

# Using Copulas to Measure Association between Air Pollution and Respiratory Diseases

Snezhana P. Kostova, Krassi V. Rumchev, Todor Vlaev, and Silviya B. Popova

**Abstract**—Air pollution is still considered as one of the major environmental and health issues. There is enough research evidence to show a strong relationship between exposure to air contaminants and respiratory illnesses among children and adults. In this paper we used the Copula approach to study a potential relationship between selected air pollutants (PM<sub>10</sub> and NO<sub>2</sub>) and hospital admissions for respiratory diseases. Kendall's tau and Spearman's rho rank correlation coefficients are calculated and used in Copula method. This paper demonstrates that copulas can be used to provide additional information as a measure of an association when compared to the standard correlation coefficients. The results find a significant correlation between the selected air pollutants and hospital admissions for most of the selected respiratory illnesses.

**Keywords**— Air pollution, Copula, Respiratory Health.

## I. INTRODUCTION

AIR quality is still considered as a major environmental and health issue worldwide. It determines the quality of our life and has a major impact on the sustainability of our lifestyle. Despite the introduction of cleaner technologies in industries, air pollution remains a major health risk and epidemiological studies have consistently demonstrated an association between air quality and adverse health effects. Some of the the major pollutants in urban areas are generally particulate matter and nitrogen dioxide. Particulate matter (PM) can be of organic or inorganic origin and include airborne dust particles, soot and hydrocarbons [7]. Airborne particulates are classified by size and particulate matter of 10µm or less in size are classified as coarse (or thoracic) particles (PM<sub>10</sub>) and those less than 2.5µm as fine particles (PM<sub>2.5</sub>) [6]. Nitrogen dioxide (NO<sub>2</sub>), an oxidised form of nitric oxide, is generated from fossil fuel combustion with emissions due mainly to motor vehicle exhausts, heating and power sources [27]. Extensive time series analysis have been used to address potential relationships between PM and NO<sub>2</sub> and hospital admissions for respiratory and cardiovascular diseases [13], [18], [23], [24]. This evidence is consistent at exposures that are currently experienced by urban populations in both developed and developing countries [27], [2], [4], [9].

S. P. Kostova is with the Institute of System Engineering and Robotics, Bulgarian Academy of Sciences, 1113, Sofia, Bulgaria, POB 79; (e-mail: kostovasp@yahoo.com)

K.V. Rumchev is with the <sup>2</sup>School of Public Health, Curtin University of Technology, Australia; (e-mail: Krassi.Rumchev@exchange.curtin.edu.au).

T. Vlaev is with the Metaskill Ltd, Sofia, Bulgaria; (e-mail: metaskill@hotmail.com)

S. B. Popova is with the Institute of System Engineering and Robotics, Bulgarian Academy of Sciences, 1113, Sofia, Bulgaria, POB 79; (e-mail: popova\_silvia2000@yahoo.com)

The relationship between air pollution and adverse health effects is very complex to study. The classical method of linear correlation coefficients is not appropriate for reproducing complex structures of dependence between random variables. The use of Pearson's correlation as a dependence measure has several pitfalls and hence application of regression prediction models based on this correlation may not be an appropriate methodology [14], [15]. Correlation coefficients can measure the degree of linear dependence however a weak correlation does not necessarily imply a weak dependence. Correlation coefficients measure the overall strength of the association, but give no information about how that varies across the distribution [3].

Copula based methodology for analysis, prediction, modeling and data simulation is an alternative approach to correlation coefficients [15]. It offers more flexibility by providing a convenient way not only to demonstrate joint distributions of two or more random variables but to indicate in which part the joint distribution is stronger. The main statistical advantage of copulas is in replicating datasets through simulation with *any type of marginal distributions that allows modeling both linear and non-linear relationships* [21], [19], [14].

This article demonstrates the application of copulas as a tool for relating different dimensions of potential relationships between air pollutants and health effects.

## II. AIR POLLUTION AND HEALTH DATA

In order to achieve the aim of this paper, time series data for air pollution and several respiratory diseases was used. The retrospective air quality data was obtained from the Department of Environment and Conservation, Perth, Western Australia and consist of daily (24h average) concentrations of PM<sub>10</sub> and nitrogen dioxide (NO<sub>2</sub>) for the period 2004 – 2008. Air quality data were collected by direct measurements as Tapered Element Oscillating Microbalance was used to obtain continuous readings of PM and the chemiluminescence method was applied for collecting data of NO<sub>2</sub>. Daily hospital admissions for respiratory illness were obtained for the same period 2004-2008 from the Health Department, Western Australia. The retrospective health data consisted of daily hospital admissions for respiratory illnesses including: acute respiratory infections (ICD codes J00-J22), respiratory symptoms (ICD codes J23-J99, excluding J45) and asthma (ICD code J45).

III. THE COPULA METHOD

The copula method was originally developed by the Industrial economics, however recently researchers from other fields became interested in applying copulas. Copulas are best described by [12]), as a multivariate distribution function that is used to bind each marginal distribution function to form the joint. The copula describes the dependence between the margins and the parameters of each marginal distribution function can be estimated separately. The main purpose of a copula is to disentangle the dependence structure of a random vector from its marginals. Copula models provide a relatively simple but powerful description of structure of dependence for mixtures.

The copula method consists of several steps which are presented in the following section. Kendall's tau and Spearman's rho rank correlation coefficients are calculated and used in the Copula method.

A. Measure of Concordance

A definition of concordance in mathematical terms was provided by Nelsen [21]: if  $(X_i, Y_i)$  and  $(X_j, Y_j)$  are used to denote two observations from a vector  $(\mathbf{X}, \mathbf{Y})$  of random variables, then:

$$\begin{aligned} & (X_i, Y_i) \text{ and } (X_j, Y_j) \text{ are concordant if } (X_i < X_j) \text{ and } \\ & (Y_i < Y_j), \text{ or if } (X_i > X_j) \text{ and } (Y_i > Y_j) \quad (1) \\ & (X_i, Y_i) \text{ and } (X_j, Y_j) \text{ are discordant if } (X_i < X_j) \text{ and } \\ & (Y_i > Y_j) \text{ or if } (X_i > X_j) \text{ and } (Y_i < Y_j) \end{aligned}$$

an alternative mathematical definition of concordance was also discussed by Embrechts et al. [10] and Nelsen [21]. the observations  $(X_i, Y_i)$  and  $(X_j, Y_j)$  are concordant if

$$\begin{aligned} & (X_i - X_j)(Y_i - Y_j) > 0 \\ \text{and discordant if} & (X_i - X_j)(Y_i - Y_j) < 0 \quad (2) \end{aligned}$$

According to Scarsini [22] "dependence" defers from "concordance" as follows: "Dependence is a matter of association of  $\mathbf{X}$  and  $\mathbf{Y}$  along any (measurable) function, i.e., the more  $\mathbf{X}$  and  $\mathbf{Y}$  tend to cluster around the graph of a function, either  $\mathbf{Y} = f(\mathbf{X})$  or  $\mathbf{X} = g(\mathbf{Y})$  the more they are dependent. The minimum dependence, as well as the minimum of monotone dependence, corresponds to independence. Concordance, on the other hand, takes into account the kind of monotonicity (whether increasing or decreasing), so that the maximum of concordance is attained when a strictly monotone increasing relation exists between the variables, and the minimum of concordance is attained when a relationship exists that is strictly monotone decreasing."

A pair of observations  $(\mathbf{X}, \mathbf{Y})$  is concordant if the observation with a larger value of  $\mathbf{X}$  has also a larger value

for  $\mathbf{Y}$ . The pair is discordant if the observation with a larger value of  $\mathbf{X}$  has a smaller value of  $\mathbf{Y}$ .

$$\Pr(\text{concordance}) = \Pr[(X_1 - X_2)(Y_1 - Y_2) > 0]$$

$$\Pr(\text{discordance}) = \Pr[(X_1 - X_2)(Y_1 - Y_2) < 0]$$

The variables may involve ties as

$$\Pr(X_1 = X_2) > 0 \quad \Pr(Y_1 = Y_2) > 0$$

$$\Pr(\text{tie}) = \Pr(X_1 = X_2 \text{ or } Y_1 = Y_2)$$

Kendall's tau and Spearman's rho are some of the most common measures of monotone dependence (concordance), used in copula method. They provide some of the best alternatives to the linear correlation coefficient as a measure of dependence for nonelliptical distributions, for which the linear correlation coefficient is inappropriate and often misleading.

B. Kendall's tau

In the bivariate case the Kendall's tau rank correlation is defined as the probability of concordance minus the probability of discordance:

$$\begin{aligned} \tau &= P[(X_1 - X_2)(Y_1 - Y_2) > 0] - P[(X_1 - X_2)(Y_1 - Y_2) < 0] \\ \tau &\in [-1, 1] \quad (3) \end{aligned}$$

of a random vector  $(X_1, Y_1)$  and a second, independent from it vector  $(X_2, Y_2)$  with the same distribution. If there is a tendency for  $X_1$  to increase in line with  $Y_1$ , then it is expected the concordance part  $P[(X_1 - X_2)(Y_1 - Y_2) > 0]$  of equation (3) to outweigh its discordance part  $P[(X_1 - X_2)(Y_1 - Y_2) < 0]$ .

C. Spearman's rho

Bouyé and Salmon [5] define the population version of Spearman's rho as:

$$\rho = \frac{\sum_{i=1}^n (R_i - \bar{R})(S_i - \bar{S})}{\sqrt{\sum_{i=1}^n (R_i - \bar{R})^2} \sqrt{\sum_{i=1}^n (S_i - \bar{S})^2}} \quad (4)$$

where  $R_i$  denotes the rank of  $X_i$  among all  $X$  values, and  $S_i$  denotes the rank of  $Y_i$  among all  $Y$  values.

D. The Copulas

The copulas provide the opportunity to study and measure relationships between random variables. A copula returns the joint probability of events as a function of the marginal probabilities of each event. This makes copulas attractive, as the univariate marginal behavior of random variables can be modeled separately from their dependence. Copulas are referred to as "dependence functions" by Galambos [11] and Deheuvels [8].

Two random variables  $X$  and  $Y$  are joined by a copula function,  $C$ , if their joint cumulative distribution function (cdf) can be written as:

$$F(X, Y) = C(F_1(X), F_2(Y))$$

where  $F_1(X)$  and  $F_2(Y)$  are *cdf* of a random variable  $X$  and  $Y$ . This result is known as Sklar's Theorem [25].

Nelsen [21] shows that if  $X$  and  $Y$  are continuous random variables whose copula is  $C$ , then the two popular copula-based concordance measures: the rank correlations Kendall's tau and Spearman's rho satisfy the properties for a measure of concordance.

$$\tau = 4 \iint_{I^2} C(u, v) dC(u, v) - 1, \tag{5}$$

with  $u = F_1(X); v = F_2(Y)$

$$\rho = 12 \iint_{I^2} (C(u, v) - uv) dudv \in [-1, 1] \tag{6}$$

where  $\rho$  measures the distance between the joint distribution of  $u$  and  $v$  (represented by  $C(u, v)$ ) and the independence copula, represented by  $uv$ . According to Embrechts P, et.al, [10] Spearman's rho is proportional to the surface, formed between the joint distribution  $C(u, v)$  and the independence copula.

It is well known result that the *cdf* of a random variable  $X$ , denoted  $F(X)$ , is uniformly distributed on the interval  $[0, 1]$ .

Also, if the joint and marginal distributions are known, the copula function can be written as

$$C(u, v) = F(F_1^{-1}(u), F_2^{-1}(v))$$

where  $u$  and  $v$  are uniformly distributed on the interval  $[0, 1]$ .

$$u = F_1(X), v = F_2(Y), X = F_1^{-1}(u), Y = F_2^{-1}(v) \tag{7}$$

Many different copulas are available in the literature to express dependence between marginal distributions including Gaussian (normal) copula, Archimedean (Clayton, Frank and Gumbel) copula, and Students copula which are useful in empirical modeling and able to capture different wide range of dependence and easy to estimate.

In the present paper the Clayton copula which belongs to the family of Archimedean copulas was used. It is defined by the copula parameter, denoted  $\theta$  which is closely related to the strength of dependence between variables.

As mentioned above, the correlation coefficient used for copula analysis must be invariant under the monotonic increasing transformations, i.e.:

$$\omega(X, Y) = \omega(u, v)$$

where  $\omega$  is a correlation measure,  $X$  and  $Y$  are the raw data and  $u$  and  $v$  are the transformed data in the interval  $[0, 1]$ . It has been shown that rank correlation measures such as  $\tau$  and  $\rho$  are invariant under monotonic transforms [1]. On the other hand, Pearson's correlation is not invariant under monotonic

transforms and is therefore not appropriate for copula analysis. The three most common methods of estimating copula parameter are  $\tau$ ,  $\rho$  and Maximum Pseudo Likelihood (MPL). Rank measures of correlation can be related to the copula density and hence to the copula parameter  $\theta$ . Parameter estimation using rank correlation is very popular because closed form solutions exist for many commonly used copulas. For example the  $\tau$  can be related to the Gumbel copula parameter by:  $\tau = 1 - \theta^{-1}$

and for the Clayton copula by:  $\tau = \frac{\theta}{\theta + 2}$

The idea behind the copula model is not to define the correlation structure between the variables of interest directly, but rather to map the variables of interest into other more manageable and define correlation structure between those variables.

The copula method consists of the following main calculations:

1. Kendall Kendall  $\tau$
2. Copula parameter -  $\theta$
3. Generator -  $\varphi(t)$
4. Generator's first derivate -  $\varphi'(t)$
5. Generator's Inverse -  $\varphi^{-1}(t)$
6. Copula function -  $C_\theta$

Copula function $C_\theta$	Generator $\varphi(t)$ ( $t=u$ , or $t=v$ )	Range $\theta$	Kendall's tau $\tau$
$[u^{-\theta} + v^{-\theta} - 1]^{-\frac{1}{\theta}}$	$t^{-\theta} - 1$	$[0, \infty)$	$\frac{\theta}{\theta + 2}$

The process of construction of copula is available in a number of statistical packages. In the current study MATLAB was applied for calculating the Copula function.

If it is assumed that all steps above, 1-6, are fulfilled and that the dependence between  $X$  and  $Y$  is represented by Clayton's copula, several analyses on the base of joint *cd* can be made and all represented by copula. These analyses are based on the following three values:

- Value of joint probability  $F_{uv}$  (given from the copula),
- Corresponding values of transformed data  $u$  and  $v$  (in the interval  $[0, 1]$ , taken from the cumulative probability diagram, or from the perspective plot of the cumulative probability diagram),
- Corresponding values of the row data  $X$  and  $Y$  (calculated from  $u$  and  $v$  by transformations (7)).

The copula method described above will be illustrated by using the data, described in Section 2 to model the relationship between the selected air pollutants and hospitalization of respiratory diseases.

IV. APPLICATION OF CLAYTON COPULA TO MEASURE A POTENTIAL RELATIONSHIP BETWEEN AIR POLLUTION AND HOSPITALIZATION WITH RESPIRATORY DISEASES

The Pearson correlation coefficient was calculated for all pair's *pollutant - respiratory disease*. This was followed by determining the rank correlation coefficients of Kendal  $\tau$  and Spearman  $\rho$  and the corresponding Clayton copula dependence parameters by using  $\tau$  and  $\rho$ , accordingly. The results are presented in Table I. The pairs are ordered according to the Kendal rank correlation coefficient (column 3 or 5, the ordering is the same; the ordering by Pearson correlation coefficients is different). Based on this information, analysis of the strength of the relationship between air pollutants and respiratory diseases can be made. The corresponding levels of significance are calculated.

TABLE I  
CORRELATION COEFFICIENTS AND RELATED COPULA PARAMETERS

1	2	3	4	5	6
	Pearson correlation coefficient*	Kendall rank corr. coefficient	Clayton copula dep. parameter-Kendall $\theta_k$	Spearman rank corr. coefficient	Clayton copula dep. parameter-Spearman $\theta_s$
Chronic bronchitis - PM <sub>10</sub>	0.2251	0.2694	0.7376	0.3559	0.6423
Acute bronchitis and bronchiolitis -NO <sub>2</sub>	0.2192	0.2309	0.6003	0.3236	0.5645
Acute upper respiratory infections - NO <sub>2</sub>	0.1966	0.1973	0.4916	0.2831	0.4742
Bronhitis-NO <sub>2</sub>	0.0906	0.1692	0.4072	0.2404	0.3867
Bronhitis- PM <sub>10</sub>	0.2113	0.1544	0.3651	0.2310	0.3684
All diseases of upper respiratory tract -NO <sub>2</sub>	0.1540	0.1339	0.3092	0.1973	0.3054
Bacterial pneumonia-NO <sub>2</sub>	0.1428	0.1306	0.3004	0.1904	0.2929
Asthma -NO <sub>2</sub>	0.0509	0.113	0.2548	0.1635	0.2458
Asthma - PM <sub>10</sub>	0.2089	0.1079	0.2418	0.1618	0.2429
Other diseases of upper respiratory tract -NO <sub>2</sub>	0.1166	0.0917	0.202	0.1346	0.1976
Other diseases of upper respiratory tract -PM <sub>10</sub>	0.0529	0.0469	0.0985	0.0695	0.097
All diseases of upper respiratory tract - PM <sub>10</sub>	0.0535	0.0443	0.0928	0.0664	0.0926

The data from Table I is grafically illustrated in Fig 1 which demonstrates the consistency between different measures with regards to the direction of the relationship between air pollution and respiratory diseases and at the same time illustrates the diference between the Pearson correlation coefficient and the copula dependence parameters.

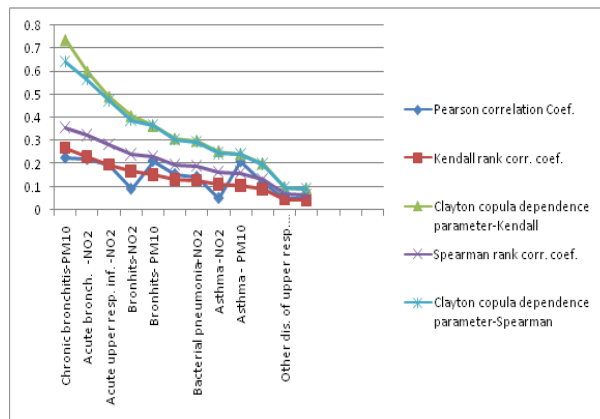


Fig. 1 Comparison of different measures of dependence

As can be seen from Table I, the air pollutants and hospital admissions for respiratory illness are concordant although the relationship is not very strong with coefficients under 0.3 (the biggest Kendall's is equal to 0.2694). This can be explained that there are some other factors in addition to NO<sub>2</sub> and PM<sub>10</sub> that may contribute to hospital admissions with respiratory symptoms. The strongest association is observed between hospital admissions with chronic bronchitis and PM<sub>10</sub> and also between hospitalization with acute bronchitis and exposure to NO<sub>2</sub>.

The Clayton copula parameters were calculated for each pair (Table I) but the Clayton copula is graphically presented for only one pair. In Fig.2 and Fig. 3 the Clayton copula and Cumulative Probability for the pair *Chronic bronchitis -PM<sub>10</sub>* are presented. Perspective plot of Cumulative Probability is presented in Fig.4 for a better understanding of Fig. 3. Original data and marginal distributions are shown in Fig.5 and the Probability density function for considered pair *Chronic bronchitis -PM<sub>10</sub>* is plotted in Fig. 6.

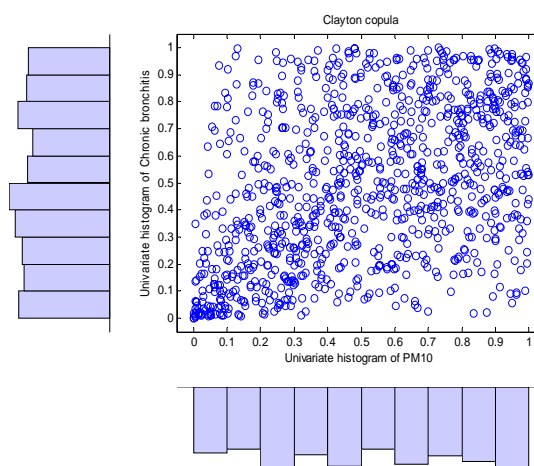


Fig. 2 Clayton copula of the pair *Chronic Bronchitis -PM<sub>10</sub>*

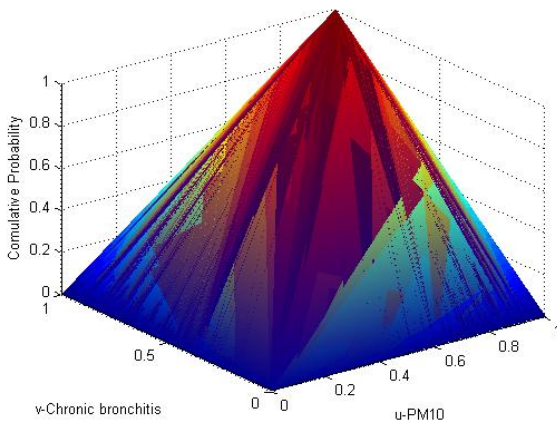


Fig. 3 Cumulative Probability of the pair *Chronic bronchitis- PM<sub>10</sub>*

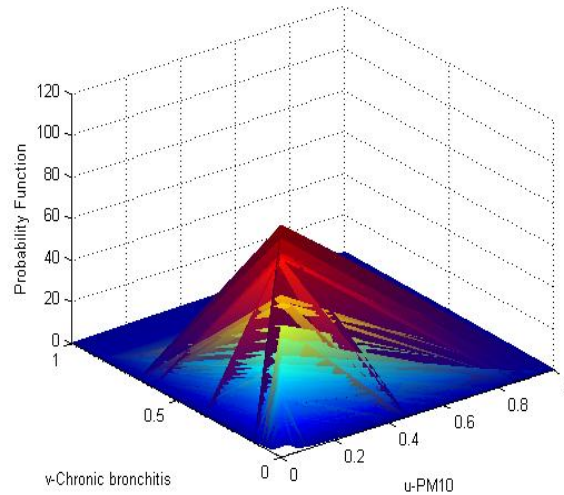


Fig. 6 Probability density function of the pair Chronic Bronchitis – *PM<sub>10</sub>*

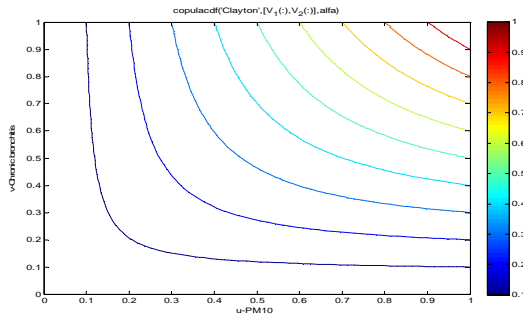


Fig. 4 Perspective plot of Cumulative Probability of the pair *Chronic Bronchitis –PM<sub>10</sub>*

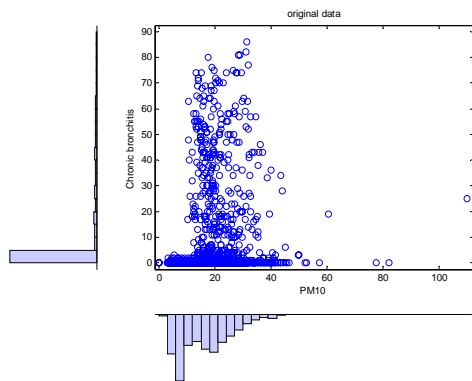


Fig. 5 Original data and marginal distributions

As previously discussed, graphical presentations can provide information in relation to both transformed variables  $u$  and  $v$  and also for the joint probability  $F_{uv}$ .

From the Clayton Copula the cases in which both investigated variables are less than the previously given limits could be established (Fig. 2). As an example, if we would like to calculate the joint probability of  $PM_{10}$  with less than given value of  $X_{limit}$  and the number of hospital admissions with less than a given value of  $Y_{limit}$ , the following steps need to be considered:

- First, by applying the copula formula (7) and using  $X_{limit}$  and  $Y_{limit}$  the corresponding values  $u_{limit}$  and  $v_{limit}$  can be calculated
- This is followed by estimating the joint probability  $F_{uv}$  using Figure 6.

Considering the above, Copulas can be applied to determine the risk for hospitalization with respiratory illnesses at certain concentrations of air pollutants. In the current paper, the highest probability for hospitalization with respiratory illness at certain exposure levels of air pollutants corresponds to the middle value of  $u$  and the highest value of  $v$  which can be determined from Figure 6. This is followed by defining the corresponding values of  $X$  and  $Y$  using Fig. 5. In the case presented in Fig 5, it can be estimated that at exposure levels of  $PM_{10}$ , between  $18 \mu\text{g}/\text{m}^3$  and  $20 \mu\text{g}/\text{m}^3$ , approximately 3 or 4 people are likely to be hospitalized with chronic bronchitis. Therefore, by estimating the risk for hospitalization at certain exposure levels to air pollutants, the copula function can be used to control the number of hospital admissions by controlling the concentrations of air pollutants.

The maximum risk for hospitalization using copulas has also been calculated for the other studied respiratory illnesses but not presented in the current paper.

### V. CONCLUSION

This article demonstrated the application of a relatively new approach to measure associations between air pollution and

adverse health effects: the Copula method. To our knowledge this is the first paper to present the application of Copulas as a measure of an association between air pollution and adverse health effects.

This study demonstrated an association between exposures to PM<sub>10</sub> and NO<sub>2</sub> and increased number of hospital admissions for selected respiratory illnesses which is consistent with the findings of other studies [17], [2], [18], [26].

The analysis presented in the current paper confirmed that copulas can be successfully applied in providing additional important information in relation to associations between two variables which cannot be provided by correlation coefficients.

In our future work we will determine the copula risk assessment of extreme events in the context of air pollution and hospitalization for respiratory illnesses.

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