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Triple Diffusive Convection in a Vertically Oscillating Oldroyd-B Liquid

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Abstract**—**The effect of linear stability analysis of triple diffusive convection in a vertically oscillating viscoelastic liquid of Oldroyd-B type is studied. The correction Rayleigh number is obtained by using perturbation method which gives prospect to control the convection. The eigenvalue is obtained by using perturbation method by adopting Venezian approach. From the study, it is observed that gravity modulation advances the onset of triple diffusive convection.

Keywords**—**Gravity modulation, Oldroyd-B liquid, triple diffusive convection, Venezian approach.

I. Introduction

IN the classical Bénard problem, the instability is determined by the difference in density caused by the variation in temperature between the upper and lower surfaces bounding the liquid. This is referred as single component convection. When the instability in a liquid is caused by two opposing density components, then it is termed as two-component convection or double diffusive convection. When the instability in a liquid is caused by three different diffusivities then the mathematical and physical situation becomes increasingly richer and such problems are termed as three-component convection or triple diffusive convection.

The minimum necessities for the occurrence of triple diffusive convection are the following:

(i) The liquid must contain three components with different molecular diffusivities. It is the differential diffusion that produces the density differences required to drive the motion.

(ii) The components must make opposing contributions to the vertical density gradient.

Pearlstein et al. [1] obtained very fascinating results in triply diffusive convection. The outcomes of Pearlstein et al. are noteworthy. They determined that, for triple diffusive convection, linear uncertainty can arise in discrete units of Rayleigh number since for few parameters the neutral curve has its oscillatory curve lying lower than that of the standard boundless stationary convection. The problems of triple diffusive convection have also been studied by Lopez et al. [2], Poulikakos [3], Sumithra [4] and Rionero [5] and recently by Sameena [6].

A significant class of natural convection is concerned with the effort in eluding the convection in the earth's gravitational field despite having the basic temperature gradient constant and ignoring the interfacial instabilities. The gravity field in an orbiting laboratory is not constant in a microgravity condition; however, it is randomly fluctuating, and this kind of fluctuating gravity is called as g-jitter or gravity modulation. In literature it is observed that the vibrations can either significantly advance or delay the heat and mass transfer and therefore thoroughly disturbs the convection. The problems with gravity modulation have been studied by Gresho and Sani [7], Siddheshwar and Pranesh [8], Rees and Pop [9] and Sameena and Pranesh [10].

In the classification of non-Newtonian liquids – viscoelastic liquid reveals both liquid and solid properties. Viscoelastic liquids with rheological equation include relaxation and retardation times. It possesses both viscosity (linked with liquids) and elasticity (linked with solids) leading to distinct instability forms which is not seen in Newtonian liquids. It has wide-ranging applications in numerous fields such as material processing, geothermal energy modeling, cooling of electronic devices, thermal insulation material, crystal growth, transport of chemical substances, solar receivers, petroleum industry, injection moulding, chemical industries, nuclear industries, bioengineering, geophysics, and so on. Many authors have considered viscoelastic liquid like Siddeshwar et al. [11], Malashetty and Swamy [12], Narayana et al. [13], Bhadauria and Kiran [14] and recently Sameena and Pranesh [15].

The problem under investigation has wide range of applications like in material processing, solidification of alloys, underground spreading of chemical pollutants, petroleum reservoirs, cooling of electronic devices, thermal insulation material, crystal growth, transport of chemical substances, solar receivers, injection moulding, chemical industries, nuclear industries, bioengineering, oceanography, meteorology, astrophysics, geophysics and so on. Therefore, the main objective of this paper is to study the effect of gravity modulation on triple diffusive convection in Oldroyd-B liquid.

II. Mathematical Formulation

Consider a layer of Oldroyd-B liquid confined between two infinite horizontal surfaces separated by a distance 'd' and is subject to the time dependent gravity acting in negative direction of z-axis. Let $\Delta T, \Delta S_1$ and ΔS_2 be the differences in temperature and solute concentrations, respectively of the liquid between the lower and the upper plates. Appropriate single-phase heat and two-phase solute transport equations are chosen with effective heat capacity ratios and effective thermal diffusivity (as shown in Fig. 1).

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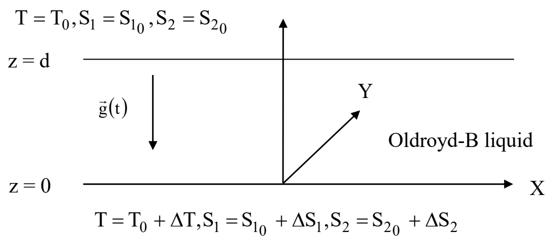

Fig. 1 Physical configuration of triple diffusive convection in Oldroyd-B liquid under gravity modulation where $\vec{g}(t) = -g_0\left(1+\delta\cos\left(\gamma t\right)\right)$

The basic governing equations are:

$$\nabla.\vec{q} = 0\,, \tag{1}$$

$$\rho_o\left[\frac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}\right] = -\nabla p - \rho g_0\left(1+\delta\cos\left(\gamma t\right)\right)+\nabla.\tau'\,, \tag{2}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\tau' = \mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\nabla\vec{q}+\nabla\vec{q}^{tr}\right)\,, \tag{3}$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T = \chi_t\nabla^2 T\,, \tag{4}$$

$$\frac{\partial S_1}{\partial t}+(\vec{q}.\nabla)S_1 = \chi_{S1}\nabla^2 S_1\,, \tag{5}$$

$$\frac{\partial S_2}{\partial t}+(\vec{q}.\nabla)S_2 = \chi_{S2}\nabla^2 S_2\,, \tag{6}$$

$$\rho = \rho_0\left[1-\alpha_t\left(T-T_0\right)+\alpha_{S1}\left(S_1-S_{10}\right)+\alpha_{S2}\left(S_2-S_{2_0}\right)\right], \tag{7}$$

where $\vec{q}$ is velocity, p is pressure, ρ_0 is constant density, ρ is density, $\vec{g}(t)$ is gravitational force, g_0 is mean gravity, δ is amplitude of modulation, γ is frequency of modulation, τ' is stress tensor, λ_1 is stress relaxation time, λ_2 is strain retardation time, μ is viscosity, T is temperature, S_1 is solute1, S_2 is solute2, χ_t is thermal diffusivity, χ_{S1} is the solute1 diffusivity, χ_{S2} is the solute2 diffusivity, α_t is coefficient of thermal expansion, α_{S1} is coefficient of solute1 expansion, and α_{S2} is coefficient of solute2 expansion.

Operating divergence on (3) and using (2), we get,

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\begin{pmatrix}\rho_o\dfrac{\partial\vec{q}}{\partial t}+(\vec{q}.\nabla)\vec{q}+\nabla p\\ -\rho g_0\left(1+\delta\cos\left(\gamma t\right)\right)\end{pmatrix} = \mu\left(1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\vec{q}\,, \tag{8}$$

The boundary conditions for temperature and mass transfer are given by:

$$\left.\begin{array}{l} T = T_0+\Delta T,\ S_1 = S_{1_0}+\Delta S_1 \text{ and } S_2 = S_{2_0}+\Delta S_2 \text{ at } z=0\\ T = T_0,\ S_1 = S_{1_0} \text{ and } S_2 = S_{2_0} \text{ at } z=d\end{array}\right\}. \tag{9}$$

The basic state of liquid is quiescent and is described by:

$$\left.\begin{array}{l}\vec{q}_b = (0,0,0),\, p = p_b(z),\, \rho = \rho_b(z),\\ T = T_b(z), S_1 = S_{1b}(z), S_2 = S_{2_b}(z)\end{array}\right\}, \tag{10}$$

where the subscript ‘b’ denotes the basic state.

The stability of the basic state is analyzed by introducing the following perturbation:

$$\left.\begin{array}{l}\vec{q} = \vec{q}_b+\vec{q}\,',\, p = p_b+p',\, \rho = \rho_b+\rho',\\ T = T_b+T', S_1 = S_{1_b}+S_1', S_2 = S_{2_b}+S_2'\end{array}\right\}, \tag{11}$$

where, the prime indicates that the quantities are infinitesimal perturbations.

Substituting (11) into (1)-(7), eliminating pressure by operating curl twice, introducing the stream functions $u' = \frac{\partial\psi}{\partial z}$ and $w' = -\frac{\partial\psi}{\partial x}$ and non-dimensionalizing the resulting equations, we get the following linearized dimensionless equations:

$$\left.\begin{array}{l}\left(1+\Lambda_1\dfrac{\partial}{\partial t}\right)\left[\begin{array}{l}\dfrac{1}{Pr}\dfrac{\partial}{\partial t}\left(\nabla^2\psi\right)-\\ \left(-Ra\dfrac{\partial\theta}{\partial x}+R_{S1}\dfrac{\partial\phi_{S1}}{\partial x}+R_{S2}\dfrac{\partial\phi_{S2}}{\partial x}\right)\\ \left(1+\delta\cos\left(\Omega t\right)\right)\end{array}\right]\\ = \left(1+\Lambda_2\dfrac{\partial}{\partial t}\right)\nabla^4\psi\end{array}\right\}, \tag{12}$$

$$\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial x} = \nabla^2\theta\,, \tag{13}$$

$$\frac{\partial\phi_{S1}}{\partial t}+\frac{\partial\psi}{\partial x} = \tau_1\nabla^2\phi_{S1}\,, \tag{14}$$

$$\frac{\partial\phi_{S2}}{\partial t}+\frac{\partial\psi}{\partial x} = \tau_2\nabla^2\phi_{S2}\,, \tag{15}$$

Here, nondimensionalizing parameters in (12)-(15) are $Pr = \frac{\mu}{\rho_0\chi}$ (Prandtl number), $\Lambda_1 = \frac{\chi_t\lambda_1}{d^2}$ (Stress relaxation parameter), $\Lambda_2 = \frac{\chi_t\lambda_2}{d^2}$ (Strain retardation parameter), $Ra = \frac{\rho_o\alpha_t g\Delta T d^3}{\mu\chi_t}$ (Rayleigh number), $R_{S1} = \frac{\rho_o\alpha_{S1}g\Delta S_1 d^3}{\mu\chi_t}$ (solute Rayleigh number1), $R_{S2} = \frac{\rho_o\alpha_{S2}g\Delta S_2 d^3}{\mu\chi_t}$ (solute

Rayleigh number2), $\tau_1 = \frac{\chi_{S1}}{\chi_t}$ (ratio of diffusivity of solute1 and heat diffusivity), $\tau_2 = \frac{\chi_{S2}}{\chi_t}$ (ratio of diffusivity of solute2 and heat diffusivity), $\Omega = \frac{d^2\gamma}{\chi_t}$ (Non-dimensional modulation frequency).

Eliminating θ, ϕ_{S1} and ϕ_{S2} from (12)-(15), we get the equation for ψ in the form of:

$$\begin{bmatrix} X_4 X_3 X_2 X_1 \nabla^2 - \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) \\ \left(Ra X_4 X_3 - R_{S1} X_4 X_2 - R_{S2} X_3 X_2\right) \\ \left(1 + \delta \cos\left(\Omega t\right)\right) \frac{\partial^2}{\partial x^2} \end{bmatrix} \psi = 0 , \tag{16}$$

where

$$X_1 = \left[\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) \frac{1}{Pr} \frac{\partial}{\partial t} - \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right) \nabla^2\right], \qquad X_2 = \left(\frac{\partial}{\partial t} - \nabla^2\right),$$

$$X_3 = \left(\frac{\partial}{\partial t} - \tau_1 \nabla^2\right) \text{ and } X_4 = \left(\frac{\partial}{\partial t} - \tau_2 \nabla^2\right).$$

In dimensionless form, the velocity boundary conditions for solving (16) are obtained in the form:

$$\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^4 \psi}{\partial z^4} = \frac{\partial^6 \psi}{\partial z^6} = \frac{\partial^8 \psi}{\partial z^8} = 0 \text{ at } z = 0,1 . \tag{17}$$

III. Method of Solution

We now seek the eigen function ψ and eigenvalue Ra of (16). Thus, the eigenvalues of the present problem differ from those of the ordinary Bénard convection by quantities of order δ. We seek the solution of (16) in the form:

$$\left(Ra, \psi\right) = \left(Ra_0, \psi_0\right) + \delta\left(Ra_1, \psi_1\right) + \delta^2\left(Ra_2, \psi_2\right) + ... \tag{18}$$

The expansion (18) is substituted in (16) and equating the coefficients of various powers of δ on either side of the equation, we get:

$$L\psi_0 = 0 , \tag{19}$$

$$L\psi_1 = \begin{bmatrix} \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[Ra_0 X_4 X_3 - R_{S1} X_4 X_2 - R_{S2} X_3 X_2\right] \\ f \frac{\partial^2 \psi_0}{\partial x^2} + \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) Ra_1 X_4 X_3 \frac{\partial^2 \psi_0}{\partial x^2} \end{bmatrix}, \tag{20}$$

$$L\psi_2 = \begin{bmatrix} \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[Ra_0 X_4 X_3 - R_{S1} X_4 X_2 - R_{S2} X_3 X_2\right] \\ f \frac{\partial^2 \psi_1}{\partial x^2} + \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) Ra_1 X_4 X_3 \frac{\partial^2 \psi_1}{\partial x^2} \\ + \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) Ra_1 X_4 X_3 f \frac{\partial^2 \psi_0}{\partial x^2} \\ + \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) Ra_2 X_4 X_3 \frac{\partial^2 \psi_0}{\partial x^2} \end{bmatrix} \tag{21}$$

where

$$f = \cos(\Omega t),$$

$$L = \begin{bmatrix} X_4 X_3 X_2 X_1 \nabla^2 \\ -\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) \begin{pmatrix} Ra_0 X_4 X_3 - R_{S1} X_4 X_2 \\ -R_{S2} X_3 X_2 \end{pmatrix} \frac{\partial^2}{\partial x^2} \end{bmatrix} . \tag{22}$$

A. Solution to the Zeroth Order Problem

The stability of the system in the absence of gravity modulation is examined by introducing vertical velocity perturbation ψ_0 corresponding to lowest mode of convection as:

$$\psi_0 = \sin\left(\pi \alpha x\right) \sin\left(\pi z\right) \exp\left(\sigma t\right) . \tag{23}$$

Substituting (23) in (19), we obtain the expression for Rayleigh number in the form:

$$Ra_0 = \frac{\begin{Bmatrix} \left(\sigma + k^2\right)\left(\sigma + \tau_1 k^2\right)\left(\sigma + \tau_2 k^2\right) \\ \left[\left(1 + \Lambda_1 \sigma\right) \frac{\sigma}{Pr} + \left(1 + \Lambda_2 \sigma\right) k^2\right] k^2 \\ + R_{S1}\left(\sigma + k^2\right)\left(\sigma + \tau_2 k^2\right) \pi^2 a^2 \\ + R_{S2}\left(\sigma + k^2\right)\left(\sigma + \tau_1 k^2\right) \pi^2 a^2 \end{Bmatrix}}{\left(\sigma + \tau_1 k^2\right)\left(\sigma + \tau_2 k^2\right) \pi^2 a^2} . \tag{24}$$

For stationary convection, σ in (24) must be real and the corresponding Rayleigh number Ra_0^{st} for stationary convection is obtained by putting $\sigma = 0$ in (24) in the form:

$$Ra_0^{st} = \frac{R_{S1}}{\tau_1} + \frac{R_{S2}}{\tau_2} + \frac{k^6}{\pi^2 a^2} , \tag{25}$$

where

$$k^2 = \pi^2 + \pi^2 a^2 .$$

To study oscillatory motions, we put $\sigma = i\omega$, where $\omega \in \Re$, in (24), we obtain the expression of Rayleigh number for oscillatory convection as

$$\left.\begin{array}{l} Ra_0^{oc} = \dfrac{R_{S1}\left(\tau_1 k^4 + \omega^2\right)}{\left(\tau_1^2 k^4 + \omega^2\right)} + \dfrac{R_{S2}\left(\tau_2 k^4 + \omega^2\right)}{\left(\tau_2^2 k^4 + \omega^2\right)} \\ + \dfrac{\left(1 + \Lambda_1 \Lambda_2 \omega^2\right)k^6}{\left(1 + \Lambda_1^2 \omega^2\right)\pi^2 a^2} + \dfrac{\left(\Lambda_1 - \Lambda_2\right)\omega^2 k^4}{\left(1 + \Lambda_1^2 \omega^2\right)\pi^2 a^2} \\ - \dfrac{\omega^2 k^2}{\pi^2 a^2 \, Pr} \end{array}\right\}, \tag{26}$$

and frequency,

$$A\omega^6 + B\omega^4 + C\omega^2 + D = 0, \tag{27}$$

where,

$$A = \Lambda_1 \Lambda_2 k^4 + \frac{\Lambda_1^2 k^4}{Pr},$$

$$B = \begin{bmatrix} R_{S1}(\tau_1 - 1)k^2\pi^2 a^2 \Lambda_1^2 + R_{S2}(\tau_2 - 1)k^2 \pi^2 a^2 \Lambda_1^2 \\ -(\Lambda_1 - \Lambda_2)k^6 + \left(1 + \dfrac{1}{Pr}\right)k^4 + \left(\dfrac{\Lambda_1^2}{Pr} + \Lambda_1 \Lambda_2\right)\left(\tau_1^2 + \tau_2^2\right)k^8 \end{bmatrix},$$

$$C = \begin{bmatrix} R_{S1}(\tau_1 - 1)k^2\pi^2 a^2 + R_{S2}(\tau_2 - 1)k^2\pi^2 a^2 \\ + R_{S1}(\tau_1 - 1)\tau_2^2 k^6 \pi^2 a^2 \Lambda_1^2 + R_{S2}(\tau_2 - 1)\tau_1^2 k^6 \pi^2 a^2 \Lambda_1^2 \\ + \left(1 + \dfrac{1}{Pr} - \Lambda_1 k^2 + \Lambda_2 k^2\right)\left(\tau_1^2 + \tau_2^2\right)k^8 \\ + \left(\dfrac{\Lambda_1^2}{Pr} + \Lambda_1 \Lambda_2\right)\tau_1^2 \tau_2^2 k^{12} \end{bmatrix},$$

$$D = \begin{bmatrix} R_{S1}(\tau_1 - 1)\tau_2^2 k^6 \pi^2 a^2 + R_{S2}(\tau_2 - 1)\tau_1^2 k^6 \pi^2 a^2 \\ + \left(1 + \dfrac{1}{Pr} - \Lambda_1 k^2 + \Lambda_2 k^2\right)\tau_1^2 \tau_2^2 k^{12} \end{bmatrix}.$$

B. Solution to the First Order Problem

Equation (20) for ψ_1 now takes the form:

$$L\psi_1 = \left\{\begin{array}{l} Ra_0 \begin{bmatrix} \left(f'' + \tau_1 k^2 f' + \tau_2 k^2 f' + \tau_1 \tau_2 k^4 f\right) \\ + \Lambda_1\left(f''' + \tau_1 k^2 f'' + \tau_2 k^2 f'' + \tau_1 \tau_2 k^4 f'\right) \end{bmatrix} \\ - R_{S1} \begin{bmatrix} \left(f'' + k^2 f' + \tau_2 k^2 f' + \tau_2 k^4 f\right) \\ + \Lambda_1\left(f''' + k^2 f'' + \tau_2 k^2 f'' + \tau_2 k^4 f'\right) \end{bmatrix} \\ - R_{S2} \begin{bmatrix} \left(f'' + k^2 f' + \tau_1 k^2 f' + \tau_1 k^4 f\right) \\ + \Lambda_1\left(f''' + k^2 f'' + \tau_1 k^2 f'' + \tau_1 k^4 f'\right) \end{bmatrix} \\ + Ra_1 \tau_1 \tau_2 k^4 \end{array}\right\} \left(-\pi^2 a^2\right)\psi_0 \tag{28}$$

If (28) has a solution then the right-hand side must be orthogonal to the null-space of the operator L. This implies that the time independent part of the RHS of (28) must be orthogonal to $sin(\pi z)$. Since f varies sinusoidal with time, the only steady term on the RHS of (28) is $-\tau_1 \tau_2 k^4 \pi^2 a^2 Ra_1$, so that $Ra_1 = 0$. It follows that all the odd coefficients i.e. $Ra_1 = Ra_3 = ... = 0$.

Using (22), we find that

$$\left.\begin{array}{l} L[sin(\pi z) sin(\pi a x) e^{-i\Omega t}] \\ = L(\Omega) sin(\pi z) sin(\pi a x) e^{-i\Omega t} \end{array}\right\}, \tag{29}$$

where

$$L(\Omega) = Y_1 + iY_2, \tag{30}$$

$$Y_1 = \frac{1}{Pr}\left\{\begin{array}{l} \pi^2 a^2 \, Pr \begin{bmatrix} -k^4\left(R_{S2}\tau_1 + \left(R_{S1} - Ra_0 \tau_1\right)\tau_2\right) \\ + \left(-Ra_0 + R_{S1} + R_{S2}\right)\Omega^2 \\ + k^2 \Lambda_1 \begin{pmatrix} R_{S1}(1 + \tau_2) \\ + R_{S2}(1 + \tau_1) \\ - Ra_0(\tau_1 + \tau_2) \end{pmatrix} \Omega^2 \end{bmatrix} \\ + k^2 \begin{bmatrix} -k^8 \, Pr \, \tau_1 \tau_2 + k^4\left(Z_2 + Pr \, Z_1\right)\Omega^2 \\ + k^6\left(\Lambda_1 \tau_1 \tau_2 + Pr \, \Lambda_2 Z_2\right)\Omega^2 \\ - \Omega^4 - k^2\left(Pr \, \Lambda_2 + \Lambda_1 Z_1\right)\Omega^4 \end{bmatrix} \end{array}\right\},$$

$$Y_2 = \frac{\Omega}{Pr}\left\{\begin{array}{l} \pi^2 a^2 \, Pr \begin{bmatrix} k^4 \Lambda_1\left(R_{S2}\tau_1 + \left(R_{S1} - Ra_0 \tau_1\right)\tau_2\right) \\ + \left(Ra_0 - R_{S1} - R_{S2}\right)\Lambda_1 \Omega^2 \\ + k^2 \begin{pmatrix} R_{S1}(1 + \tau_2) + R_{S2}(1 + \tau_1) \\ - Ra_0(\tau_1 + \tau_2) \end{pmatrix} \end{bmatrix} \\ + k^2 \begin{bmatrix} k^8 \, Pr \, \Lambda_2 \tau_1 \tau_2 + k^6\left(\tau_1 \tau_2 + Pr \, Z_2\right) \\ - k^2\left(Pr + Z_1\right)\Omega^2 \\ - k^4\left(Pr \, \Lambda_2 Z_1 + \Lambda_1 Z_2\right)\Omega^2 + \Lambda_1 \Omega^4 \end{bmatrix} \end{array}\right\}$$

$$Z_1 = 1 + \tau_1 + \tau_2,$$
$$Z_2 = \tau_1 + \tau_2 + \tau_1 \tau_2.$$

The particular solution of (28) is

$$\psi_1 = -\frac{\pi^2 a^2}{\left|L(\Omega)\right|^2} Re\left[\left(M_9 + iM_{10}\right)e^{-i\Omega t}\right]\psi_0, \tag{31}$$

where

$$M_1 = 1 + \Lambda_1(\tau_1 + \tau_2)k^2, \; M_2 = (\tau_1 + \tau_2)k^2 + \Lambda_1 \tau_1 \tau_2 k^4,$$

$M_3 = 1 + \Lambda_1 (1+\tau_2) k^2$, $M_4 = (1+\tau_2) k^2 + \Lambda_1 \tau_2 k^4$,
$M_5 = 1 + \Lambda_1 (1+\tau_1) k^2$, $M_6 = (1+\tau_1) k^2 + \Lambda_1 \tau_1 k^4$,

$$M_7 = \begin{bmatrix} Ra_0 \left(M_1 \Omega^2 + \tau_1 \tau_2 k^4 \right) - R_{S1} \left(-M_3 \Omega^2 + \tau_2 k^4 \right) \\ -R_{S2} \left(-M_5 \Omega^2 + \tau_1 k^4 \right) \end{bmatrix},$$

$$M_8 = \begin{bmatrix} Ra_0 \left(\Lambda_1 \Omega^3 - M_2 \Omega \right) - R_{S1} \left(\Lambda_1 \Omega^3 - M_4 \Omega \right) \\ -R_{S2} \left(\Lambda_1 \Omega^3 - M_6 \Omega \right) \end{bmatrix},$$

$M_9 = Y_1 M_7 + Y_2 M_8$, $M_{10} = Y_1 M_8 - Y_2 M_7$.

The equation for ψ_2, then becomes

$$L\psi_2 = \begin{bmatrix} \left(Z_3 f''' + Z_4 f'' + Z_5 f' + Z_6 f \right) \left(-\pi^2 a^2 \right) \psi_1 \\ -\tau_1 \tau_2 k^4 \pi^2 a^2 Ra_2 \psi_0 \end{bmatrix}, \quad (32)$$

where

$Z_3 = (Ra_0 - R_{S1} - R_{S2}) \Lambda_1$, $Z_4 = Ra_0 M_1 - R_{S1} M_3 - R_{S2} M_5$,
$Z_5 = Ra_0 M_2 - R_{S1} M_4 - R_{S2} M_6$,
$Z_6 = Ra_0 \tau_1 \tau_2 k^4 - R_{S1} \tau_2 k^4 - R_{S2} \tau_1 k^4$.

Equation (32) is not solved but is used to determine Ra_2 (denoted by R_{2C}). For the existence of a solution of (32), it is necessary that the steady part of its right-hand side is orthogonal to $sin(\pi z)$. This gives,

$$\int_0^1 \begin{bmatrix} \left(Z_3 f''' + Z_4 f'' + Z_5 f' + Z_6 f \right) \\ \left(-\pi^2 a^2 \right) \psi_1 - \tau_1 \tau_2 k^4 \pi^2 a^2 Ra_2 \psi_0 \end{bmatrix} sin(\pi z) dz = 0,$$

Taking time average and from Reynold's time average, we get,

$$R_{2C} = -\frac{\pi^2 a^2 Z_6 M_9}{2 \left| L(\Omega) \right|^2 \tau_1 \tau_2 k^4}. \quad (33)$$

IV. Results and Discussions

In this paper, the effect of gravity modulation on triple diffusive convection in viscoelastic liquid using Oldroyd-B model, heated and solute from below is made. The behavior of various parameters like stress relaxation parameter Λ_1, strain retardation parameter Λ_2, solute Rayleigh number1 R_{S1}, solute Rayleigh number2 R_{S2}, ratio of diffusivity of solute1 and heat diffusivity τ_1, ratio of diffusivity of solute2 and heat diffusivity τ_2 and Prandtl number Pr on the onset of convection are analyzed. The expression for the correction Rayleigh number R_{2C} is computed as function of the frequency of the modulation Ω.

The validity of the results obtained here depends on the value of the modulating frequency Ω. When $\Omega \ll 1$, the period of modulation is large and hence the disturbance grows to such an extent that it makes finite amplitude effects important. When $\Omega \to \infty$, $R_{2C} \to 0$, thus the effect of modulation becomes small. In view of this, we have chosen only moderate values of Ω in our study.

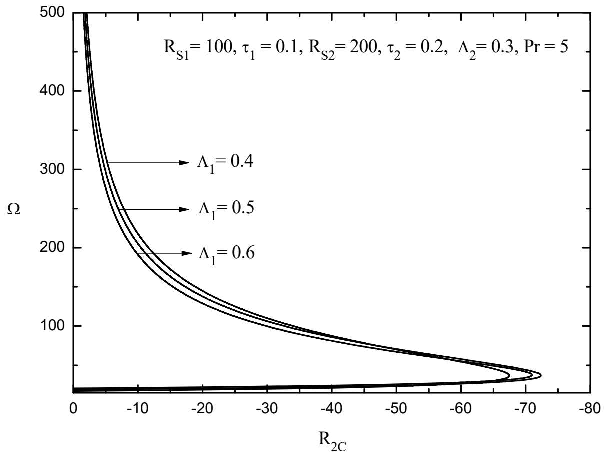

Fig. 2 Plot of correction Rayleigh number R_{2C} versus frequency of modulation Ω for different values of stress relaxation parameter Λ_1

Fig. 2 is the plot of correction Rayleigh number R_{2C} versus frequency of modulation Ω for different values of stress relaxation parameter Λ_1. From Fig. 2, we observe that the effect of increase in stress relaxation parameter, decreases the magnitude of correction Rayleigh number. Λ_1 is an elastic parameter and represents the relaxation of stress while having its influence on liquid, indicating when $\Omega < \Omega^*$ (around 50), the gravity modulation destabilizes and for $\Omega > \Omega^*$ gravity modulation stabilizes the system.

Fig. 3 is the plot of R_{2C} versus Ω for different values of strain retardation parameter Λ_2. From Fig. 3, we observe that the effect of increase in strain retardation parameter, increases the magnitude of correction Rayleigh number. Λ_2 is also an elastic parameter and it represents the relaxation in liquid in responding to the stress. It is also observed that when $\Omega < \Omega^*$ (around 50), the gravity modulation destabilizes and for $\Omega > \Omega^*$ gravity modulation stabilizes the system.

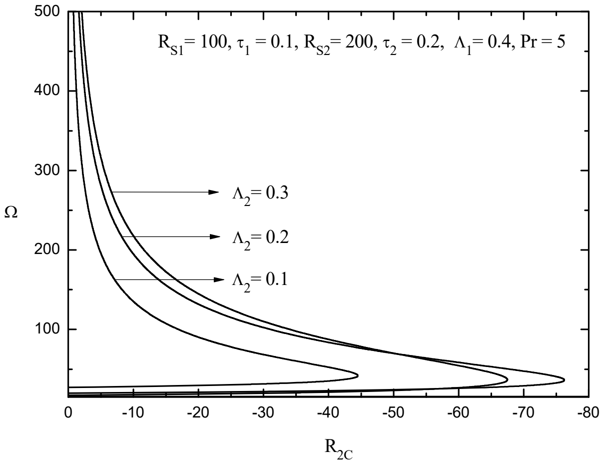

Fig. 3 Plot of R_{2C} versus Ω for different values of strain retardation parameter Λ_2

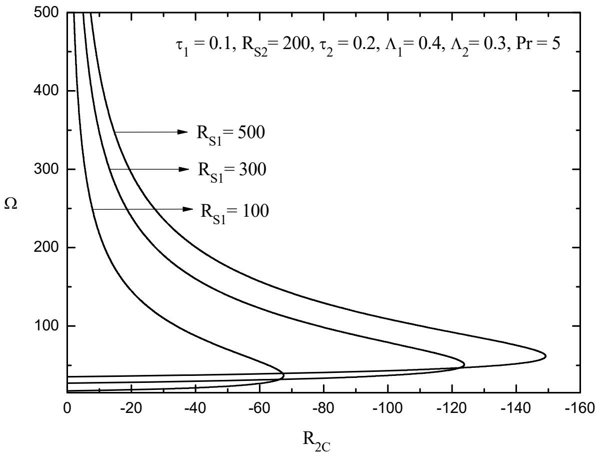

Fig. 4 Plot of R_{2C} versus Ω for different values of solute Rayleigh number1 R_{S1}

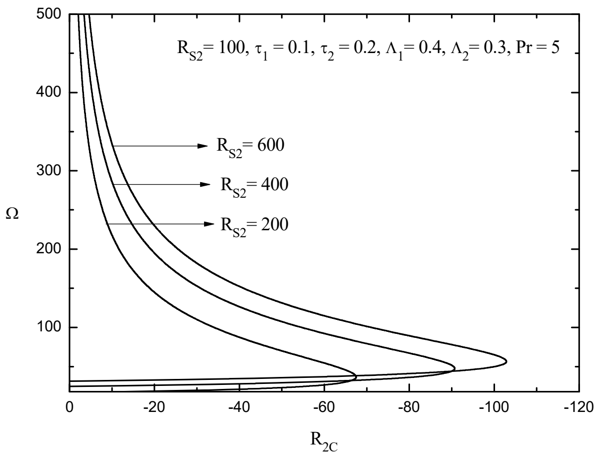

Fig. 5 Plot of R_{2C} versus Ω for different values of solute Rayleigh number2 R_{S2}

Figs. 4 and 5 are the plots of R_{2C} versus Ω for different values of solute Rayleigh number1 R_{S1} and solute Rayleigh number2 R_{S2}, respectively. Here we observe that, the effect of increase in R_{S1} and R_{S2} increases the magnitude of correction Rayleigh number. This is due to the fact that when we add solutes from below, the solutes concentration stays at the lower wall and do not disturb the system. It is also observed that when $\Omega < \Omega^*$ (around 50), the gravity modulation destabilizes and for $\Omega > \Omega^*$ gravity modulation stabilizes the system.

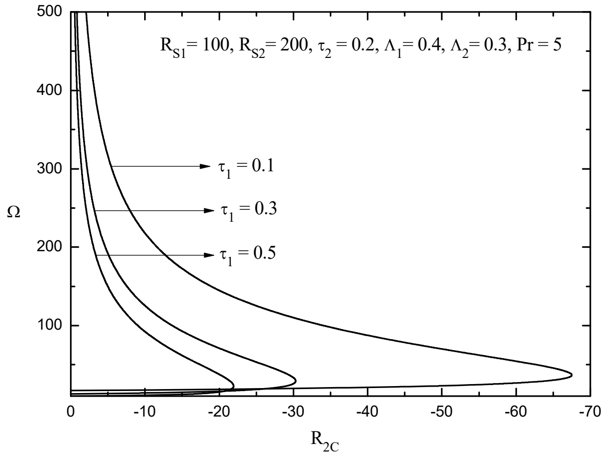

Fig. 6 Plot of R_{2C} versus Ω for different values of ratio of diffusivity of solute1 and heat diffusivity τ_1

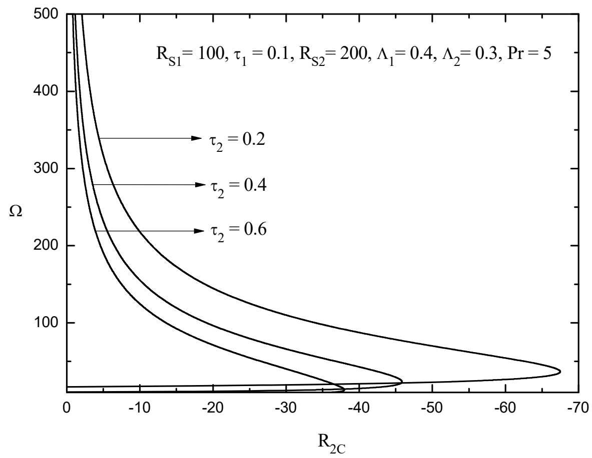

Fig. 7 Plot of R_{2C} versus Ω for different values of ratio of diffusivity of solute2 and heat diffusivity τ_2

Figs. 6 and 7 are the plots of R_{2C} versus Ω for different values of ratio of diffusivity of solute1 and heat diffusivity τ_1 and ratio of diffusivity of solute2 and heat diffusivity τ_2, respectively. It is observed that increasing the values of τ_1

and τ_2, decreases the magnitude of correction Rayleigh number. This is because, the diffusivity of heat is more than the diffusivity of solutes. It is also observed that when $\Omega < \Omega^*$ (around 50), the gravity modulation stabilizes and for $\Omega > \Omega^*$ gravity modulation destabilizes the system.

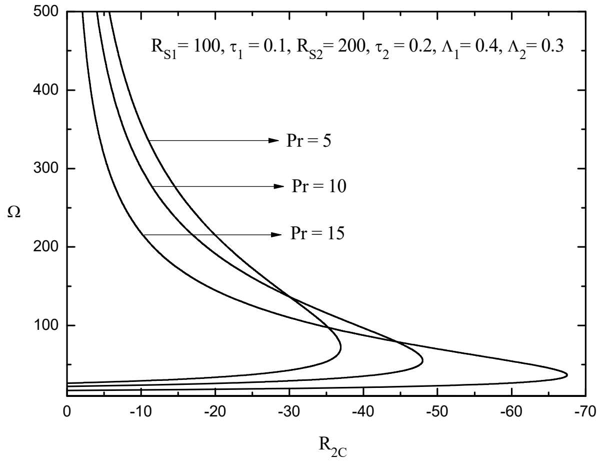

Fig. 8 Plot of R_{2C} versus Ω for different values of Prandtl number Pr

Fig. 8 is the plot of R_{2C} versus Ω for different values of Prandtl number Pr. It is observed that increase in the values of Prandtl number, decreases the magnitude of correction Rayleigh number. This means that the liquids with suspended particles are more susceptible to stabilization by modulation than clean liquid. It is also appropriate to note that Pr does not affect the Ra_0 – part of Ra, it effects only R_{2C}. It is also observed that when $\Omega < \Omega^*$ (around 100), the gravity modulation stabilizes and for $\Omega > \Omega^*$ gravity modulation destabilizes the system.

V. Conclusion

It is observed that since correction Rayleigh number is always negative for all frequencies, gravity modulation destabilizes the onset of triple diffusive convection.

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