

# Unified Fusion Approach with Application to SLAM

Xinde Li, Xinhan Huang and Min Wang

**Abstract**—In this paper, we propose the pre-processor based on the *Evidence Supporting Measure of Similarity* (ESMS) filter and also propose the unified fusion approach (UFA) based on the general fusion machine coupled with ESMS filter, which improve the correctness and precision of information fusion in any fields of application. Here we mainly apply the new approach to *Simultaneous Localization And Mapping* (SLAM) of Pioneer II mobile robots. A simulation experiment was performed, where an autonomous virtual mobile robot with sonar sensors evolves in a virtual *world* map with obstacles. By comparing the result of building map according to the general fusion machine (here DSMT-based fusing machine and PCR5-based conflict redistributor considered) coupling with ESMS filter and without ESMS filter, it shows the benefit of the selection of the sources as a prerequisite for improvement of the information fusion, and also testifies the superiority of the UFA in dealing with SLAM.

**Keywords**—DSMT, ESMS filter, SLAM, UFA

## I. INTRODUCTION

THE study on exploration of entirely unknown environment for intelligent mobile robots has being a popular and difficult subject for experts in robots' field for a long time, especially SLAM [1] is a very challenging work, which is compared as the puzzle *chicken and egg*. How to manage and fuse the information of multi-sources, which successfully will solve the puzzle more efficiently? Evidence reasoning have become more and more popular in the community of information fusion, since DST was proposed by Dempster & Shafer in 1976 [2], especially since Prof. Smets proposed Transferable Belief Model (TBM) [3] and gave an explain on it. Recent DSMT based on DST and Bayesian theory proposed by Jean Dezert & Florentin Smarandache is a new, general, and flexible theory [4]. Presently a more general theory for unification of fusion theories (UFT) is proposed by Prof. Smarandache [5]. DST, DSMT, and UFT all offer interesting issues to combine uncertain sources of information expressed

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in term of belief functions. In these frameworks many fusion rules have been proposed by different authors, the most common ones being Dempster's rule [2], Yager's rule [6], Dubois and Prade's rule [7] and the most recent ones based on proportionalization like minC [8] and PCR1-PCR5 rules [9]. However, each theory and rule works well for some application, but not for all. Here we propose the unified fusion approach based on ESMS filter [10] coupled with the general fusion machine. Due to existing ESMS filter to select the *best* subset of sources of information to combine with respect to a pre-defined criteria also called measure of similarity (or consistency) between sources, by a proper choice of consistent sources, we can decrease the conflict before applying the fusion rule (whatever the rule we choose), and thus will improve the correctness and precision of the fusion result, as it will be shown in our application in the sequel. Moreover, UFA fits to any model and rule, overcomes the self limitation of the fusion machine, not only enlarges the range of the application, but also save the amount of computing and improve the efficiency of information fusion. At last, we give an application of UFA considering DSMT-based fusion machine and PCR5-based conflict redistributor in dealing with robot's SLAM, and get a very good result.

## II. THE GENERAL FUSION MACHINE

### A. General principle

The general principle of a general fusion machine consists in  $k$  sources of evidences (i.e. the inputs) providing basic belief assignments over a propositional (fusion) space generated by elements of a frame of discernment and set operators endowed with eventually a given set of integrity constraints, which depend on the nature of elements of the frame. The set of belief assignments must then be combined with a fusion operator. Since in general the combination of uncertain information yields a degree of conflict, says  $K$ , between sources, this conflict must be managed by the fusion operator/machine. The way the conflict is managed is the key of the fusion step and makes the difference between the fusion machines. The fusion can be performed globally/ optimally (when combining the sources in one derivation step all together) or sequentially (one source after another as in Fig.1). The sequential fusion processing (well adapted for temporal fusion) is natural and simpler than the global fusion, but in general remains only sub optimal if the fusion rule chosen is not associative, which is the case for most of fusion rules, but Dempster's rule. For the convenience of our application in mobile robot, we give the

general sequential fusion machine shown in Fig.1.

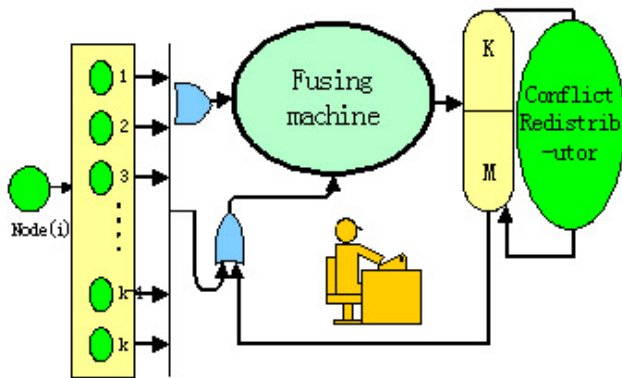


Fig. 1 General sequential fusion machine

### B. Propositional (Fusion) space

Supposed there is a discernment frame  $\Theta = \{\theta_1, \dots, \theta_n\}$ , here  $n > 2$ , then suppose its general hyper-power sets [4,11,12] to be  $G^\ominus$ , if  $G^\ominus = 2^\ominus$ , then the DS model is adopted, if  $G^\ominus = D^\ominus$ , then the DSm model is adopted, and if  $G^\ominus = S^\ominus$ , then the UFT model is adopted. For UFT [5], its hyper-power sets  $S^\ominus = \{\Theta, \cup, \cap, \ell\}$ , that is,  $\Theta$  closed under these three operations: union, intersection, and complementation of sets respectively, forms a Boolean algebra, we define a general basic belief assignment (gbba) as a mapping  $m(\bullet): S^\ominus \in [0,1]$  associated to a given source, say  $S$ , of evidence as  $m(\phi) \geq 0$ , and  $\sum_{A \in S^\ominus} m(A) = 1$ ,  $m_s(A)$  is the gbba of  $A$  committed by the sources. Of course, in DS and DSm model,  $m(\phi) = 0$ . So hyper-power sets  $S^\ominus$  is more general. But for an engineering application, system designer often adopts DS model and DSm model.

### C. Fusing machine

Here the fusing machine is included in the general fusion machine. The system designer may select combination rule through a switcher according to the physical application. The basic principle of choosing combination rules is listed as follows:

- 1) If all sources of evidence are reliable, then apply the conjunctive rule, which means consensus between them (or their common part).
- 2) If some sources are reliable and others are not, but we don't know which ones are unreliable, apply the disjunctive rule as a cautious method (and no transfer or normalization is needed).
- 3) If only one among all sources is reliable, but we don't know which one, under this situation, we adopt the exclusive disjunctive rule.
- 4) If a mixture of the previous three cases, in any possible way, use the mixed conjunctive - disjunctive rule.
- 5) If we know which resource of evidence is unreliable, moreover, we know the unreliable degree, then applying

the discounting method. But if all sources of evidence are fully unreliable, then the result of fusion will keep the other basic belief mass being zero, except the basic belief mass of total ignorance, i.e.  $m(\Theta) = 1$ . The solution becomes more uncertain and occurs the phenomenon of entropy increasing. Under this situation, we must find some other evidential sources (at least, no full unreliable) to fuse them again.

- 6) According to the physical requirement, we may choose a special model, So if we choose DS model, then we get the power sets  $2^\ominus$ , if DSm model, then get the hyper power-sets  $D^\ominus$ , if UFT model, then get hyper-power sets  $S^\ominus$ ,  $S^\ominus$  includes  $2^\ominus$  and  $D^\ominus$ . Here without loss of generality, we denotes  $G^\ominus$  the general power set on which will be defined the basic belief assignments (or masses), i.e.  $G^\ominus = 2^\ominus$ , when DST is adopted or  $G^\ominus = D^\ominus$ , when DSmT [11], [12] is adopted, also or  $G^\ominus = S^\ominus$ , when UFT model is adopted.

### D. Conflict redistributors

The other important loop is conflict redistributors in the general fusion machine, that is, when we have known the model, and found out the contradictions (i.e. empty intersection) or consensus (i.e. non-empty intersections) of the problem/ application. Of course, if an intersection  $A \cap B$  is not empty, we go on keeping the mass  $m(A \cap B)$  on  $A \cap B$ , which means consensus (common part) between the two hypotheses  $A$  and  $B$ . In fact, when contradiction occurs between the two hypotheses  $A$  and  $B$ . that is, the intersection  $A \cap B = \phi$ , we need to redistribute the conflict mass according to the following principle:

- 1) If we know one of which is right between the two hypotheses  $A$  and  $B$ , but we don't know which one, to be cautious of doing this, then we transfer the mass  $m(A \cap B)$  to  $m(A \cup B)$ , since  $A \cup B$  means at least one is right.
- 2) If we know that between the two hypotheses  $A$  and  $B$  one is right and the other is false, and we exactly know which one is right, here supposed  $A$  is right,  $B$  is false, then we can precisely transfer all the conflict mass  $m(A \cap B)$  to  $m(A)$ , that is, nothing is transferred to  $B$ .
- 3) If we don't know much about them, but we have an optimistic view on hypotheses  $A$  and  $B$ , then we transfer the conflicting mass  $m(A \cap B)$  to  $A$  and  $B$  according to proportional conflict redistribution rule (PCR). If we have a pessimistic view on hypotheses  $A$  and  $B$ , then one transfer the conflicting mass  $m(A \cap B)$  to  $m(A \cup B)$  (the more pessimistic the further we get in the Specificity Chain:  $(A \cap B) \subset A \parallel B \subset (A \cup B) \subset I$ ), if we have the most pessimistic view on the hypotheses  $A$  and  $B$ , then we transfer the conflict mass to the total ignorance in closed world, or to the empty set in an open world.
- 4) If we think both of two hypotheses  $A$  and  $B$  are false, we

must transfer the conflict mass to the other non-empty set (not A, B, and  $A \cup B$ ). Because both are false, the other non-empty set will have a higher probability to occur. Of course, if we consider that none of the hypotheses A, B is right and no other hypotheses exists in the frame of discernment, then in open world, transfer the mass to the empty set.

Of course, whether the fusion rule or conflict redistribute rule can be extended for any intersection of two or more sets i.e.  $A \cap B \cap C$ , etc. and even for mixed sets:  $A \cap (B \cup C)$ , etc.

### III. THE ESMS FILTER

To improve the performances of the fusion machine/fusion processor by setting up a pre-processing task, in order to select only a subset of sources, called consistent sources, and combine among all sources available at each time step of the process. Such idea is very general since it doesn't depend on the application neither on the fusion machine/rule itself (while the belief function framework is used).

#### A. Some definitions and theorems

**Definition 1** (measure of similarity):

Let's consider any three gbba, say  $m_1(\bullet)$ ,  $m_2(\bullet)$  and  $m_3(\bullet)$  defined over same space  $G^\Theta$ , the mapping  $N(.,.): G^\Theta \times G^\Theta \rightarrow [0,1]$  is called an Evidence Support Measure of Similarity (ESMS) or a similarity function for short, if the three following conditions are satisfied:

1.  $\forall m_1(\bullet), m_2(\bullet), N(m_1, m_2) = N(m_2, m_1)$
2.  $\forall m(\bullet)$  defined over  $G^\Theta$ ,  $N(m, m) = 1$
3.  $N(m_1^X, m_2^Y) = 0$ , if  $X \neq Y$

Where  $m_s^X$ ,  $s=1,2$  represents a belief assignment total focused on  $X$ ,  $X \in G^\Theta$ ,  $m_s^X$  is defined by  $m_s^X(X) = 1$ , and  $m_s^X(Y) = 0$  for all  $Y \neq X$ .

If  $N(m_1, m_2) > N(m_1, m_3)$ , then  $m_2$  is said more similar to  $m_1$  than  $m_3$ ,  $N(m_1, m_2)$  is called as the evidence supporting measure of similarity between  $m_1(\bullet)$  and  $m_2(\bullet)$ .

**Theorem 1:** If there exists an unitary n dimensional vector (i.e. a basic belief assignment)  $m_1(\bullet)$  and an enough small positive real number  $\varepsilon$  is given, then no less than one unitary n-dimensional vector  $m_2(\bullet)$  exist and satisfy the condition of some distance measure<sup>1</sup>  $d(m_1, m_2) \leq \varepsilon$ .

Proof (by contradiction): Let suppose there doesn't exist the vector  $m_2(\bullet)$  satisfying the condition  $d(m_1, m_2) \leq \varepsilon$ , then we may let  $m_2(\bullet)$  to be equal to  $m_1(\bullet)$ , so it is known that  $d(m_1, m_2) = 0$ , but by assumption  $\varepsilon > 0$ , then  $d(m_1, m_2) \leq \varepsilon$ . So it is in conflict with the assertion of theorem and thus this

<sup>1</sup> Here we don't specify the distance measure and keep it only as a generic distance. Actually  $d(.,.)$  can be any distance measure. In practice, the Euclidean distance is frequently used.

completes the proof by contradiction.

**Definition 2** (agreement of evidence): If there exist two basic belief assignments  $m_1(\bullet)$  and  $m_2(\bullet)$  defined over the same space  $G^\Theta$ , such that  $d(m_1, m_2) \leq \varepsilon$ ,  $\varepsilon > 0$ , for some distance  $d(.,.)$ , then  $\varepsilon$  is called the agreement of evidence supporting measure between  $m_1(\bullet)$  and  $m_2(\bullet)$  with respect to distance  $d$ .  $m_1(\bullet)$  and  $m_2(\bullet)$  are  $\varepsilon$ -consistent with respect to distance  $d$ .

**Theorem 2:** If there exist two basic belief assignments  $m_1(\bullet)$  and  $m_2(\bullet)$  defined over the same space  $G^\Theta$ , then the following sufficient and necessary condition holds: if  $m_1(\bullet)$  and  $m_2(\bullet)$  are  $\varepsilon$ -consistent ( $d(m_1, m_2) \leq \varepsilon$ ), then they satisfy Theorem 1.

Proof: We first prove the sufficient condition. Since  $m_1(\bullet)$  and  $m_2(\bullet)$  are assumed  $\varepsilon$ -consistent, then from definition 2, there exists a small positive real number  $\varepsilon$ , such that  $d(m_1, m_2) \leq \varepsilon$ . So satisfies the theorem1. Secondly, we prove the necessary condition. From theorem 1, if a basic belief assignment  $m_1(\bullet)$  and a small positive real number  $\varepsilon$  are given, then there must exist a basic belief assignment vector  $m_2(\bullet)$ , which keeps  $d(m_1, m_2) \leq \varepsilon$ , and thus satisfies the definition 2, we think the evidence supporting measure between  $m_1(\bullet)$  and  $m_2(\bullet)$  is consistent.

**Theorem 3:** Smaller  $\varepsilon > 0$  is, nearer the distance between  $m_1(\bullet)$  and  $m_2(\bullet)$  is, that is, more similar or consistent  $m_1(\bullet)$  and  $m_2(\bullet)$  are.

Proof: According to the theorem 2, if the evidence measure between  $m_1(\bullet)$  and  $m_2(\bullet)$  is  $\varepsilon$ -consistent, then  $d(m_1, m_2) \leq \varepsilon$ . Let's take  $\varepsilon = 1 - N(m_1, m_2)$ ; when  $\varepsilon$  becomes smaller and smaller,  $N(m_1, m_2)$  becomes greater and greater, according to the definition of ESMS and thus more similar or consistent  $m_1(\bullet)$  and  $m_2(\bullet)$  become. Finally, if  $\varepsilon = 1 - N(m_1, m_2) = 0$ , then  $m_1(\bullet)$  and  $m_2(\bullet)$  are totally consistent.

From the previous definitions and theorems, we propose to use a pre-processing/ threshold technique based on ESMS as an efficient tool to weight the agreement measure of two sources of evidence.

#### B. A simple ESMS function

**Definition 3:** Let's  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$  ( $n > 1$ ),  $m_1(\bullet)$  and  $m_2(\bullet)$  defined over the same space  $G^\Theta$ ,  $X_i$  the  $i$ -th (generic) element of  $G^\Theta$  and  $|G^\Theta|$  the cardinality of  $G^\Theta$ . A simple ESMS function considered in this work is defined by

$$N(m_1, m_2) = 1 - \frac{1}{\sqrt{2}} \left( \sum_{i=1}^{|G^\Theta|} (m_1(X_i) - m_2(X_i))^2 \right)^{1/2} \quad (1)$$

Remark: ESMS function is not unique and other functions (see [13]) could be used. The one proposed here is simple

enough to be used easily in our simulator. The purpose of this paper is not to justify a specific ESMS function, but to show the advantage of ESMS filter for improving performances of the fusion machine.

**Theorem 4:**  $N(m_1, m_2)$  defined in (1) is an ESMS function.

Proof:

1. Let's first prove that  $N(m_1, m_2) \in [0, 1]$ . One has  $N(m_1, m_2) \leq 1$ , since if one assumes  $N(m_1, m_2) > 1$ , from (1)

one would get  $\frac{1}{\sqrt{2}} \sqrt{\sum_{i=1}^{|G^\ominus|} (m_1(X_i) - m_2(X_i))^2} < 0$  which is

impossible. Now let's prove  $N(m_1, m_2) \geq 0$ . If one assumes  $N(m_1, m_2) < 0$ , then from (1) it would yield

$\sum_{i=1}^{|G^\ominus|} (m_1(X_i) - m_2(X_i))^2 > 2$ . Since  $m_1(\bullet)$  and  $m_2(\bullet)$  are

basic belief masses,  $\sum_{i=1}^{|G^\ominus|} m_s(X_i) = 1, s = 1, 2$ , to prove

$\sum_{i=1}^{|G^\ominus|} (m_1(X_i) - m_2(X_i))^2 > 2$ , we enlarge the left side of the

inequality by supposing a  $|G^\ominus|$  dimensional zero vector  $O$ , so

we have the following inequality

$\sum_{i=1}^{|G^\ominus|} (m_1(X_i) - O_i)^2 > \sum_{i=1}^{|G^\ominus|} (m_1(X_i) - m_2(X_i))^2 > 2$ , so

equivalently,  $\sum_{i=1}^{|G^\ominus|} (m_1(X_i))^2 > 2$ , then we know

$(\sum_{i=1}^{|G^\ominus|} m_s(X_i))^2 > \sum_{i=1}^{|G^\ominus|} (m_1(X_i))^2 > 2$  from the definition of

basic belief mass, that is,  $1 > 2$ , which is impossible, conflicting with the assumption. So complete the proof.

2. It is easy to check that  $N(m_1, m_2)$  satisfies the first condition of Definition 1.

3. If  $m_1(\bullet) = m_2(\bullet)$ , then  $N(m_1, m_2)$  because

$\sum_{i=1}^{|G^\ominus|} (m_1(X_i) - m_2(X_i))^2 = 0$ . Thus the second condition of

Definition 1 is also satisfied.

4. If there exist  $m_1^X$  and  $m_2^Y$  for some  $X, Y \in G^\ominus$  such that  $X \neq Y$ , then according to (1), one gets

$\sum_{i=1}^{|G^\ominus|} (m_1(X_i) - m_2(X_i))^2 = [m_1^X(X)]^2 + [m_1^Y(Y)]^2 = 2$  and

thus one has  $N(m_1^X, m_2^Y) = 1 - \sqrt{2}/\sqrt{2} = 0$  so that  $N_E(\dots)$

verifies the third condition of Definition 1.

5. According to the definition of  $N(m_1, m_2)$ , we can easily check that  $N(m_1, m_2)$  is a distance measure between  $m_1(\bullet)$  and  $m_2(\bullet)$ , since according to theorem 3, if there exists  $m_3(\bullet)$  such that  $N(m_1, m_2) > N(m_1, m_3)$ , then  $m_2(\bullet)$  is more similar to  $m_1(\bullet)$  than  $m_3(\bullet)$  essential indispensable necessary.

*C. Barycentre of belief masses*

We introduce here the barycentre of belief masses, which will be used in ESMS filter. ESMS filter will reject all sources

having an ESMS value below a pre-defined rejection threshold chosen by the system designer. We must distinguish four cases for the derivation of barycentre of belief mass depending on the characteristics of imperfect information sources.

Case 1: Equireliable sources

Let's denote  $k = |G^\ominus|$  the cardinality of  $G^\ominus$  and let's consider  $S$  independent<sup>2</sup> sources of evidence. If we assume all sources equireliable, the barycentre of belief masses of the  $S$  sources is expressed as follows:  $\forall j = 1, \dots, k$

$$\bar{m}(X_j) = \frac{1}{S} \sum_{s=1}^S m_s(X_j) \tag{2}$$

**Theorem 5:** The vector of barycentres of belief masses is also a belief mass, i.e.  $\sum_{j=1}^k \bar{m}(X_j) = 1$

Proof: Let's consider  $S$  sources with their gbba over  $G^\ominus$ , and then the matrix representation  $M$  can be used for the set of all gbba where the index of each row of  $M$  corresponds to a specific source and the index of each column of  $M$  corresponds to a specific element of  $G^\ominus$ . For example, for  $S$  sources with  $k = |G^\ominus|$ , one has

$$M = \begin{bmatrix} m_1(X_1) & m_1(X_2) & \dots & m_1(X_k) \\ m_2(X_1) & m_2(X_2) & \dots & m_2(X_k) \\ \vdots & \vdots & \dots & \vdots \\ m_s(X_1) & m_s(X_2) & \dots & m_s(X_k) \end{bmatrix}$$

The sum of each row of  $M$  is one since each source is characterized by its own (normalized) gbba, so that the total sum of all elements of  $M$  is  $S$ . Therefore, the sum of all the sums of each column of  $M$  is also  $S$  and thus the total sum of normalized sums (normalization by  $S$ ) of each column (i.e. the sum of barycentres of each column) is one, which completes the proof.

Case 2: Unreliable sources

In this second case, we consider each source, says, with its own reliability factor  $a_s \in [0, 1]$ . Then the gbba of each source is discounted according to its reliability factor based on classical discounting approach [2]-[4],[7], i.e. for  $s = 1, \dots, S$

$$\begin{cases} m_s'(X) = a_s m_s(X) \forall X \in G^\ominus, X \neq \Theta \\ m_s'(\Theta) = (1 - a_s) + a_s m_s(\Theta) \end{cases}$$

where  $\Theta$  denotes here the proposition reflecting the total ignorance, i.e.  $\Theta = X_1 \cup X_2 \cup \dots \cup X_n$ .

The barycentre of (discounted) gbba is expressed as previously, i.e.  $\forall j = 1, \dots, k$

$$\bar{m}'(X_j) = \frac{1}{S} \sum_{s=1}^S m_s'(X_j) \tag{3}$$

Similarly to Theorem 5, one has  $\sum_{j=1}^k \bar{m}'(X_j) = 1$ . In fact,

<sup>2</sup> In fact, only independent sources are considered in this work. Although independence is a difficult concept to define in all theories managing epistemic uncertainty, we consider that two sources (or more) of evidence are independent (i.e. distinct and non interacting) if each leaves one totally ignorant about the particular value the other will take.

we may regard easily the same reliable degree as a special instance, when discounting/reliability factor  $a_s$  is equal to one.

Case 3: Incomplete and paraconsistent sources

For incomplete information, the sum of these bba components can be less than 1 (not enough information known), however for paraconsistent information, the sum can exceed 1 (overlapping contradictory information). Under the two situations, all of the components can be normalized (i.e. keeping the sum of their components to be one). So we can derive the barycentre of belief masses as follows:  $s = 1, \dots, S$ ,

$$\forall i = 1, \dots, k, \quad \text{we let } M_s = \sum_{i=1}^k m(X_i),$$

$$m'_s(X_i) = m_s(X_i) / M_s, \text{ then the barycentre of belief masses is expressed as previously, i.e. } \forall j = 1, \dots, k$$

$$\bar{m}'(X_j) = \frac{1}{S} \sum_{s=1}^S m'_s(X_j) \quad (4)$$

Similarly to Theorem 5, one has  $\sum_{j=1}^k \bar{m}'(X_j) = 1$ . The vector of barycentres of belief masses are also a belief mass.

Case 4: Mutually complementary sources

For mutually complementary sources, generally speaking, refer to heterogeneous multi-sources information system, or single sensor's difference system of time or space. Because each source gives a precise description from some a view, all sources generally supplies with a self-contained description of those discernable objects. Even sometimes because of the limitation of its own physical characteristics of sensors, some conflicting information also might happen between mutually complementary sources. Under this situation, to avoid filtering some useful information, we generally only filter the homogeneous information sources, and currently don't filter the heterogeneous information sources, and even leave the highly conflicting information to be fused in the fusion machine.

D. Principle of ESMS filter

Let's consider a frame  $\Theta$ , and  $n$  discernable objects in the system and  $S > 1$  sources of evidence. The principle of ESMS filter, corresponding to the left block in the figure Fig. 3 shown in next section, is represented by the Figure 2 where  $S_1, S_2 \dots S_S$  represent the  $S$  sources of evidence available as inputs to the preprocessor, i.e. the ESMS filter.  $o_1, o_2 \dots o_n$  express  $n$  discernable objects,  $S_i(o_j)$  represents the belief mass assignment defined on  $G^\Theta$  provided by the  $i$ -th source of evidence about the  $j$ -th discernable object.  $S_{ai}$  represents the vector of barycentres of each discernable object computed from belief masses of the sources of evidences relative to the  $i$ -th object.  $S_{ai} \equiv \bar{m}(X)$  is computed according to (2) (or (3) ,(4) if sources are considered as inquirable or incomplete and paraconsistent ).  $N_{Ei}(o_j)$  represents the ESMS between the barycentre of belief mass and the belief mass  $m_i(\bullet)$  of the  $i$ -th source committed to the  $j$ -th discernable object.  $N_{Ei}(o_j)$  is

computed according to (1).  $N(o_j) = 1 - \varepsilon$  is the rejection threshold for ESMS filter (i.e. the  $\varepsilon$ -consistency tuning parameter), which is a positive real number in  $[0, 1]$  chosen by the system designer. When the condition  $N_{Ei}(o_j) > N(o_j)$  is satisfied, the pre-processed information can pass through the ESMS filter and thus the corresponding source can feed the fusion machine (right block in Figure 2). Node(i) expresses the connecting node between the ESMS filter and the fusion machine.

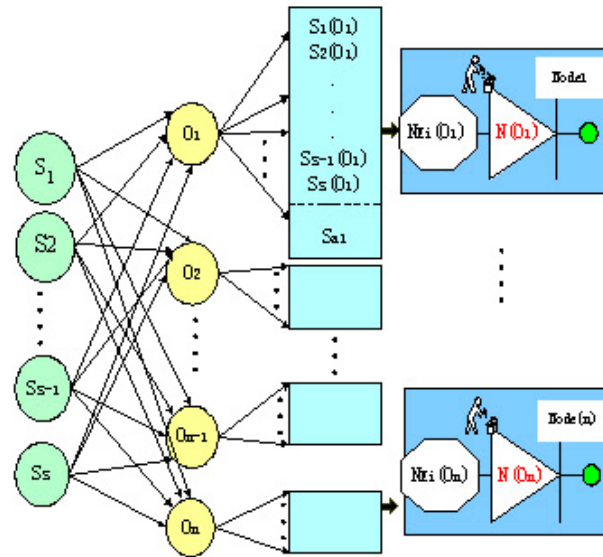


Fig. 2 Principal of ESMS filter

IV. UNIFIED FUSION APPROACH

In this section, we integrate pre-processor of fusion based on ESMS filter with the fusion machine, and propose unified fusion approach of serial mechanism owning the characteristics of information agreement. The unified fusion approach is shown in Fig.3.

Some characteristics of unified fusion approach:

- 1) Reduce the amount of computing. Although add ESMS filter in the pre-processor, it seems as if it increase the amount of computing. In fact, from the point of view of the fusion machine, due to the existence of filter, discard some evidence sources, which don't pass the filter, because of their high conflict, the computing amount of unified fusion approach is very little.
- 2) Improve the precision and correctness of fusion. Due to filtering the conflicting information or misleading information, reduce the influence of its on the fusion machine.
- 3) Improve the popularity of evidence reasoning. Enlarge the range of application for some models, as an example, for DS model, if the conflicting factor is equal to one, then the denominator is zero, and the result tends towards  $\infty$ . For existing filter, no high conflicting mass exists.
- 4) Extend the fusion space, i.e. from the closed world to open

world, from the application of single combination rule and conflict redistribution rule to the integration of all rules, improve the fusion level and cognition eyeshot.

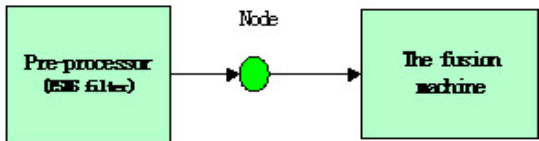


Fig. 3 Unified fusion approach

V. APPLICATION OF UFA TO SLAM

SLAM is a very hot and difficult subject, some one compares it as the puzzle *egg and chicken*. To evaluate the benefit of ESMS filtering technique proposed in this paper, we have carried out a set of simulation (with and without ESMS filter) for solving the SLAM problem with a virtual Pioneer II mobile robot and DSMT-based fusion machine running PCR5 fusion rule). Here Self-localization mainly depends on the odometer on the robot in our simulation experiment. Of course, we also may choose other sensors (i.e. sonar, laser, and vision, etc.) in real environment with respect to the imprecise odometer reading to improve the precision of localization. Map building is an important loop in SLAM. Here we mainly compare the result of map building with and without ESMS filter.

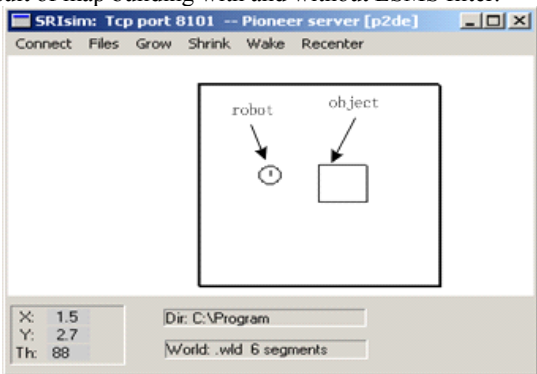


Fig. 4 A world map opened in the SRIsim

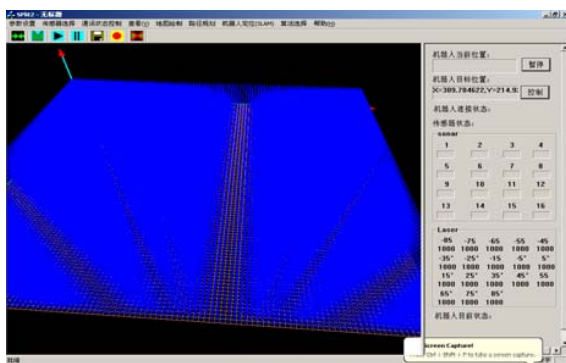


Fig. 5 the platform for simulation or real experiment

The experiment consists in simulating the autonomous navigation of a virtual Pioneer II Robot carrying 16 simulated sonar detectors in a 5000mm× 5000mm square array with an

unknown obstacle/object. The map building with sonar sensors on the mobile robot is done from the simulator of SRIsim (shown in Fig.4) of ActivMedia company and our self-developing experimental or simulation platform together. (shown in fig.5) together. Here the platform developed with the tool software of visual c++ 6.0 and OpenGL servers as a client end, which can connect the sever end (also developed by ourselves, which connects the SRIsim and the client). When the virtual robot runs in the virtual environment, the sever end can collect many information (i.e. the location of robot, sensors reading, velocity, etc.) from the SRIsim. Through the protocol of TCP/IP, the client end can get any information from the sever end and fuse them. The Pioneer II Robot may begin to run at arbitrary location; here we choose the location (1500mm, 2700mm) with an 88 degrees angle the robot faces to. We let the robot move at speeds of trans-velocity 100mm/s and turning-velocity 50degree/s around the object in the *world* map plotted by the Mapper (a simple plotting software), which is opened in the SRIsim shown in fig.4.

We adopt grid method to build map. The global environment is divided into 50× 50 lattices (which of size are same). The object in fig.4 is taken as a regular rectangular box, when the virtual robot runs around the object, through its sonar sensors, we can clearly recognize the object and know its appearance, and even its location in the environment.

DSMT-based fusion machine coupled with PCR5-based Conflict redistributors compose a kind of general sequential fusion machine (in fact, the comparison in map building between DSMT and DST can also be seen in [14]). According to the DSMT, frame of discernment including two focal elements is given in this experiment, that is,  $\Theta = \{\theta_1, \theta_2\}$ , here  $\theta_1$  means grid is empty,  $\theta_2$  means occupied, and then we can get its hyper-power set  $D^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$ , then exists a set of map of  $m(\cdot): D^\Theta \in [0,1]$ , here we can define the general basic belief assignment (gbbaf)  $m(\cdot)$  which are computed according to [14],[15] as follows:  $m(\theta_1)$  is defined as the gbbaf for grid-unoccupied (empty).  $m(\theta_2)$  is defined as the gbbaf for grid-occupied.  $m(\theta_1 \cap \theta_2)$  is defined as the gbbaf for holding grid-unoccupied and occupied simultaneous (conflict).  $m(\theta_1 \cup \theta_2)$  is defined as the gbbaf for grid-ignorance due to the restriction of knowledge and present experience (here referring to the gbbaf for these grids still not scanned presently), it reflects the degree of ignorance of grid-unoccupied or occupied. PCR5 is chosen as a conflict redistributors, which can redistribute proportionally the conflict mass  $m(\theta_1 \cap \theta_2)$  to  $m(\theta_1)$  and  $m(\theta_2)$ .

ESMS filter is taken as pre-processor of fusion, which can filter the inconsistent sources and make evidence more consistent. In this experiment, the ESMS threshold of the ESMS filter had been set to 0.7. The setting of ESMS threshold depends highly on the real system.

To describe the experiment more clearly, the main steps of procedure are given as follows:

- 1) Initialize the parameter of robot (i.e. initial location, moving velocity), etc. Acquire 16 sonar readings, and robot's location, when the robot is running. (here we set the first timer, of which interval is 100 ms).

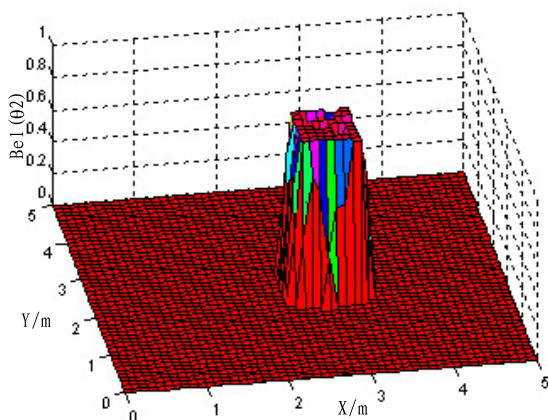


Fig. 6 Map reconstruction without ESMS filter

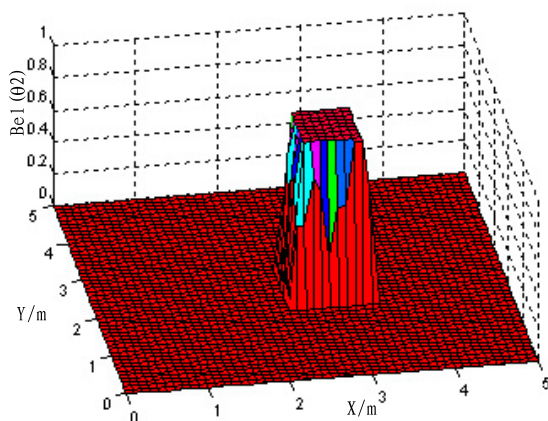


Fig.7 Map reconstruction with ESMS filter

- 2) Compute gamma of the fan-form area detected by each sonar sensor.
- 3) Whether some grids are scanned more than 5 times by sonar sensors (same sonar in different location, or different sonar sensors, of course, here we suppose each sonar sensor has the same characteristics)? If yes, go to next step; otherwise, go to step 2.
- 4) All of 5 evidence sources enter into the ESMS filter. The inconsistent information is filtered, then the remaining consistent sources of evidence feed the sequential fusion machine if at least two sources are consistent enough, otherwise go back to step 2 to acquire new sources of evidence.
- 5) Compute the credibility of occupancy  $bel(\theta_2)$  of some grids, which have been fused.
- 6) Rebuild the map of the environment. (here we set the second timer, of which interval is 100 ms) Whether all the grids have been fused? Yes, stop robot and exist. Otherwise, go to step 2.

Finally, we rebuild the map shown in the Fig.7 with ESMS filter coupled with general sequential fusion machine, also rebuild it without ESMS filter shown in Fig.6. According to our first results presented in Figures 6 and 7, it can be concluded that:

- 1) Recognition rate of the unified fusion approach is higher than only after-processor. Seen from the Fig.7, the brim of object in world map is clearer and more regular than that in Fig.6.
- 2) Fusion time is short, although in the simulation experiment, the time spent by the two methods is not obviously distinct, this is because there are not enough evidence sources, although the system may supply with many evidence sources. When we fuse them, in order to compute simply, we restrict the maximum numbers of evidence sources is less than 5. In fact, if we don't restrict the number of evidence sources in the robot system, or increase the number, then I believe, the fusion time will be obviously short.
- 3) Since the number of sources of evidence which can pass through the ESMS filter is less than the feeding input number, especially when many inconsistent sources are filtered and discarded, the computing amount is very low; The fusion machine only fuses useful/consistent information.

## VI. CONCLUSION

In this paper, we propose unified fusion approach, which extends the range of application of fusion theories and methods, and also improves the precision and correctness of information fusion. Although PCR5 and DSMT coupling with ESMS filter provides very good results in our application, we expect in next works to improve the performances of SLAM using a PCR6-based fusion machine coupled with ESMS filter, since the PCR6 fusion rule proposed recently by Martin and Osswald is said to be more precise [16].

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