petri nets. It allows the modeling of any unspecified parameter in a manufacturing process.

An example of assembly process mechanic is presented at the end, for illustrate the interest of the proposed modelling and analysis.

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II. Modeling of the Systems of Assembly

In many assembly manufacturing systems, production of parts proceeds in stages. Each stage may be seen as a production/inventory system composed of a manufacturing process and an output buffer. The manufacturing process may consist of a single machine or a subnet work of several machines. It contains parts which are currently being processed in the stage (either being waiting for or receiving processing) and are referred to as the work in process of the stage. A system of assembly, the manufacturing process is organized in various stages which group together one or several elementary operations which can be made in parallel. These operations, of transformation or assembly, must be executed in a specific order called range of manufacturing. Let us describe the assembly manufacturing systems we are interested in. Figure 1 illustrates the topology of a system having assembly levels. This topology is a tree structure. The first root of the tree represents the finished product P. The last level contains the materials first $MP1$ and $MP2$ necessary for the manufacturing of the product. $S1$ et $S2$ is components which go into the final assembly of the product while composing them. $R1$ and $R2$ represent elements obtained by transformations of the raw material. The factors which appear on branches represent the number of components necessary for the manufacturing of one unit of product (are thus needed 2 components $S1$ and 2 components $S2$ to make a unit of P).

In this paper, we are interested in the problem of synthesis of command of a manufacturing process by considering that the determining parameter, other one than the constraints of

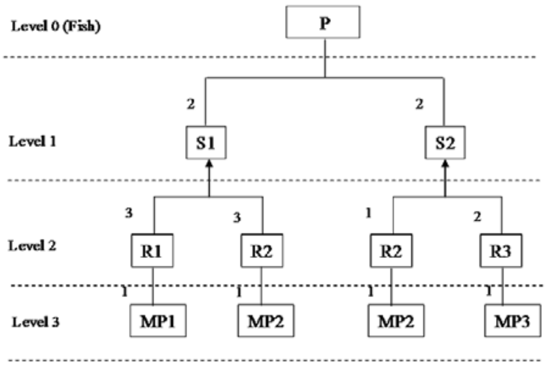

Fig. 1. Topology for assembly manufacturing systems

synchronization and cooperation of the manufacturing process which are imperative, is not the time. This parameter however has to belong in a very strict way to intervals of validity. The command will then have to give guarantees onto the respect for these specifications. That is why an appropriate model, rich in analytical properties, is necessary for the synthesis of the piloting. Indeed, the operations of assembly impose the respect for the constraints of specification on elements to be assembled such as the intervals of tolerance on the quotations and / or on the geometrical forms.

The purpose of this paper is to provide a common modelling approach based on Petri nets completed by fuzzy logic modelling and for control systems in the case of assembly mechanical systems.

The control system presented is modelled using Interval Constrained Petri Nets (ICPN) [3], [4]. This model is a sub-class of High Level Petri Nets with Abstract Marking (AM-HLPN) [5] [6]. ICPN allow one to model and guarantee a constraint on any parameter of a manufacturing process. Indeed, this modeling tool is considered as a significant research way for the determination and evaluation of robustness [7].

A. Extensions of the models PN

Classical Petri Nets are defined as a structure

$N = \prec P, P, F \succ$ where, $P = P1, P2, , Pn$ means set of places, $T = T1, T2, , Tm$ is the finite set of transitions,

$\mathrm{F} = (\mathrm{P} \times \mathrm{T}) \cup (\mathrm{T} \times \mathrm{P})$ is the set of arcs from places to transactions and from transactions to places,

$\mathrm{w} : \mathrm{F} \rightarrow \{1, 2, 3, ...\}$ is the weight function on the arcs. Graphical representation is set up by following symbols:

- Places - rings
- Transitions - rectangle
- Relations - pointers between transitions and places or places and transitions.

We won't describe any more into details the idea and properties of basic Petri Nets and for deeper understanding of this problem we recommend literature.

Several classes Petri networks have been developed, each trying to describe a "view" of production systems, their design and conduct. Among those classes, we include models that integrate the dimension of time. The extension covers the modeling systems whose behaviour depends on an explicit values time.

However, in the class of modeling tools, fuzzy Petri nets, [8], [9], [10], are very interesting tools for discrete event systems characterized by an imprecise knowledge.

B. Interval Constrained Petri Nets

Petri Nets are known to be a well fitted tool to model synchronization and parallelism for Discrete Event Systems (DES). High Level Petri Nets (HLPN) has the ability of both powerful modelling and integration of external parameters. Some works are led in order to get ISO normalization for HLPN [ISB]. The existence of maximal durations drastically modifies the performance evaluation in DES and the classical formulas used for Cmax computing can not be used. A dedicated tool was introduced in 1996 in order to integrate the staying time constraints in a Petri Net model: the P-time Petri Nets. From a specification point of view, they can be presented as a sub-class of mathematical abstraction of HLPN: Abstracted Marking Petri Nets (AM-HLPN) [5], Many industrial problems concern the control of vague values ("small, "big", position, weight,). Consequently, there was a need for a functional generalisation of the mathematical properties of P-time PN. Interval Constrained PN (ICPN) was presented to meet this need [7].

Definition 1: An ICPN is a t-uple <R, M, IS,D,Val,Val_0,X, X_0>; where:

- R is an unmarked PN,
- M being an application associating token to places as:
  m is a vector indexed on the set of places P
  Let $m(p)$ be a place marking
  Let V be a non empty set of rational variables
  Let μV be a multiset defined on V,
- IS:P$\rightarrow$ $Q \cup \{-\infty, +\infty\} \times Q \cup \{-\infty, +\infty\}$ defines the intervals associated to places
  Q is the set of rational numbers
  pi$\rightarrow$ $ISi = [\alpha_i, \beta_i]$;where $\alpha_i \leq \beta_i$,
- $D : \{M(p) \times \{p\} \, | p \in P\} \rightarrow V$
  $\forall i, 1 \leq i \leq n, n = Card(P)$
  Let k be a token, k $\in m(pi)$
  $k \rightarrow qi | \alpha_i \leq qi \leq \beta_i$
  D associates a rational local parameter to each token in a place,
- Val be an application: $M(P) \rightarrow \mu V$
  $(k \in m(p), p) \rightarrow v \in \mu V;$ where k is a given token in p
  Val associates a multi set of parameters to each token
  This multi-set is carried by the token thorough the net,
- Val_0 corresponds to initial values associated to tokens,
- X defines the evolution of the local parameter associated to each token in a place
  X: V $\rightarrow Q$
  $v \rightarrow q \in Q$,
- X_0 is the vector of initial value of variables.

A mark in the place pi is taken into account in transition validations when it has reached a value comprised between ai and bi. When the value is greater than bi the mark is said to be dead. Logically, in the firing of an upstream transition, token are generated in the output places and their associated variable are equal to:

$$Val(k) + qi(k)$$

C. Fuzzy IF - THEN Rules

Fuzzy IF-THEN rule is a concept used for describing of logical dependence between variables of the following form:

IF X_1 is A_1 AND AND X_n is An THEN Y is B where $A_1, ..., A_n$ and B are certain predicates characterizing the variables $X_1, ..., X_n$ and Y . They are often specified linguistically. We will work with specific kind of linguistic expressions as defined. The part before THEN is called the antecedent and the part after it the succedent. The variables $X_1, ..., X_n$ are called input, or independent variables. The variable Y (in general, there may be more of them, but we will limit oneself to only one) is called output, or dependent variable.

The fuzzy IF-THEN rules are usually put together to form the linguistic description

$R_1 =$ IF X_1 is A_{11} AND ...AND X_n is A_{1n}THEN $Y$is B_1

....

Rm= IF $X_1$is A_{m1}AND ...AND X_n is A_{mn}THEN $Y$is B_m

Mapping between IFTHEN rules and fuzzy Petri nets is obvious. Any IFTHEN rule of the previous form:

IF X_1 is A_1 AND ... AND X_n is An THEN $Y$is B

can be expressed by the following petri net:

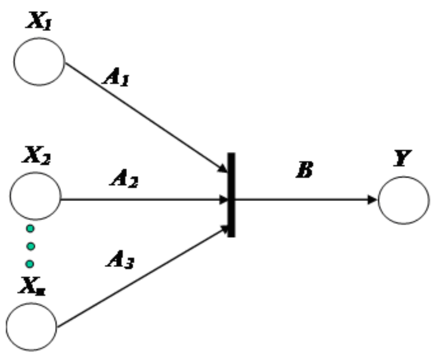

Fig. 2. Each transition of the result fuzzy Petri net corresponds to one rule of linguistic description

III. APPLICATION: CASE OF ASSEMBLY SYSTEMS

A. Workshop model

In an assembly system, we find two types of operations: the operations of transformation, which aim at the manufacturing of components, and at the operations of assembly, which combine these components to obtain more complex components or finished product. For our application, we consider the simple example of the figure 2. It is about an assembly of four parts.

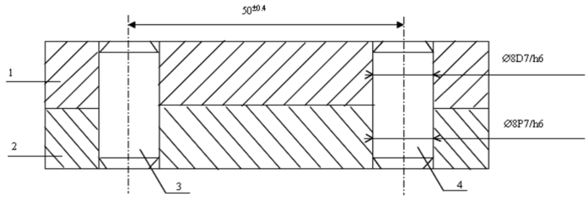

Fig. 3. Assembly parts

TABLE I
TOLERANCES OF THE OPERATIONS

Parts/dist.	Operations	Dimentions	tolerance interval
1	$O_{11} - O_{12}$	$\emptyset 8D7$	[8.044, 8.055]
2	$O_{21} - O_{22}$	$\emptyset 8D7$	[7.976, 7.991]
3	O3	$\emptyset 8h6$	[-0.2, 0.4]
4	O4	$\emptyset 8h6$	[-0.2, 0.4]
5	D5	50 ± 0.4	[49.96, 50.04]

This corresponds to the following conditions [ISO]: The minimum and the maximum tolerances of the operations are as follows:

The concerned systems are the manufacturing systems of flow-shop type. In this paragraph we are going to separately deal with the problems of flow robustness and quality robustness.

B. Process planning

Our objective is to represent the processing planning of our application by means of a Petri Net. Knowing that in the simple case where the product is obtained by a continuation of operations. For each operation is associated a validity interval. Its lower bound indicates the minimum value needed to execute the operation. The non respect of this value means that the operation was not conforming. The upper bound fixes the maximum time to not exceed otherwise the quality of the product is deteriorated. Such systems have a robustness property in order to maintain product quality when there are value disturbances. The robustness is defined as the ability of the system to preserve the specifications facing some expected or unexpected variations. So the robustness characterizes the capacity to deal with disturbances. Such a process planning will be constituted by a continuation of transitions separated by places.

In our case, the manufacturing also requires operations of assembly. The planning process is not linear any more. Such a planning process is represented by a tree called nomenclature. The figure 4 represents the nomenclature of the assembly parts (fig. 3). Parts $C1, C2, C3$ and $C4$ undergo respectively, $O11-O12$, $O21-O22$, $O3$ and $O4$. Then, parts $C2, C3$ and $C4$ are assembled to obtain a semi finished product (PS). The semi finished product and $C1$ is assembled to obtain the finished product (P).

C. Construction of ICPN models

- *ICPN model*

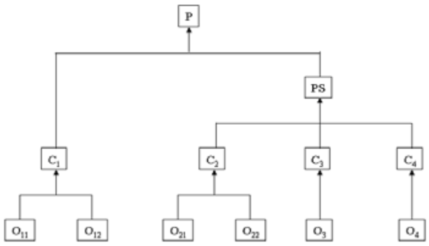

Fig. 4. Nomenclature

The list of the figure 4 can be by the network model Interval Constrained PN (ICPN) of the figure 5. In this model, place $P1$ and $P4$ represents stocks to the entries of machines. Place $P2, P5$ and $P7$ are, respectively, the operations to execute on pins, the detail 2 and the detail 1. Places are $P3, P6$ and $P8$ represents components with the finished quotations. The transitions $(t1, t4, t10)$ and $(t2, t5, t7)$ represent, respectively, the load and the unloading of machines. Place $P9$ and $P20$ is places of assembly of details. Places $P10, P12, P14$ and $P16$ represent the constraints subjected on the variation of the quotations during manufacturing.

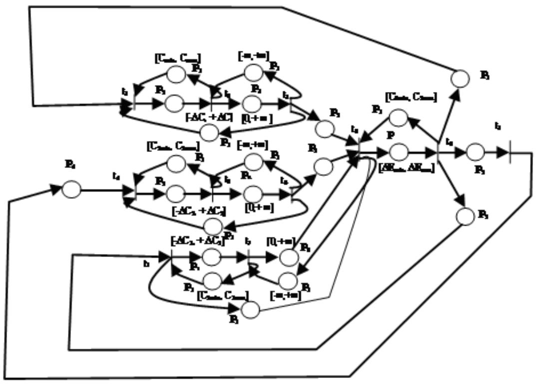

Fig. 5. Interval Constrained PN (ICPN)

- *Fuzzy ICPN model*

The fuzzy ICPN model is structured numerical estimators. It starts from highly formalized insights bout the structure of categories found in the real word and then articulate fuzzy IF-THEN rules. A token is to move among two nodes from place (or transition) to transition (or place) while transaction happens, the moving path of token is decided to the inference rules of transition Fig 6.

D. Computing the next step

We presented in the paragraph (2.2.2) an approach for a continuation of regulations allowing guaranteeing the respect for the dimensional and geometrical constraints of details to assemble in spite of the fluctuations in the process the variations on the tolerances. Indeed, in every abnormality we look for a regulation which can manage the new value to remain meanwhile of validity predefined. In this paragraph

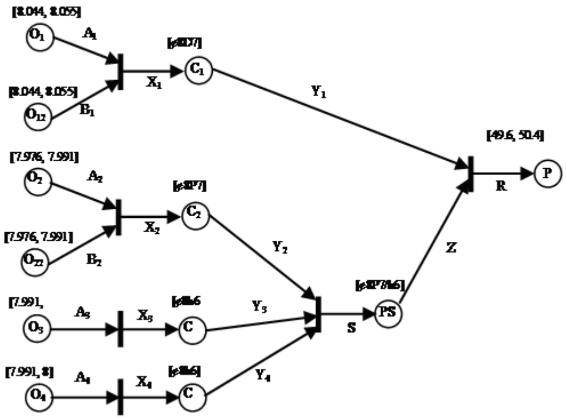

Fig. 6. Fuzzy Interval Constrained PN (FICPN)

TABLE II
SETTING INTERMEDIARIES

Step	1	2	3
Δ	[-0.4, 0.4]	[-0.35, 0.35]	[-0.3, 0.4]
Step	4	5	6
Δ	[-0.25, 0.4]	[-0.25, 0.35]	[-0.2, 0.4]
Step	7	8	9
Δ	[-0.1, 0.4]	[-0.05, 0.4]	[0, 0.4]

we an application of this proposed approach will be presented. For that purpose, we consider, for example, the variation of the quotation C5. This operation is modelled by the primary circuit $< t8, P9, t9, P16 >$ of the network of Petri of the figure 6. We have $C5 = 50$ with the constraint of $C5 \in [49.96, 50.04]$ Let us suppose that the values described by the graph of the network are the variations with regard to an average. For the first step, we find: $C5 = [-0.4, 0.4]$. The application of the theorem for various no passing regulations gives us the results described by the table II. In this case, Fixing $C = 1$, the obtained developed graph is described on the figure 6.

IV. COMPUTING THE ROBUST CONTROL

Finally, when the ICPN model of the process is completely defined, it is possible to analyse the structural properties. It was proved that the most of the structural properties of P-time PN can be extended to ICPN [4]. An internal robustness analysis of the ICPN model of the presented process is published in [6]. Finally, using the production data information, a computing methodology was applied in order to build the validity intervals of the ICPN model. This approach uses only a sub-part of the information, because we only want to find critical tests which are needed for The Designs of experiments applying in the production data. developed graph

A. Control maintaining constant dimensions

The effective value of parameter can be calculated with polynomial algorithms. This can be done because the above algorithm is only based upon the structural properties of P-time Petri Net. In this case, it was shown that, under some

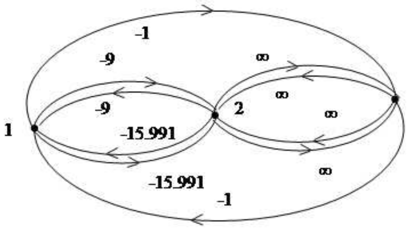

Fig. 7. Developed graph

TABLE III
FUZZY RELATION TABLE

C5		C1		
		S	M	L
PS	S	M	L	L
	M	S	M	L
	M	S	S	L

particular assumptions, the property may be extended to ICPN. A graph G' is associated with the Strongly Connected P-time Event Graph G in 1-periodic functioning mode of C period: the nodes of G' are the transitions of G, the arcs of G' are obtained from the places of G. two arcs are associated with each place p:

- the first one from p to p is valued by:

$$vp = ap - C.m(p)$$

- the second one from p to p is valued by:

$$v'p = -bp + C.m(p)$$

A periodic control of the parameters is obtained with the following algorithm:

- choose a transition ts, associate

$$Sts(1) = 0$$

- associate with each transition tu

$$Stu(1)_{l_{su}} = \max \sum_{p \in l_{su}} v_p$$

where lsu is an elementary directed path from s to u. This last algorithm is in $O(n3)$.

B. Capability verifying

Our objectif is to verify the capability of command computing in sec. 4. This command must to respect the constraint of $C5$ and predict the state change.

C5 depends upon two input variables, the dimension $C1$ and the dimension PS. Those variables are departed three degrees, which are small (S), middle (M), and large (L) defined in table I.

The inference engineer is constructed of fuzzy rule. According to the able fuzzy rule: "If $\prec inputvariable \succ and \prec inputvariable \succ THEN \prec outputvariable \succ$ ".The rages of the variables are shown in figure 8.

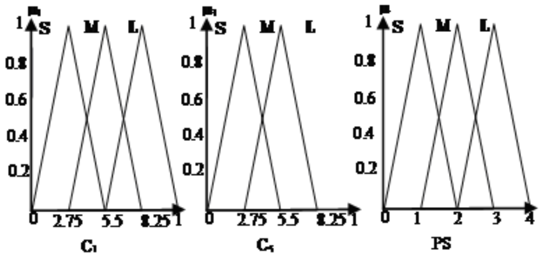

Fig. 8. Membership functions of $C1$, PS and $C5$

The relation of $C1$, PS and $C5$ variables are listed in Table II.

The real dimension $C5$ is defuzzied by the center of area (COA) defuzzification formula. The $C5$ is calculated as following steps,

$$\mathrm{zco_a} = \sum \mu \mathrm{c}(\mathrm{z_j})\mathrm{z_j} / \sum \mu \mathrm{c}(\mathrm{z_j})$$

, where $j = 1, ..., n$; n is the number of quantization levels of the output, z_j is the number of control output at the quantization level j, $\mu C(z_j)$ represents its membership value in the output fuzzy set. Refer to Figure 9 and exploit the formula of COA, we get the fuzzy set of the result $C5$ exploit dimension is shown in figure 8.

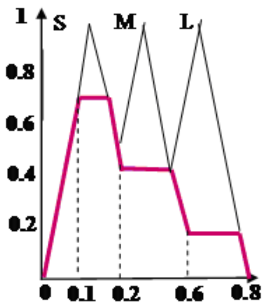

Fig. 9. The result of C5 dimension

V. CONCLUSION

In this paper, we attempt mixing two different characteristics theories, The fuzzy system and Petri net system, the ICPN model concerns processes where the conformity of the finished product depends on the value of the dimension parts by a produced unit. This value must belong to a certain validity interval. Outside this interval product is considered as rejection. To improve production performance within such a process, the control of the quality constraint must be able to adjust fluctuations that affect the system's entries. This adjustment has to be done through intermediate regulations that do not alter specifications. Regulations concern operations that influence directly the value of dimensions. In our case four values must be controlled. We also have completed ICPN model with an FICPN model, in which the transition nodes have the capability to infer and predict the state change. The AND logic operation is explicate to the fuzzy logic.

One must note that an adjustment near maximal or minimal bounds might increase rejections rates in the case of noised entries. A multicriteria evaluation, in the context of workshop presenting interval validity constraints, could resolve such a problem.

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