Two Stage Fuzzy Methodology to Evaluate the Credit Risks of Investment Projects

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Abstract**—**The work proposes a decision support methodology for the credit risk minimization in selection of investment projects. The methodology provides two stages of projects' evaluation. Preliminary selection of projects with minor credit risks is made using the Expertons Method. The second stage makes ranking of chosen projects using the Possibilistic Discrimination Analysis Method. The latter is a new modification of a well-known Method of Fuzzy Discrimination Analysis.

Keywords**—**Expert valuations, expertons, investment project risks, positive and negative discriminations, possibility distribution.

I. Introduction

A financial activity of banks, investment funds comes with the risk of the loss, especially in the sphere of crediting. Hence the issue of increasing the effectiveness of credit policies and lowering credit risks becomes very topical [10], [17]-[19].

The investment decision-making usually uses special methods, such as logistic regression, discriminant analysis, various machine learning techniques, etc. The methods also can be based on the possibility analysis [2], [20].

Along with traditional statistical techniques, new credit scoring models are developed to support credit decisions. The investment decision-making is influenced by the various uncertainty factors and, the need to formalize and process fuzzy, insufficient and, mainly, expert data. Ignoring the above mentioned factors results in inadequate and non-acceptable decisions. Correct processing of such data is provided with application of fuzzy-set approach [1], [5], [8]-[18], [21]-[23].

Literature, published for the past decade, proposes application of fuzzy-statistical models, neural and fuzzy-neural networks and genetic algorithms when evaluating credit risks [6], [18], [23]. All of the approaches mentioned above are based on the objective databases and expert data.

The authors of this paper are experienced in applying heuristic methods to the decision-making problems which are based on the objective and expert data [5]-[7], [11]-[19]. They propose the methodology that combines two fuzzy-statistical methods and provides the means of evaluating risks of investment decisions. The methodology uses expert data provided by the members of the investment fund experts' commission.

To support the first stage, the Kaufmann's Expertons Method is used [2]-[4]. The method uses interval (pessimistic–optimistic) evaluations defined by the experts in order to reduce a possibly large number of investment projects requesting for credit. The expert knowledge is thereby condensed and compatibility levels on the set of possible risks for each investment project are constructed. The method is described in Section II.

In the second stage the chosen projects are compares and their ranking is made using the modified Possibilistic Discrimination Analysis Method. This method represents a possibilistic generalization of the known Fuzzy Discrimination Analysis [9] and is the modification of Possibilistic Discrimination Analysis Method previously proposed by the author's [18]. Using the expert knowledge and experience, the possibility distribution is constructed on the set of all possible decisions (projects), which is used for projects ranking. The description of the modified method is presented in Section III.

In addition to the proposed new methodology, by the authors is developed software package, which has already been successfully used in the investment tender. An example of investment decision making clearly illustrating the work of the proposed methodology is given in Section IV.

II. Description of the Expertons Method

The use of the expertons theory allows the investment fund experts to evaluate the competition results of investment projects, represent these results in general form, condense data and obtain optimal evaluations. The aim of the expert methodology is to accumulate subjective estimates provided by the expert commission members in our specially created model of the decision-making system. The system then acts as an adviser that assists the expert commission in selecting minimum risk applications.

An experton is the generalization of probability when cumulative probabilities are replaced by monotonically decreasing intervals. These intervals are statistically defined by a group of experts. The concept of the expertons theory can be briefly described as follows.

Let D be a set of decisions (possible risks in our case). The group of r experts is requested to express their subjective opinion regarding each alternative from D in the form of a confidence interval:

$$\forall d \in D: [a_*^j(d), a_j^*(d)] \subset [0,1]$$

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where j is the number of an expert.

We consider statistics when to each element $d \in D$ both the lower and the upper bounds of confidence intervals are assigned. The cumulative distribution law $F_*(\alpha, d)$ is then given by the expressions $a_*^j(d)$, and, $F^*(\alpha, d)$ is given by the expressions $a_j^*(d)$. Thus, we obtain

$$\forall d \in D \quad \forall \alpha \in [0,1]: \quad \tilde{A}(d) = \left[F_*(\alpha, d), F^*(\alpha, d)\right],$$

where $\tilde{A}$ denotes an experton.

Let be given some set of monotone levels: $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_\ell < 1$. The experton, which is used as a possibility to choice possible decision d, is reduced to a possibility distribution on the set of decisions by the averaging of the middle points of interval expertons:

$$\delta(d) = \left(\sum_{i=1}^{\ell}\left(F_*(\alpha_i, d) + F^*(\alpha_i, d)\right)/2\right)\Big/\ell .$$

III. Description of the Modified Possibilistic Discrimination Analysis Method

The modified Possibilistic Discrimination Analysis (presented in this Section) is a modification of the generalized method [7], [17]-[19] of the Fuzzy Discrimination Analysis [9] purposed to process expert data.

Let the set of all possible factors which acted on the possible decisions be $\Omega = \{\omega_1, \omega_2, ..., \omega_m\}$. The factors are determined by the group of experts representing expert commission of investment fund.

The set of decisions that represent all project-competitors with minimum risks selected by the expertons method is denoted by $D = \{d_1, d_2, ..., d_n\}$. Unlike a generalized method [8], [18], here we consider the weighting vector $w = \{w_1, w_2, \ldots, w_m\}$, the component w_i of which defines significance level of ω_i factor for a decision: $w_i \in [0,1]$, $\sum_{i=1}^{m} w_i = 1$. The vector of weights is also determined by experts.

"Classic" variant of Fuzzy Discrimination Analysis Method is based on so-called frequency numerical-tabular knowledge base { f_{ij} }, where f_{ij} designates the fraction of decisions d_j that were correct when ω_i factor was exhibited. Such a knowledge base can be built, if databases of statistical information on successfully implemented investment projects exist. However, in developing countries (instance, in Georgia) such a databases with statistical (objective) data or not exists, or they are, but information in them is insufficient. Therefore the values f_{ij} can only be obtained by the psychometric survey of the experts. Then f_{ij} will designate the fraction of the experts who consider d_j being correct when ω_i factor was present. So, if N experts participate in the psychometric survey, then

$$f_{ij} = N_{ij}/N , \tag{1}$$

where N_{ij} is the number of experts who supported decision d_j when for the d_j competitor ω_i factor was present. And then instead of the frequency tabular-numeric knowledge base { f_{ij} } build so called possibilistic tabular-numeric knowledge base { π_j^i } [7], [9], [11], [17]-[19]. For instance, by normalizing each column of { f_{ij} } we obtain

$$\pi_j^i = f_{ij}\Big/\max_{j=1,n} f_{ij} . \tag{2}$$

The algorithm of a modified Possibilistic Discrimination Analysis Method is following:

The algorithm of a modified Possibilistic Discrimination Analysis Method is following:

1) Transform the possibilistic distribution table to the probabilistic distribution table [2], [11], [19]:

For $\forall \omega_i$ ($i = 1,2,...,m$) let $\pi_{j_1}^i \geq \pi_{j_2}^i \geq \cdots \geq \pi_{j_n}^i$, then the conditional probability p_j^i corresponding to the possibility π_j^i is expressed by the formula

$$p_{j_s}^i = \sum_{\ell=s}^{n} \frac{1}{\ell}\left(\pi_{j_\ell}^i - \pi_{j_{\ell+1}}^i\right), s = 1, 2, \ldots, n , \ \pi_{j_{n+1}}^i \equiv 0 ; \tag{3}$$

2) Build positive and negative discriminations on $\Omega \times D$ and calculate the concrete compatibility levels which define how much the ω_i factor influence (positive discrimination) and how much it does not influence (negative discrimination) the decision d_j as compared with other decisions:

$$p_{ij} = \frac{1}{n+1}\left\{1 + \sum_{k:\,p_k^i < p_j^i}\left(p_j^i - p_k^i\right)^{\alpha_1} + \frac{1}{1 + \sum_{k:\,p_k^i > p_j^i}\left(p_k^i - p_j^i\right)^{\alpha_2}}\right\}, \tag{4}$$

$$n_{ij} = \frac{1}{n+1}\left\{1 + \sum_{k:\,p_k^i > p_j^i}\left(p_k^i - p_j^i\right)^{\alpha_1} + \frac{1}{1 + \sum_{k:\,p_k^i < p_j^i}\left(p_j^i - p_k^i\right)^{\alpha_2}}\right\}, \tag{5}$$

$$\alpha_s > 0, \quad s = 1,2.$$

Remark 1: clearly, positive and negative discriminations are not the notions which complement each other (in general, $p_{ij} + n_{ij} \neq 1$) and therefore the influence of the factor ω_i on the decision d_j is represented in the form of pairs (p_{ij} ; n_{ij}).

Remark 2: if π_j^i are numbers of "nearly" the same order, then for the "spectral decomposition" of p_{ij} and n_{ij} we will take values $\alpha_s < 1$;

3) Build the following weighted average positive and negative discriminations on the set of decisions D :

$$\pi_j = \sum_{i=1}^{m} p_{ij} w_i \ , \quad \nu_j = \sum_{i=1}^{m} n_{ij} w_i \ , \qquad (6)$$
$$j = 1,2,\ldots,n ;$$

4) Build possibility distribution $\forall j = 1,2,...,n$ on the possible decisions D:

$$\delta_j = \left(\pi_j^{\beta} + (1-\nu_j)^{\beta}\right)/2 \, , \quad \beta > 0 \, ; \qquad (7)$$

Remark 3: if π_j and ν_j are numbers of "nearly" the same order, then for the "spectral decomposition" of δ_j we will take values $\beta < 1$;

5) Perform the ranking of possible decisions by sorting their possibility levels in decreasing order: $d_i \succcurlyeq d_j$, if $\delta_i \geq \delta_j$, where $\succcurlyeq$ is the preference relation on D.

Remark 4: if necessary, regard the decision δ_{j_0} which has a maximum value on the possibility distribution $\{ \delta_j \}$ as the most convincing decision:

$$\delta_{j_0} = \max_j \delta_j \, .$$

IV. An Example of the Application of Fuzzy Methodology to Select Candidates with Minimal Credit Risk

Processing the information with the expertons method allows for selecting only those applicants whose profile provides either insignificant- or, possibly, low - credit risk.

Let's presume that the possible risk estimates for a given applicant, i.e. the possible decisions (crediting risks) are: d_1: crediting with an insignificant risk; d_2 : crediting with a low risk; d_3: crediting with an average risk; d_4 : crediting with a high risk.

A. Preliminary Selection with the Expertons Method

Assume that the members of the expert commission consider 4 possible decisions (i.e. the levels of credit granting risks for the concrete competitor) d_1, d_2, d_3, d_4. Instead of expressing their opinion by value $\alpha \in [0,1]$, they provide confidence intervals which are included in the interval $[0,1]$: $[a_1, a_2] \subset [0,1]$, where a_1 is the pessimistic level of given risk and a_2 is the optimistic level of the risk.

The aggregate table of experts' estimates may have the following form:

TABLE I
THE AGGREGATE TABLE OF EXPERTS' ESTIMATES

Experts j	Possible decisions d_i: d_1	d_2	d_3	d_4
1	[0.3,0.5]	[0.6,0.7]	[0.3,0.4]	0.5
2	[0.5,0.6]	[0.4,0.6]	[0,0.1]	[0.2,0.4]
3	[0.4,0.7]	[0.8,0.9]	[0.1,0.4]	[0.1,0.3]
4	[0.3,0.4]	1	[0.2,0.5]	0
5	0.6	[0.7,0.9]	[0.1,0.4]	[0.5,0.7]
6	[0.8, 1]	[0.2,0.3]	0.4	0.3
7	[0.4,0.8]	[0,0.1]	[0.3,0.7]	[0.6,0.7]
8	[0.4,0.5]	1	[0.8,1]	0.4
9	[0,0.2]	[0.8,1]	0	[0.3,0.5]
10	[0.6,0.8]	[0.4,0.7]	0.5	[0.2,0.6]

Let us consider 11 α -cuts from 0 to 1, and for each of the possible decisions $d_j, j = 1,\ldots,4$ calculate two statistics for each cut: one for the lower boundary of an interval and, the other, for the upper boundary. By extending these statistics to the set of levels $\{ 0, 0.1, 0.2, \ldots, 0.9, 1 \}$, we obtain experton (see Table II):

TABLE II
EXPERTON

Level	Possible decisions: d_1	d_2	d_3	d_4
0	1	1	1	1
0.1	[0.9, 1]	[0.9, 1]	[0.8, 0.9]	0.9
0.2	[0.9, 1]	0.9	[0.6, 0.8]	[0.8, 0.9]
0.3	[0.9, 0.9]	[0.8, 0.9]	[0.5, 0.8]	[0.6, 0.9]
0.4	[0.7, 0.9]	0.8	[0.3, 0.8]	[0.4, 0.7]
0.5	[0.4, 0.8]	[0.6, 0.8]	[0.2, 0.4]	[0.3, 0.5]
0.6	[0.3, 0.6]	[0.6, 0.8]	[0.1, 0.2]	[0.1, 0.3]
0.7	[0.1, 0.4]	[0.5, 0.7]	[0.1, 0.2]	[0, 0.3]
0.8	[0.1, 0.3]	[0.4, 0.5]	0.1	0
0.9	[0, 0.1]	[0.2, 0.5]	[0, 0.1]	0
1	[0, 0.1]	[0.2, 0.3]	[0, 0.1]	0

An experton $\tilde{A}$ is then transformed as follows:

- an averaged experton is calculated by taking a mean arithmetic value of each interval boundaries;
- the averaged experton is reduced to a fuzzy set by calculating mean values;
- if necessary, a nonfuzzy set, the closest to the fuzzy one, is found.

In a given example we calculate averaged experton:

TABLE III
THE AVERAGED EXPERTON

Level	Possible decisions: d_1	d_2	d_3	d_4
0	1	1	1	1
0.1	0.95	0.95	0.85	0.90
0.2	0.95	0.90	0.70	0.85
0.3	0.95	0.85	0.65	0.75
0.4	0.90	0.80	0.55	0.55
0.5	0.80	0.70	0.30	0.40
0.6	0.60	0.70	0.15	0.20
0.7	0.45	0.60	0.15	0.10
0.8	0.25	0.45	0.10	0
0.9	0.05	0.35	0.05	0
1	0.05	0.25	0.05	0

After calculating the mean values for each d_i on $D=\{d_1,d_2,d_3,d_4\}$, we obtain the possibility distribution $\delta(d_i)$ of identified risks of the considered competitor:

$$\{\, d_1/0.56364,\, d_2/0.68636,\, d_3/0.41364,\, d_4/0.43182 \,\}.$$

To receive unique decision we apply the principle of a maximum: $\delta_{i_0}=\max_i \delta(d_i)$.

This means that in accordance to the common opinion of the experts the experton gives preference to the decision d_2, i.e. considered competitor has a low crediting risk. Such a decision, of course, permits the applicant to participate at the second stage of the competition.

B. Final Decision with the Possibilistic Discrimination Analysis

The second stage of the decision making methodology chooses from the number of the selected candidates by evaluating certain factors characteristic to these candidates. In our tender, after processing data by the expertons method, for further consideration only six applicants remained from thirty-four possible candidates considered at the first stage. Their data then was processed by modified possibilistic discrimination analysis.

Let us determine main $\omega_k, k=1,2,\ldots,9$ factors, by which all of the tender commission experts will score the candidate juridical person seeking the credit.

The following factors influencing the decisions will be considered [11], [19]:

ω_1: business profitability; ω_2: purpose of the credit; ω_3: pledge guaranteeing repayment of the credit; ω_4: credit amount (monetary value); ω_5: payment of interest ; ω_6: credit granting date; ω_7: credit repayment date; ω_8: monthly payment of a portion of the principal and accrued interest (repayment scheme); ω_9: percent ratio of the pledge to the credit monetary amount.

In our case, the value f_{ij} describes the level of the i factor for the j participant. Thus, the expert commission consists of 10 members, the factors to be evaluated are $\omega_k, k=1,2,\ldots,9$, and, after the preliminary selection, the number of competitors equals to 6 (d_j, $j=1,2,\ldots,6$).

Suppose that the aggregate table f_{ij} looks like (see Table IV):

TABLE IV
THE AGGREGATE TABLE OF f_{ij} VALUES

Ω	d_1	d_2	d_3	d_4	d_5	d_6
ω_1	0.6	0.5	0.7	0.1	0.3	0.4
ω_2	0.4	0.8	0.1	0.5	0.2	0.3
ω_3	0.1	0.5	0.2	0.4	0.6	0.3
ω_4	0.4	0.3	0.6	0.1	0.2	0.7
ω_5	0.5	0.4	0.8	0.3	0.7	0.6
ω_6	0.6	0.1	0.3	0.4	0.5	0.2
ω_7	0.3	0.3	0.2	0.4	0.4	0.2
ω_8	0.5	0.2	0.3	0.6	0.4	0.3
ω_9	0.2	0.6	0.4	0.5	0.8	0.2

Firstly, using (2) we calculate the table of π_j^i conditional possibilistic distribution (see Table V):

TABLE V
THE TABLE OF π_j^i POSSIBILISTIC DISTRIBUTION

Ω	d_1	d_2	d_3	d_4	d_5	d_6
ω_1	0.86	0.71	1.00	0.14	0.43	0.57
ω_2	0.50	1.00	0.13	0.63	0.25	0.38
ω_3	0.17	0.83	0.33	0.67	1.00	0.50
ω_4	0.57	0.43	0.86	0.14	0.29	1.00
ω_5	0.63	0.50	1.00	0.38	0.88	0.75
ω_6	1.00	0.17	0.50	0.67	0.83	0.33
ω_7	0.75	0.75	0.50	1.00	1.00	0.50
ω_8	0.83	0.33	0.50	1.00	0.67	0.50
ω_9	0.25	0.75	0.50	0.63	1.00	0.25

By converting it to the table of conditional probabilistic distribution p_j^i, we receive (see (3) and Table VI):

TABLE VI
THE TABLE OF CONDITIONAL PROBABILISTIC DISTRIBUTION p_j^i

Ω	d_1	d_2	d_3	d_4	d_5	d_6
ω_1	0.236	0.164	0.379	0.024	0.081	0.117
ω_2	0.119	0.556	0.021	0.181	0.046	0.077
ω_3	0.028	0.242	0.061	0.158	0.408	0.103
ω_4	0.136	0.088	0.279	0.024	0.052	0.421
ω_5	0.119	0.087	0.348	0.063	0.223	0.160
ω_6	0.408	0.028	0.103	0.158	0.242	0.061
ω_7	0.146	0.146	0.083	0.271	0.271	0.083
ω_8	0.228	0.056	0.089	0.394	0.144	0.089
ω_9	0.042	0.208	0.104	0.146	0.458	0.042

Further, to calculate the tables of positive and negative discriminations, we take the values $\alpha_1=0.25$, $\alpha_2=0.75$ (chosen empirically for the “spectral decomposition” of the values p_{ij} and n_{ij}) as the coefficients of α_s, $s=1,2$. As a result, (see (4), (5)) we receive the table of positive and negative discriminations (see Table VII):

TABLE VII
THE TABLE OF POSITIVE AND NEGATIVE DISCRIMINATIONS

	D					
	d_1	d_2	d_3	d_4	d_5	d_6
	0.603	0.472	0.789	0.205	0.288	0.369
	0.448	0.876	0.207	0.569	0.270	0.350
	0.207	0.610	0.276	0.473	0.806	0.364
	0.457	0.355	0.641	0.206	0.272	0. 811
p_{ij}	0.361	0.279	0.77	0.214	0.591	0.463
	0.806	0.207	0.364	0.473	0.610	0.276
	0.386	0.386	0.221	0.644	0.644	0.221
	0.599	0.213	0.285	0.799	0.454	0.285
	0.211	0.584	0.368	0.462	0.833	0.211
	0.218	0.262	0.175	0.469	0.366	0.311
	0.294	0.171	0.462	0.255	0.394	0.339
	0.463	0.221	0.385	0.269	0.174	0.323
	0.287	0.335	0.215	0.464	0.393	0.173
n_{ij}	0.304	0.360	0.175	0.428	0.216	0.258
	0.174	0.463	0.323	0.270	0.221	0.385
	0.274	0.274	0.403	0.184	0.184	0.403
	0.222	0.433	0.351	0.174	0.272	0.351
	0.441	0.236	0.319	0.276	0.173	0.441

We proceed with calculating π_j and ν_j representing the weighted average values of positive and negative discriminations for the j th applicant (see (6)). The weighted vector

$$w = \{0.125, 0.125, 0.125, 0.11, 0.125, \\ 0.0775, 0.0775, 0.11, 0.123\}$$

is defined by expert commission.

Taking the coefficient value equal to $\beta = 0.95$ (chosen empirically for the "spectral decomposition" of the values δ_j), we determine the possibility distribution on $D = \{d_1, d_2, \dots, d_8\}$ (see (7) and Table VIII):

TABLE VIII
THE POSITIVE AND NEGATIVE DISCRIMINATION'S WEIGHTED AVERAGE VALUES AND THE POSSIBILITY DISTRIBUTION ON *D*

D	π_j	ν_j	δ_j
d_1	0.43735	0.30574	0.58143
d_2	0.46102	0.29780	0.59697
d_3	0.44837	0.30794	0.58582
d_4	0.43759	0.31749	0.57586
d_5	0.52554	0.26969	0.64230
d_6	0.37874	0.32766	0.54170

Table VIII shows that ranking of considered projects according to possibility distribution δ_j is the following:

$$d_5 \succ d_2 \succ d_3 \succ d_1 \succ d_4 \succ d_6.$$

V. CONCLUSION

We developed the methodology of experts information processing and synthesis. The methodology is the combination of Kaufmann's expertons method and the possibilistic discrimination analysis proposed in the present work. The methodology provides experts with the opportunity to manifest intellectual activity of a high level. Securing the freedom of experts' subjective evaluations, the methodology, however, allows for developing experts' joint decision on granting credits. The latter distinguishes our methodology from the others.

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