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Two Spherical Three Degrees of Freedom Parallel Robots 3-RCC and 3-RRS Static Analysis

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***Abstract*—**The main purpose of this study is static analysis of two three-degree of freedom parallel mechanisms: 3-RCC and 3-RRS. Geometry of these mechanisms is expressed and static equilibrium equations are derived for the whole chains. For these mechanisms due to the equal number of equations and unknowns, the solution is as same as 3-RCC mechanism. A mathematical software is used to solve the equations. In order to prove the results obtained from solving the equations of mechanisms, the CAD model of these robots has been simulated and their static is analysed in ADAMS software. Due to symmetrical geometry of the mechanisms, the force and external torque acting on the end-effecter have been considered asymmetric to prove the generality of the solution method. Finally, the results of both softwares, for both mechanisms are extracted and compared as graphs. The good achieved comparison between the results indicates the accuracy of the analysis.

***Keywords*—**Robotic, Static analysis, 3-RCC, 3-RRS

I. Introduction

A PARALLEL robot has some chains which are parallel together. They are from base to end-effector and they move the end-effector to a certain point [1]. They have many advantages in comparison to serial mechanisms. MacCallion and Pham [2] have suggested using parallel devices. Then, after a systematic review of the different kinematic possibilities for parallel manipulators, a few architectures [3] of fully parallel devices emerged as the most promising designs [4]. These mechanisms usually have short, simple and rigid chains which make them more stable than serial mechanisms against unwanted movements. One advantage of the parallel manipulators is that in these mechanisms the actuators are mounted on the ground which makes them much lighter. This reduction makes the arms lighter with faster movements. This also makes a great decrease in total moment of inertia which is very important for mobile and haptic robots.

All these features result in mechanisms with a wide range of motion ability. As their speed of action is often constrained by their stiffness rather than sheer power, they can be fast-acting, in comparison to serial ones. Most robot applications require this stiffness. Using high-quality rotary joints provide them this ability and permit movements in one axis but make rigidity against movements outside this.

Parallel mechanisms workspace is one of their important drawbacks in comparison to serial manipulators. These manipulators have the limitation emanated from geometrical and mechanical limits of the design as like as serial manipulators. But the important thing is that they are also limited by the existence of singularities, which are positions where, for some trajectories of the movement, the variation of the lengths of the legs is infinitely smaller than the variation of the position.

The singular points in the workspace of a parallel manipulator [5] are composed of continuous and differentiable sub manifolds in the workspace of the mechanism. While moving along a specified path, if the manipulator comes close to a singular point, the leg actuator forces increase extremely. If the manipulator is controlled quasistatically [6], before achieving the singularity, the actuator forces always attain their limits [7].

Another drawback of parallel manipulators is their nonlinear behaviour: Location of end-effector has an important effect on the order which is required to get a movement of that. This nonlinearity is difficult and it is the cause that parallel manipulators are not yet used in high precision machining, despite their advantages. Most of studies in this context are on synthesis of parallel robots. Some systematic methods were presented to synthesis these mechanisms, Such as screw theory [8], group of rigid body displacements theory [9], and virtual chain approach [10]. With these methods parallel robots with desired motions of the end-effector can be designed. According to end-effector motion pattern, 3-DOF parallel robots are divided to four types; three transitions DOF, three rotations DOF, two transitions and one rotation DOF and two rotation and one transition DOF [11]. 3-$\underline{R}$CC and 3-$\underline{R}$RS have both three rotations DOF. Statics is a branch of mechanics which is concerned with the analysis of loads (force and torque, or moment) on physical systems in static equilibrium, a state where the relative positions of subsystems do not vary over time, or where components and structures are at a constant velocity. In static equilibrium, the system is either at rest, or its center of mass moves at a constant velocity. By Newton's first law, this situation implies that the net force and net torque (also known as moment of force) on every part of the system is zero. From this constraint, such quantities as stress or pressure can be derived. The net force equalling zero is known as the first condition for equilibrium, and the net torque

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equalling zero is known as the second condition for equilibrium.

Pham and Chen proposed a 2-DOF flexure parallel mechanism and studied its kinematic and static analysis [12]. Lu et al. explored static analyzing of a parallel 3-DOFs robot by two methods [13]. Tale Masouleh et al. explored the direct kinematic of parallel robot 5-RPUR. They also suggested two ways to solve the direct kinematic problem in parallel robots [14]. Masouleh et al. also proposed two ways to solve forward kinematic of parallel robot based on study parameters [15]. According to previous researches on the static of parallel robots and because of these little researches on 3-DOF over constraint robots, static analysis of these robots are limited.

In this paper geometric structure of 3-RCC and 3-RRS robots are described and their static equilibrium has been presented. To solve these equations due to their equal unknowns and equations, exact answers could be found. Also the cad model of these robots is simulated in ADAMS software and the results are compared with the results extracted from MATLAB software and these comparisons are brought in some diagrams.

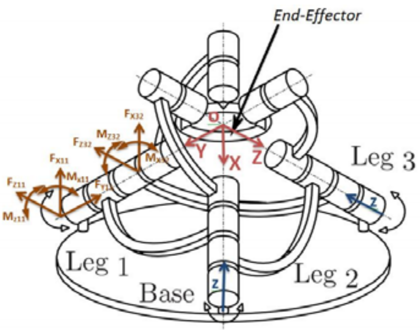

Fig. 1 CAD Model of 3-RCC [16]

II. Geometry of the Robots

In this section, the geometry of each robots and type of joints used in them are described. It is noteworthy throughout this paper, revolute, spherical, and cylindrical joints are shown by R, S, and C respectively. Also, a line is drawn under the active joints, which distinguishes them from the passive joints.

A. 3-RCC

This robot was introduced with One degree of rotational freedom and two of movement pattern. Fig. 1 depicts the 3-RCC robot. A revolute joint with the axis of rotation z, organizes input and active joint in each chain. The joint axes of the second and third revolute joints in each kinematic chain are intersecting and perpendicular to each other and also are perpendicular to the axis of rotation of active joints. The rotation axis of joints connected to the end-effector and joints connected to the base are intersecting and perpendicular to each other.

B. 3-RRS

The 3-RRS robot is represented in Fig. 2. A revolute joint with the axes of rotation z, organizes input and active joint in each chain. The third joint in each chain is a spherical joint which rotates about three perpendicular axes. As the 3-RCC robot, the axis of rotation of joints connected to the end-effector and joints connected to the base are intersecting and perpendicular to each other.

III. Static Analysis of Robots

In this section the static equilibrium equations for each chain of robots are extracted.

A. 3-RCC

This robot consists of the end-effector, the base and three arms. There are one revolute and two cylindrical joints in each arm. Each rotational joint has a rotational degree of freedom along its axis of rotation. There is no moment on the joints in this direction. But they have two moments in two other directions which are perpendicular to the axis of rotation and they have some forces along all of three directions. So there are five unknowns for each of the rotational joints. Each cylindrical joint has four unknowns (two moments and two forces). Also one moment on each motor is unknown in the static analysis. Consequently there are 3*(5+4+4) + 3 =42 unknowns in this mechanism. There are 6 static equilibrium equations in each arm. The end-effector has 6 equilibrium equations. In sum-up this robot has 6+ (3*12) =42 equations.

General form of static equilibrium equations are:

$$\sum \boldsymbol{f} = 0 \tag{1}$$

$$\sum \boldsymbol{m} = 0 \tag{2}$$

In which f and m illustrate force and moment respectively. The external force (f) has exerted on the end-effector which is downward and perpendicular to it. Also external moment (m) has exerted about the same direction. This direction is the same that the end-effector weight is exerted to it. In equations p_e denotes summation of the decomposition of the external force and the end-effector weight in each of the three directions x, y and z. The distance between the origin (O) to the location of the first, second and third joint in each arm and along their rotation axis are represented by r_1, r_2 and r_3 respectively. The decomposition of the weight of the proximal and distal links in each arm and in mentioned directions are shown by p_1 and p_2. The distance between mass center of proximal links to the location of the first joint is shown by l_{ij} in which i illustrates the number of arm and j shows the direction. For example l_{1x} is the distance between mass center of the first arms proximal joint to the location of the first joint of this arm along the x axis. Similarly distance between the mass center of distal link to the location of the second joint in each arm is shown by s_{ij}. The moment on the motors installed on active joints are shown by m_{xt2}, m_{yt1} and m_{zt3}. F_{ijk} demonstrates force in joint k and arm j in i direction. In the same way, m_{ijk} illustrates moment in joint k and arm j about i direction.

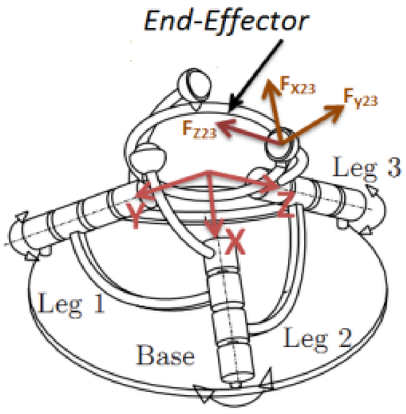

Fig. 2 CAD Model of 3-RRS [15]

TABLE I
DIMENSIONS FROM CAD MODEL OF 3-RCC

Symbol	Dimension (mm)	Symbol	Dimension (mm)
r_1	0.195	l_{3y}	0.07483
r_2	0.162	l_{3z}	-0.09129
r_3	0.082	s_{1x}	-0.08686
p_1	0.1646	s_{1y}	0
p_2	0.1565	s_{1z}	-0.0533
l_{1x}	0.07483	s_{2x}	0
l_{1y}	-0.09129	s_{2y}	-0.0533
l_{1z}	0	s_{2z}	-0.08686
l_{2x}	-0.09129	s_{3x}	-0.0533
l_{2y}	0	s_{3y}	-0.08686
l_{2z}	0.07483	s_{3z}	0
l3x	0	f_x	2*cos(t)
f_y	4*sin(t)	f_z	6*sin(t)
a_x	3*sin(t)	a_y	5*sin(t)
a_z	7*cos(t)		

$$\boldsymbol{f}_{jk} = \begin{bmatrix} f_{xjk} \\ f_{yjk} \\ f_{zjk} \end{bmatrix} \tag{3}$$

$$\boldsymbol{m}_{jk} = \begin{bmatrix} m_{xjk} \\ m_{yjk} \\ m_{zjk} \end{bmatrix} \tag{4}$$

Equilibrium equations for end-effector and the first arm are:

$$\begin{aligned} -f_{x13} - f_{x23} + p_{ex} &= 0 \\ -f_{y13} - f_{y33} + p_{ey} &= 0 \\ -f_{z23} - f_{z33} + p_{ez} &= 0 \end{aligned} \tag{5}$$

$$\begin{aligned} -f_{y13}r_3 + f_{z23}r_3 - m_{x13} - m_{x23} + a_x &= 0 \\ -f_{z33}r_3 + f_{x13}r_3 - m_{y13} - m_{y33} + a_y &= 0 \\ -f_{x23}r_3 + f_{y33}r_3 - m_{z23} - m_{z33} + a_z &= 0 \end{aligned} \tag{6}$$

$$\begin{aligned} -f_{x11} + p_2 &= 0 \\ -f_{y11} - f_{y12} + p_2 &= 0 \\ -f_{z11} - f_{z12} + p_2 &= 0 \end{aligned} \tag{7}$$

$$\begin{aligned} -m_{x11} + f_{z12}r_1 + p_2 l_{1y} - p_2 l_{1z} &= 0 \\ -m_{y12} + f_{z12}r_2 - p_2 l_{1x} + p_2 l_{1z} + m_{yt1} &= 0 \\ -m_{z11} - m_{z12} - f_{y12}r_2 + p_2 l_{1x} - p_2 l_{1y} &= 0 \end{aligned} \tag{8}$$

$$\begin{aligned} f_{x13} + p_1 &= 0 \\ f_{y12} + f_{y13} + p_1 &= 0 \\ f_{z12} + p_1 &= 0 \end{aligned} \tag{9}$$

$$\begin{aligned} m_{x13} + f_{y13}r_3 + p_1 s_{1y} - p_1 s_{1z} &= 0 \\ m_{y13} + m_{y12} - f_{x13}r_3 + p_1 s_{1z} - p_1 s_{1x} &= 0 \\ m_{z12} - f_{y13}r_2 + p_1 s_{1x} - p_1 s_{1y} &= 0 \end{aligned} \tag{10}$$

Similarly equations for the links of the first, second, and third arms are:

$$\begin{aligned} -f_{x21} - f_{x22} + p_2 &= 0 \\ -f_{y21} - f_{y22} + p_2 &= 0 \\ -f_{z21} + p_2 &= 0 \end{aligned} \tag{11}$$

$$\begin{aligned} -m_{x22} + f_{y22}r_2 + p_2 l_{2y} - p_2 l_{2z} + m_{xt2} &= 0 \\ -m_{y21} - m_{y22} - f_{x22}r_2 - p_2 l_{2x} + p_2 l_{2z} &= 0 \\ -m_{z21} + f_{y22}r_1 + p_2 l_{2x} - p_2 l_{2y} &= 0 \end{aligned} \tag{12}$$

$$\begin{aligned} f_{x22} + f_{x23} + p_1 &= 0 \\ f_{y22} + p_1 &= 0 \\ f_{z23} + p_1 &= 0 \end{aligned} \tag{13}$$

$$\begin{aligned} m_{x22} + m_{x23} - f_{z23}r_3 + p_1 s_{2y} - p_1 s_{2z} &= 0 \\ m_{y22} - f_{x23}r_2 + p_1 s_{2z} - p_1 s_{2x} &= 0 \\ m_{z23} + f_{x23}r_3 + p_1 s_{2x} - p_1 s_{2y} &= 0 \end{aligned} \tag{14}$$

$$\begin{aligned} -f_{x31} - f_{x32} + p_2 &= 0 \\ -f_{y31} + p_2 &= 0 \\ -f_{z31} - f_{z32} + p_2 &= 0 \end{aligned} \tag{15}$$

$$\begin{aligned} -m_{x31} - m_{x32} - f_{z32}r_2 + p_2 l_{3y} - p_2 l_{3z} &= 0 \\ -m_{y31} + f_{x32}r_1 + p_2 l_{3z} - p_2 l_{3x} &= 0 \\ -m_{z32} + f_{x32}r_2 + p_2 l_{3x} - p_2 l_{3y} + m_{zt3} &= 0 \end{aligned} \tag{16}$$

$$\begin{aligned} f_{x32} + p_1 &= 0 \\ f_{y33} + p_1 &= 0 \\ f_{z32} + f_{z33} + p_1 &= 0 \end{aligned} \tag{17}$$

$$\begin{aligned} m_{x32} - f_{z33} r_2 + p_1 s_{3y} - p_1 s_{3z} &= 0 \\ m_{y33} + f_{z33} r_3 + p_1 s_{3z} - p_1 s_{3x} &= 0 \\ m_{z32} + m_{z33} - f_{y33} r_3 + p_1 s_{3x} - p_1 s_{3y} &= 0 \end{aligned} \tag{18}$$

Dimensions have been obtained from the CAD model are written in Table I.

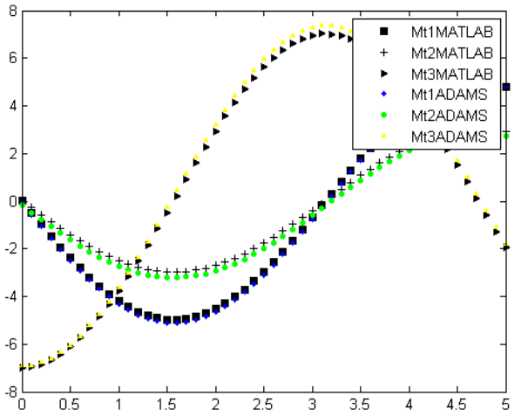

Fig. 3 Diagram of Torque – Time in 3-RCC

Set of equations can be written as:

$$[A][x]=[b] \tag{19}$$

Matrix [A] is a 42*42 matrix named coefficient matrix, [X] is a 42*1 matrix expresses unknown matrix and [B] is the constants matrix and it is 42*1. Number of equations and unknowns are equal, so there is exact solution for it.

This set of equations was solved in MATLAB software and the results are extracted for the force of active joints along their rotation axis. The cad models of these mechanisms have been simulated in ADAMS software and analyzed statically.

B. 3-RRS

This robot has an end-effector, a fixed base and three arms. Each arm has two revolute joints and one spherical joint. As you see in the static analysis of a 3-RCC robot, there are 5 unknowns in each revolute joint. Each spherical joint has three forces which are perpendicular to each other but it has no moment, so it can rotate about each axis and this set of equations has (5+3+5)*3=39 unknowns. As mentioned before each motor has a moment unknown, therefore it has 42 unknowns. Each link has six static equilibrium equations, thus each arm has 12. The end-effector has six static equilibrium equations, consequently this robot has (12*3) +6=42 equations. The equations for end-effector are similar to 3-RCC end-effector. Static equilibrium equations for the first, second and third arm of this robot are:

$$\begin{aligned} -f_{x11} - f_{x12} + p_2 &= 0 \\ -f_{y11} - f_{y12} + p_2 &= 0 \\ -f_{z11} - f_{z12} + p_2 &= 0 \end{aligned} \tag{20}$$

$$\begin{aligned} -m_{x11} + f_{z12} r_1 + p_2 l_{1y} - p_2 l_{1z} &= 0 \\ m_{yt1} + f_{z12} r_2 - p_2 l_{1x} + p_2 l_{1z} - m_{y12} &= 0 \\ -m_{z11} - m_{z12} - f_{x12} r_1 - f_{y12} r_2 + p_2 l_{1x} - p_2 l_{1y} &= 0 \end{aligned} \tag{21}$$

$$\begin{aligned} f_{x12} + f_{x13} + p_1 &= 0 \\ f_{y12} + f_{y13} + p_1 &= 0 \\ f_{z12} + f_{z13} + p_1 &= 0 \end{aligned} \tag{22}$$

TABLE II
DIMENSIONS FROM CAD MODEL OF 3-RRS

Symbol	Dimension (mm)	Symbol	Dimension (mm)
r_1	0.195	l_{3y}	0.07483
r_2	0.162	l_{3z}	-0.09129
r_3	0.082	s_{1x}	-0.08686
p_1	0.1646	s_{1y}	0
p_2	0.1565	s_{1z}	-0.0533
l_{1x}	0.07483	s_{2x}	0
l_{1y}	-0.09129	s_{2y}	-0.0533
l_{1z}	0	s_{2z}	-0.08686
l_{2x}	-0.09129	s_{3x}	-0.0533
l_{2y}	0	s_{3y}	-0.08686
l_{2z}	0.07483	s_{3z}	0
l_{3x}	0	f_x	-2*cos(t)
f_y	-4*cos(t) + 3*sin(t)	f_z	sin(t) - cos(t)
a_x	3*sin(t) + cos(t)	a_y	-5*sin(t) + cos(t)
a_z	-sin(t) + 2*cos(t)		

$$\begin{aligned} -m_{x31} - m_{x32} - f_{z32} r_2 - f_{y32} r_1 + p_2 l_{3y} - p_2 l_{3z} &= 0 \\ -m_{y31} + f_{x32} r_1 + p_2 l_{3z} - p_2 l_{3x} &= 0 \\ -m_{z32} + f_{x32} r_2 + p_2 l_{3x} - p_2 l_{3y} + m_{zt3} &= 0 \end{aligned} \tag{23}$$

$$\begin{aligned} m_{x22} + f_{y22} r_2 + p_2 l_{2y} - p_2 l_{2z} + m_{xt2} &= 0 \\ -m_{y21} - m_{y22} - f_{z22} r_1 - f_{x22} r_2 - p_2 l_{2x} + p_2 l_{2z} &= 0 \\ -m_{z21} + f_{y22} r_1 + p_2 l_{2x} - p_2 l_{2y} &= 0 \end{aligned} \tag{24}$$

$$\begin{aligned} f_{x22} + f_{x23} + p_1 &= 0 \\ f_{y22} + f_{y23} + p_1 &= 0 \\ f_{z22} + f_{z23} + p_1 &= 0 \end{aligned} \tag{25}$$

$$\begin{aligned} m_{x22} - f_{z23} r_3 + f_{y23} r_2 + p_1 s_{2y} - p_1 s_{2z} &= 0 \\ f_{x23} r_3 + p_1 s_{2x} - p_1 s_{2y} &= 0 \\ m_{y22} - f_{x23} r_2 + p_1 s_{2z} - p_1 s_{2x} &= 0 \end{aligned} \tag{26}$$

$$\begin{aligned} -f_{x31} - f_{x32} + p_2 &= 0 \\ -f_{y31} - f_{y32} + p_2 &= 0 \\ -f_{z31} - f_{z32} + p_2 &= 0 \end{aligned} \tag{27}$$

$$\begin{aligned} -m_{x31} - m_{x32} - f_{z32} r_2 - f_{y32} r_1 + p_2 l_{3y} - p_2 l_{3z} &= 0 \\ -m_{y31} + f_{x32} r_1 + p_2 l_{3z} - p_2 l_{3x} &= 0 \\ -m_{z32} + f_{x32} r_2 + p_2 l_{3x} - p_2 l_{3y} + m_{zt3} &= 0 \end{aligned} \tag{28}$$

$$f_{x32} + f_{x33} + p_1 = 0$$
$$f_{y32} + f_{y33} + p_1 = 0 \quad (29)$$
$$f_{z32} + f_{z33} + p_1 = 0$$

$$f_{y13}r_3 + p_1 s_{1y} - p_1 s_{1z} = 0$$
$$m_{y12} + f_{z13}r_2 - f_{x13}r_3 + p_1 s_{1z} - p_1 s_{1x} = 0 \quad (30)$$
$$m_{z12} - f_{y13}r_2 + p_1 s_{1x} - p_1 s_{1y} = 0$$

Dimensions obtained from CAD model are written in Table II. To solve these equations, there are:

$$[\boldsymbol{A}]_{42\times42}[\boldsymbol{x}]_{42\times1} = [\boldsymbol{b}]_{42\times1} \quad (31)$$

$$[\boldsymbol{x}] = [\boldsymbol{A}]^{-1}[\boldsymbol{b}] \quad (32)$$

This set of equations has been solved similar to 3-RCC in MATLAB and ADAMS software.

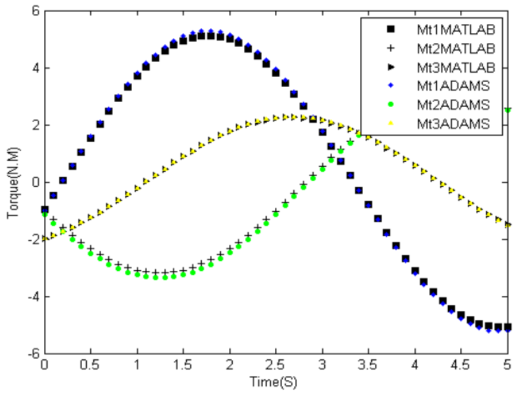

Fig. 4 Diagram of Torque – Time in 3-RRS

IV. Conclusion

Results obtained from ADAMS software are compared with the results taken from MATLAB software in the diagrams. These diagrams are brought in Fig. 3 and 4. As you can see, these results have an acceptable error which is because of errors in dimensions and other software errors. These diagrams show the correctness of the analysis. It can be concluded that static analyzing of these robots could make a better feel to use them in the robotic uses. That is because of the fact that static analyzing is the first step of studies on robots and determining that which robot is good for each place. So, it is an important start for using them in a lot of places.

References

[1] G. O. Young, "Synthetic structure of industrial plastics (Book style with paper title and editor)," in *Plastics*, 2nd ed. vol. 3, J. Peters, Ed. New York: McGraw-Hill, 1964, pp. 15–64.
[2] H. McCallion, and D. T Pham. "The analysis of a six degree of freedom work station for mechanized assembly." Proc. 5th World Congress on Theory of Machines and Mechanisms. Vol. 611. 1979.
[3] K. H. Hunt, "Structural kinematics of in-parallel-actuated robot-arms." Journal of Mechanical Design 105.4 (1983): 705-712.
[4] C. Gosselin, "Determination of the workspace of 6-DOF parallel manipulators." Journal of Mechanical Design 112.3 (1990): 331-336.
[5] E. F. Fichter, "A Stewart platform-based manipulator: general theory and practical construction." International Journal of Robotics Research, 1986, 5(2), 157.
[6] D. E. Whitney, "Force feedback control of manipulator fine motions." Transactions of ASME, Journal of Dynamic Systems, Measurement and Control, 1972, Dec, 303±309.
[7] S. Bhattacharya, H. Hatwal, and A. Ghosh. "Comparison of an exact and an approximate method of singularity avoidance in platform type parallel manipulators." Mechanism and Machine Theory 33.7 (1998): 965-974.
[8] X. Kong, C.M. Gosselin, "Generation of Parallel Manipulators with Three Translational Degrees of Freedom using Screw Theory." In Proceedings of the CCToMM Symposium on Mechanisms 2001.
[9] J. Hervé, "The Lie group of rigid body displacements, a fundamental tool for mechanism design." Mechanism and Machine Theory, Vol. 34, No. 5, pp. 719-730, 1999.
[10] X. Kong, C. M. Gosselin, "Type synthesis of 4-DOF SP-equivalent parallel manipulators: A virtual chain approach." Mechanism and Machine theory, Vol. 41, No. 11, pp. 1306-1319, 2006.
[11] X. Kong, C. M. Gosselin, "Type Synthesis of Parallel Mechanisms." Springer, pp. 141-157, 2007.
[12] H. H. Pham, I.-M. Chen, "Kinematics, workspace and static analyses of a 2-DOF flexure parallel mechanism," In Proceeding of IEEE, 2002, pp. 968-973.
[13] G. Lu, A. Zhang, J. Zhou, S. Cui, L. Zhao, "The Statics Analysis and Verification of 3-DOF Parallel Mechanism Based on Two Methods." International Journal of Automation Technology, Vol. 7, No. 2, pp. 237-244, 2013.
[14] M. T. Masouleh, C. Gosselin, M. H. Saadatzi, X. Kong, H. D. Taghirad, "Kinematic analysis of 5-RPUR (3T2R) parallel mechanisms." Meccanica, Vol. 46, No. 1, pp. 131-146, 2011.
[15] M. T. Masouleh, C. Gosselin, M. Husty, D. R. Walter, "Forward kinematic problem of 5-RPUR parallel mechanisms (3T2R) with identical limb structures." Mechanism and Machine Theory, Vol. 46, No. 7, pp. 945-959, 2011.
[16] X. Kong and C. Gosselin. "Type synthesis of parallel mechanisms." Vol. 33. Heidelberg: Springer, 2007.