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Transverse Vibration of Non-Homogeneous Rectangular Plates of Variable Thickness Using GDQ

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Abstract**—**The effect of non-homogeneity on the free transverse vibration of thin rectangular plates of bilinearly varying thickness has been analyzed using generalized differential quadrature (GDQ) method. The non-homogeneity of the plate material is assumed to arise due to linear variations in Young's modulus and density of the plate material with the in-plane coordinates x and y. Numerical results have been computed for fully clamped and fully simply supported boundary conditions. The solution procedure by means of GDQ method has been implemented in a MATLAB code. The effect of various plate parameters has been investigated for the first three modes of vibration. A comparison of results with those available in literature has been presented.

Keywords**—**Bilinear thickness, generalized differential quadrature (GDQ), non-homogeneous, Rectangular.

I. Introduction

THE study of non-homogeneous materials is of great interest to the researchers in the various field of engineering because in many engineering applications the mechanical properties of the material are not homogeneous and display spatial variation. Plywood, timber and fiber-reinforced plastic etc. are the examples of non-homogeneous materials. Nowadays some high-strength lightweight premium composites, fabricated by mixing two or more materials such as carbon fiber and epoxies are being used for aerospace applications and in high performance sporting goods. The non-homogeneity of a structure is characterized by a number of factors governing its structural features. For plate type structure these features are geometrical imperfections, inclusion of foreign materials and reinforcements of various types [1]-[3]. Sometimes plate type structural elements have to work under high temperature environment which causes non-homogeneity in the material, particularly in aerospace industry, modern missile technology and microelectronics. These rectangular plates with appropriate thickness variation have significantly greater efficiency for vibration as compared to the plates of uniform thickness and also provide the advantage of material saving and hence the cost requirement. Thus their design requires an accurate analysis for their vibration characteristic. Various models for the non-homogeneity of the plate material have been proposed in the literature and a detailed discussion is given by Lal and Dhanpati [4], [5]. In these papers, it is considered that non-homogeneity of the plate material arises due to change in only one space variable.

The present study analyze the effect of non-homogeneity on the free transverse vibration of thin rectangular plates of varying thickness employing generalized differential quadrature (GDQ) method with the two boundary conditions namely, fully clamped and fully simply supported. The thickness of the plate is taken bilinear along both the directions. Non-homogeneity of the plate material is assumed to arise due to linear variation in Young's modulus and density of the plate material with both the in-plane coordinates. The effect of various parameters on the natural frequencies has been investigated for the first two modes of vibration. A comparison of results has been presented.

II. Mathematical Formulation

Referred to a Cartesian coordinates (x, y, z), the configuration of a non-homogeneous isotropic rectangular plate of length a, breadth b, thickness $h(x,y)$ and density $\rho(x,y)$ is shown in Fig. 1. The x- and y-axes are taken along the edges of the plate, the axis of z is perpendicular to the xy-plane. The middle surface being $z=0$ and origin is at the one of the corners of the plate. The differential equation governing the transverse vibration of such plates, is given by

$$\nabla^2(D\nabla^2 w)-(1-\nu)\Big(\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-2\left(\frac{\partial^2 D}{\partial x\,\partial y}\right)\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)+\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\Big)-\rho h\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

where $D=Eh^3/12(1-\nu^2)$ is the flexural rigidity, $w(x,y,t)$ is the transverse displacement, E is the Young's modulus, ν is the Poisson ratio, $\rho(x,y)$ is the density.

For a harmonic solution, the displacement w is assumed to be

$$w(x,y,t)=\overline{w}(x,y)e^{i\omega t} \tag{2}$$

where ω is the circular frequency in radians.

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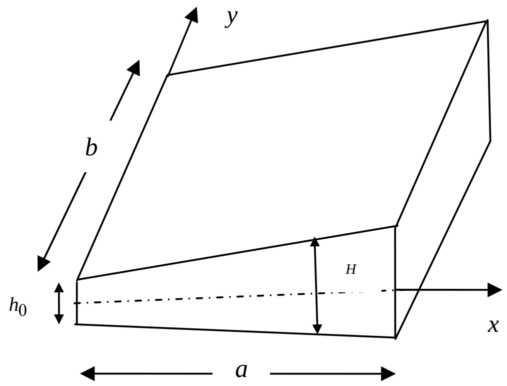

Fig. 1 Geometry of the rectangular plate

Using (2), (1) reduces to

$$D\left(\frac{\partial^4 \overline{w}}{\partial x^4}+\frac{\partial^4 \overline{w}}{\partial y^4}\right)+2D\frac{\partial^4 \overline{w}}{\partial x^2\partial y^2}+2\frac{\partial D}{\partial x}\left(\frac{\partial^3 \overline{w}}{\partial x^3}+\frac{\partial^3 \overline{w}}{\partial x\partial y^2}\right)$$
$$+2\frac{\partial D}{\partial y}\left(\frac{\partial^3 \overline{w}}{\partial y^3}+\frac{\partial^3 \overline{w}}{\partial x^2\partial y}\right)+\frac{\partial^2 \overline{w}}{\partial x^2}\left(\frac{\partial^2 D}{\partial x^2}+\nu\frac{\partial^2 D}{\partial y^2}\right) \tag{3}$$
$$+\frac{\partial^2 \overline{w}}{\partial y^2}\left(\nu\frac{\partial^2 D}{\partial x^2}+\frac{\partial^2 D}{\partial y^2}\right)+2(1-\nu)\frac{\partial^2 D}{\partial x\partial y}\frac{\partial^2 \overline{w}}{\partial x\partial y}-\rho h\omega^2\overline{w}=0$$

Introducing the non-dimensional variables $X = x/a,\ Y = y/b,\ H = h/a,\ W = \overline{w}/a$ and assuming that Young's modulus and density of the plate material vary with the space co-ordinates by the functional values

$$E = E_0(1+\alpha_1 X+\alpha_2 Y),\ \rho = \rho_0(1+\beta_1 X+\beta_2 Y) \tag{4}$$

and thickness of the plate varies linearly in both $X-$ and $Y-$ directions [14], given by

$$H = h_0(1+\gamma_1 X)(1+\gamma_2 Y) \tag{5}$$

where E_0, ρ_0 and h_0 are the Young's modulus, density and thickness of the plate at $X=0, Y=0$, α_1 and α_2 are non-homogeneity parameters and β_1 and β_2 are the density parameters respectively. Equation (3) now, reduces to

$$A_0\left(\frac{\partial^4 W}{\partial X^4}+2\lambda^2\frac{\partial^4 W}{\partial X^2\partial Y^2}+\lambda^4\frac{\partial^4 W}{\partial Y^4}\right)+A_1\left(\frac{\partial^3 W}{\partial X^3}+\lambda^2\frac{\partial^3 W}{\partial X\partial Y^2}\right)$$
$$+A_2\left(\lambda^4\frac{\partial^3 W}{\partial Y^3}+\lambda^2\frac{\partial^3 W}{\partial X^2\partial Y}\right)+A_3\left(\frac{\partial^2 W}{\partial X^2}+\nu\lambda^2\frac{\partial^2 W}{\partial Y^2}\right) \tag{6}$$
$$+A_4\left(\nu\lambda^2\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Y^2}\right)+A_5\frac{\partial^2 W}{\partial X\,\partial Y}-A_6 W=0$$

where

$\lambda = a/b$ $A_0 = (1+\alpha_1 X+\alpha_2 Y)(1+\gamma_1 X)^2(1+\gamma_2 Y)^2$

$A_1 = (6\gamma_1(1+\alpha_1 X+\alpha_2 Y)+2\alpha_1(1+\gamma_1 X))\ (1+\gamma_1 X)(1+\gamma_2 Y)^2$

$A_2 = (6\gamma_2(1+\alpha_1 X+\alpha_2 Y)+2\alpha_2(1+\gamma_2 Y))\ (1+\gamma_1 X)^2(1+\gamma_2 Y)$

$A_3 = (6\gamma_1^2(1+\alpha_1 X+\alpha_2 Y)+6\alpha_1(1+\gamma_1 X)\ (1+\gamma_2 Y)^2$

$A_4 = (6\gamma_2^2(1+\alpha_1 X+\alpha_2 Y)+6\alpha_2(1+\gamma_2 Y)\ (1+\gamma_1 X)^2$

$A_5 = 2(1-\nu)\lambda^2(9\gamma_1\gamma_2(1+\alpha_1 X+\alpha_2 Y)+3\alpha_1\gamma_2(1+\gamma_1 X)$
$\qquad +3\alpha_2\gamma_1(1+\gamma_2 Y)\ )(1+\gamma_1 X)(1+\gamma_2 Y)$

$A_6 = \Omega^2(1+\beta_1 X+\beta_2 Y),\ \Omega^2 = 12\rho_0(1-\nu^2)\omega^2/aE_0h_0^2$

Equation (6) is a fourth order partial differential equation of variable coefficients with respect to X and Y. It requires two boundary conditions at each edge. The combinations of following boundary conditions are considered in the present paper. For clamped edge:

$W = \frac{dW}{dX} = 0,\ W = \frac{dW}{dY} = 0$, at $X=0$ or $X=1$, and $Y=0$ or $Y=1$, respectively.

For simply supported edge:

$W = \frac{d^2W}{dX^2} = 0,\ W = 0\frac{d^2W}{dY^2} = 0$, at $X=0$ or $X=1$, and $Y=0$ or $Y=1$, respectively.

III. Generalized Differential Quadrature Method

According to Generalized differential quadrature (GDQ) method, the derivative of a function, with respect to a space variable at a given grid point, is approximated as a weighted linear sum of the function values at all of the grid points in the computational domain of that variable [7] .

The computational domain of a rectangular plate is $0 \le X \le 1, 0 \le Y \le 1$ Let $X_1, X_2, \ldots, X_N$ and $Y_1, Y_2, \ldots, Y_M$ are grid points in X and Y directions respectively. In this method, the n^{th} and m^{th} order derivatives of $W(X,Y)$ with respect to X, Y and its mixed derivative with respect to X and Y are approximated as

$$\frac{\partial^n W(X_i,Y_j)}{\partial X^n} = \sum_{l=1}^{N} a_{il}^{(n)}\ W(X_l,Y_j)$$
$$\frac{\partial^m W(X_i,Y_j)}{\partial Y^m} = \sum_{l=1}^{M} b_{jl}^{(m)}\ W(X_i,Y_l)$$
$$\frac{\partial^{m+n} W(X_i,Y_j)}{\partial X^n\,\partial Y^m} = \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(n)}\,b_{jl_2}^{(m)}\ W(l_1,l_2) \tag{7}$$

$i = 1,2,\ldots,N;\ j = 1,2,\ldots,M;\ n = 1,2,\ldots,N-1;\ m = 1,2,\ldots,M-1;$

where $a_{il}^{(n)}$ and $b_{jl}^{(m)}$ are the weighting coefficients associated with n^{th} and m^{th} order derivatives with respect to X and Y

respectively. The weighting coefficient of first order derivative are determined as

$$a_{ij}^{(1)} = \begin{cases} \dfrac{P^{(1)}(X_i)}{(X_i - X_j)\, P^{(1)}(X_j)}, j \neq i, \; -\sum\limits_{j=1,\, j\neq i}^{N} a_{ij}^{(1)}, j = i, \end{cases} \tag{8}$$

for

$$i,\ j = 1, 2, \ldots, N$$

where

$$P^{(1)}(X_i) = \prod_{j=1,\, j\neq i}^{N} (X_i - X_j)$$

Similarly, for the second and higher order derivatives the recurrence relationships are obtained as follows

$$a_{ij}^{(n)} = \begin{cases} n\left(a_{ii}^{(n-1)} a_{ij}^{(1)} - \dfrac{a_{ij}^{(n-1)}}{(X_i - X_j)}\right) & j \neq i, \\ -\sum\limits_{j=1,\, j\neq i}^{N} a_{ij}^{(n)} & j = i, \end{cases} \tag{9}$$

for

$$i,\ j = 1,\ 2, \ldots, N, \qquad n = 2,\ 3, \ldots, N-1$$

The corresponding coefficients $b_{jl}^{(m)}$ associated with derivatives with respect to y required can be similarly determined [7]. Discretizing (6) at the internal grid points (X_i, Y_j), with $3 \le i \le N-2$ and $3 \le j \le M-2$, it reduces

$$\begin{aligned} & A_0(i,j)\left(\sum_{l=1}^{N} a_{il}^{(4)} W_{l,j} + 2\lambda^2 \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(2)} b_{jl_2}^{(2)} W_{l_1,l_2} + \lambda^4 \sum_{l=1}^{M} b_{lj}^{(4)} W_{i,l}\right) \\ & + A_1(i,j)\left(\sum_{l=1}^{N} a_{il}^{(3)} W_{l,j} + \lambda^2 \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(1)} b_{jl_2}^{(2)} W_{l_1,l_2}\right) \\ & + A_2(i,j)\left(\lambda^4 \sum_{l=1}^{M} b_{lj}^{(3)} W_{i,l} + \lambda^2 \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(2)} b_{jl_2}^{(1)} W_{l_1,l_2}\right) \\ & + A_3(i,j)\left(\begin{matrix} \sum\limits_{l=1}^{N} a_{il}^{(2)} W_{l,j} \\ + \nu\lambda^2 \sum\limits_{l=1}^{N} b_{lj}^{(2)} W_{l,j} \end{matrix}\right) + A_4(i,j)\left(\begin{matrix} \nu\lambda^2 \sum\limits_{l=1}^{N} a_{il}^{(2)} W_{l,j} \\ + \sum\limits_{l=1}^{N} b_{lj}^{(2)} W_{l,j} \end{matrix}\right) \\ & + A_5(i,j) \sum_{l_1=1}^{N}\sum_{l_2=1}^{M} a_{il_1}^{(1)} b_{jl_2}^{(1)} W_{l_1,l_2} - A_6(i,j) W_{i,j} = 0 \end{aligned} \tag{10}$$

where N, M are the number of grid points in the X and Y directions and $a_{il}^{(n)}$, $b_{il}^{(m)}$ are the weighting coefficients in the X and Y directions, respectively. Similarly, the boundary conditions can be non-dimensionalized and then discretized by using GDQ. Here, the grid points chosen for collocation are the zeroes of shifted Chebyshev polynomial with orthogonality range [0, 1] given by

$$X_{i+1} = \frac{1}{2}\left[1 + \cos\left(\frac{2i-1}{N-2}\frac{\pi}{2}\right)\right], Y_{j+1} = \frac{1}{2}\left[1 + \cos\left(\frac{2j-1}{M-2}\frac{\pi}{2}\right)\right] \tag{11}$$
$$i = 1, 2, \ldots, N-2, \quad j = 1, 2, \ldots, M-2$$

IV. Numerical Results and Discussion

Equation (10) together with boundary conditions form a standard eigenvalue problem [7], which has been solve numerically using generalized differential quadrature method to obtain the frequency parameter Ω for various values of plate parameters. The values of various plate parameters are taken as follows: Non-homogeneity parameters α_1, α_2 = (-0.5(0.1)0.5), density parameters β_1, β_2 = 0.5(0.1)0.5), thickness parameter γ_1, γ_2 = (-0.5(0.1)0.5), aspect ratio a/b = (0.25(0.25)2.0) and Poisson's ratio ν =0.3.

To choose an appropriate number of grid points (N, M), convergence studies have been carried out for various set of plate parameters until the first six significant digits had converged. The convergence of frequency parameter Ω for the first three modes of vibration for a particular set i.e. $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \gamma_1 = \gamma_2 = 0.5,\ a/b = 1$ is shown in Table I. The values of both the grid points N and M have been fixed as 15 for both the boundary conditions.

A comparison of frequency parameter Ω for homogeneous $(\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0)$ square plate with those results obtained by other methods has been presented in Table II. A close agreement of results is obtained.

Fig. 2 depicts the behavior of frequency parameter Ω with non-homogeneity parameter α_1 for $\gamma_1 = \gamma_2 = 0.5$ $\alpha_2 = \pm 0.5,\ \beta_1 = \pm 0.5,\quad \beta_2 = 0.5$ and $a/b = 1$ for the first three modes of vibration. It is observed that the frequency parameter Ω increases with the increasing values of non-homogeneity parameter α_1. Further it is increases with the increasing values of α_2 while decreases with the increasing values of β_1 keeping all other parameters fixed. The rate of increase of Ω with α_1 is in the order of the boundary conditions CCCC>SSSS for both the values of α_2 and β_1 for the first three modes of vibration whatever be the values of other parameters.

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TABLE I
CONVERGENCE STUDY FOR THE FIRST THREE FREQUENCIES FOR $\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \gamma_1 = \gamma_2 = 0.5, a/b = 1$

MODE	I	II	III	I	II	III
No. of grid points ($N = M$)		CCCC			SSSS	
8	55.3769	111.501	112.291	30.7124	76.1069	76.9258
10	55.3821	111.950	112.831	30.7187	75.5726	76.4308
12	55.3816	111.943	112.825	30.7170	75.5727	76.4290
14	55.3816	111.943	112.825	30.7169	75.5731	76.4297
15	55.3816	111.943	112.825	30.7169	75.5730	76.4297
16	55.3816	111.943	112.825	30.7169	75.5730	76.4297

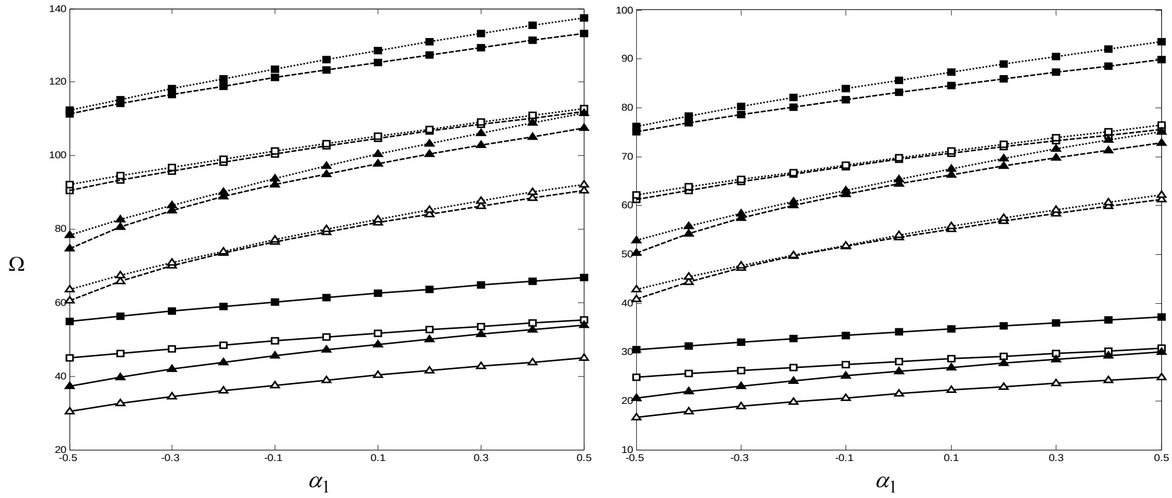

Fig. 2 Frequency parameter Ω for $\beta_2 = \gamma_1 = \gamma_2 = 0.5, a/b = 1$. first mode : ——; second mode: -------; third mode: ……; □, $\alpha_2 = 0.5,\ \beta_1 = 0.5$;■, $\alpha_2 = 0.5\ \ \beta_1 = -0.5$; Δ, $\alpha_2 = -0.5,\ \beta_1 = 0.5$; ▲, $\alpha_2 = -0.5,\ \beta_1 = -0.5$

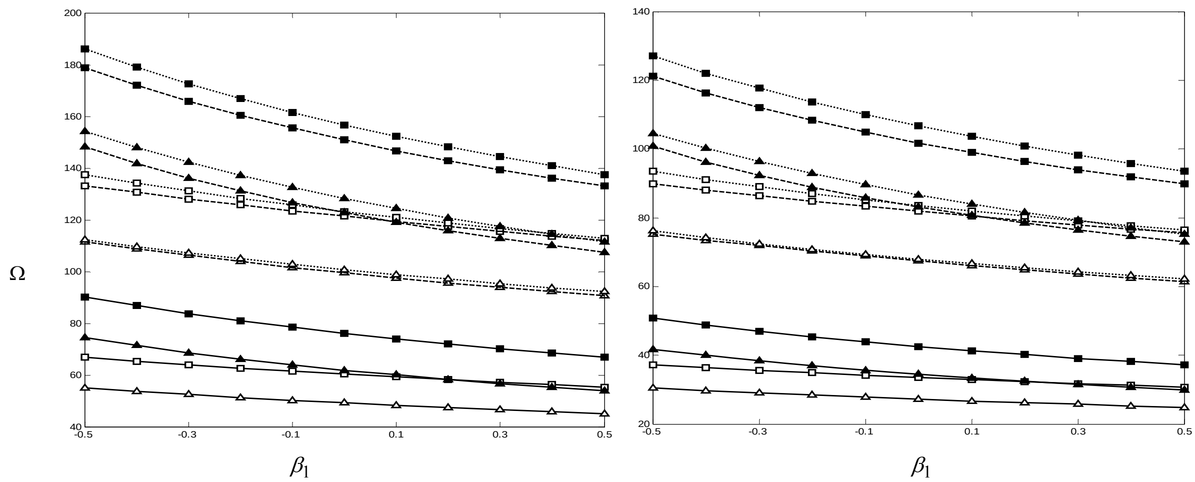

Fig. 3 Frequency parameter Ω for $\alpha_2 = 0.5, \gamma_1 = \gamma_2 = 0.5, a/b = 1$. first mode : ——; second mode: ------; third mode: ……; □, $\alpha_1 = 0.5,\ \beta_2 = 0.5$;■, $\alpha_1 = 0.5\ \ \beta_2 = -0.5$; Δ, $\alpha_1 = -0.5,\ \beta_2 = 0.5$; ▲, $\alpha_1 = -0.5,\ \beta_2 = -0.5$

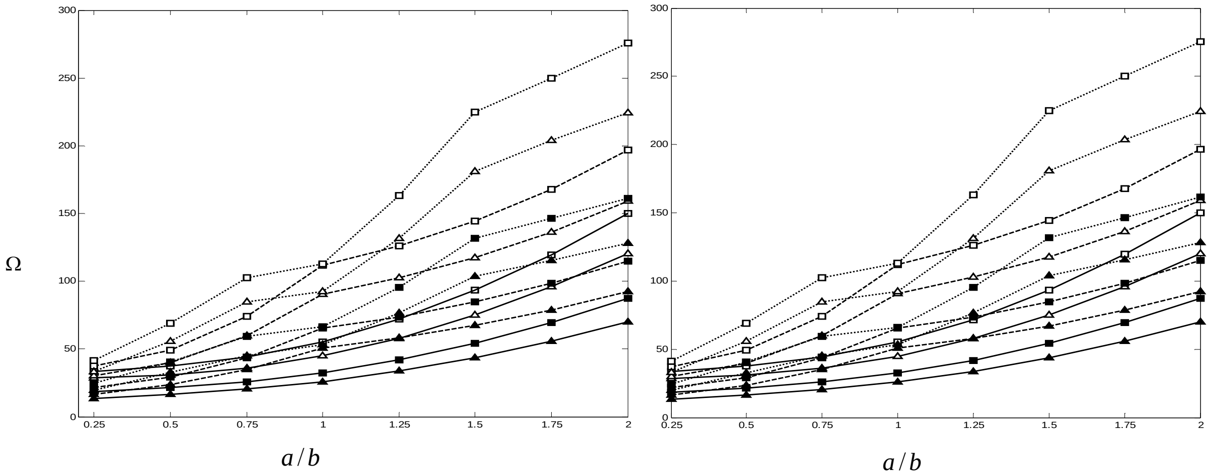

Fig. 4 Frequency parameter Ω for $\alpha_1 = \beta_1 = \beta_2 = \gamma_1 = 0.5$. first mode : ——; second mode:------; third mode: ……;□, $\alpha_2 = 0.5,\ \gamma_2 = 0.5$;■, $\alpha_2 = 0.5\ \ \gamma_2 = -0.5$; Δ, $\alpha_2 = -0.5,\ \gamma_2 = 0.5$; ▲, $\alpha_2 = -\ 0.5,\ \gamma_2 = -0.5$

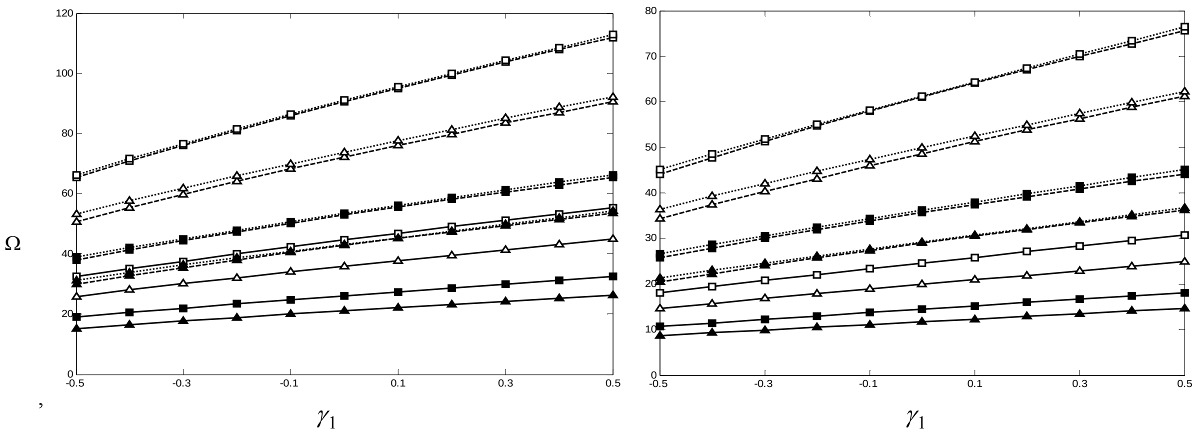

Fig. 5 Frequency parameter Ω for $\alpha_2 = 0.5, \beta_1 = \beta_2 = 0.5, a/b = 1$. first mode : ——; second mode: ------; third mode: ……; □, $\alpha_1 = 0.5,\ \gamma_2 = 0.5$; ■, $\alpha_1 = 0.5\ \ \gamma_2 = -0.5$; Δ, $\alpha_1 = -0.5,\ \gamma_2 = 0.5$; ▲, $\alpha_1 = -\ 0.5,\ \gamma_2 = -0.5$

TABLE II
COMPARISON OF FREQUENCY PARAMETER Ω FOR HOMOGENEOUS ($\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0$) SQUARE ($a/b = 1$) PLATE

Ref.	γ_1	γ_2	CCCC			SSSS		
			I	II	III	I	II	III
[8]	0.0	0.0	35.992	73.413	73.413	19.7392	49.3480	49.3480
[9]	0.0	0.0	35.986	73.395	73.395	19.739	49.348	49.348
[10]	0.0	0.0	35.986	73.395	73.395	19.7392	49.3481	49.3481
[11]	0.0	0.0	35.99	73.41	--	19.74	49.35	49.35
[12]	0.0	0.0	35.99	73.419	73.419	19.74	49.35	49.35
[13]	0.0	0.0	35.989	73.399	73.399	19.739	49.349	49.349
[6]	0.0	0.0	35.9855	73.3954	73.3954	----	-----	-----
Present	0.0	0.0	**35.9852**	**73.3938**	**73.3938**	**19.7392**	**49.3480**	**49.3480**
[11]	0.2	0.0	39.5097	80.5194	80.5857	21.6920	54.1607	54.2047
[6]	0.2	0.0	39.5097	80.5201	80.5859	-----	-----	-----
[14]	0.2	0.0	39.513	80.525	80.591	21.692	54.165	54.209
Present	0.2	0.0	**39.5094**	**80.5184**	**80.5842**	**21.692**	**54.1606**	**54.2038**
[14]	-0.5	-0.5	19.209	38.327	39.238	10.812	25.970	26.832
Present	-0.5	-0.5	**19.2072**	**38.3218**	**39.2272**	**10.8119**	**25.9245**	**26.8264**
[14]	0.5	0.5	55.208	111.72	112.66	30.553	75.300	76.211
Present	0.5	0.5	**55.2019**	**111.710**	**112.643**	**30.5526**	**75.2625**	**76.187**

Fig. 3 shows the behavior of the frequency parameter Ω with the density parameter β_1 for $\alpha_1 = \pm 0.5, \beta_2 = \pm 0.5,$ $\alpha_2 = 0.5,\ \gamma_1 = \gamma_2 = 0.5$ and $a/b = 1$ for the first three modes of vibration. It is found that the frequency parameter Ω decreases with the increasing values of density parameter β_1. The value of Ω increases with the increasing values of α_1. The rate of decrease of Ω with β_1 increases with the increasing values of α_1 while it is decreases with the increasing values of β_2. The rate of decrease of Ω with β_1 for CCCC plate is higher than that for SSSS plates when α_1 and β_2 changes from -0.5 to 0.5.

Fig. 4 illustrates the behavior of frequency parameter Ω with the increasing values of aspect ratio a/b for $\alpha_2 = \beta_2 = \pm 0.5$, $\alpha_1 = 0.5,\ \gamma_1 = \gamma_2 = 0.5$ and $\beta_1 = 0.5$ for the first three modes of vibration. It is clear that the frequency parameter Ω increases with the increasing values of aspect ratio a/b. The rate of increase of frequency parameter Ω with aspect ratio a/b is in the order of the boundary conditions CCCC>SSSS for both the values of α_2 and β_2 keeping other parameters fixed. This rate of increase is much higher for $a/b > 1$ as compared to $a/b < 1$.

The effect of thickness parameter γ_1 on the frequency parameter Ω for $\alpha_2 = 0.5,\ \beta_1 = \beta_2 = 0.5$, $a/b = 1, \alpha_2 = \pm 0.5$ and $\gamma_2 = \pm 0.5$ for the first three mode of vibration has been shown in Fig. 5. It is seen that the frequency parameter Ω increases with the increasing values of γ_1 for both the boundary conditions. The rate of increase of Ω with γ_1 is in the order of boundary condition CCCC>SSSS for the fixed values of other parameters.

V. Conclusions

The effect of non-homogeneity and thickness variation on the vibration characteristics of isotropic rectangular plates with varying aspect ratios has been studied on the basis of classical plate theory using generalized differential quadrature method. The thickness of the plate is taken bilinear along both the directions. The non-homogeneity of the plate material is assumed to arises due to the linear variations in Young's modulus and density of the plate material with in-plane co-ordinates x and y. Numerical results show that the frequencies for a CCCC plate are higher than that for a SSSS plate. It is observed that the values of frequency parameter Ω increases with the increasing values of non-homogeneity parameters α_1 and α_2, aspect ratio a/b while it decreases with the increasing values of density parameter β_1 and β_2 for both the boundary conditions keeping other plate parameters fixed. The frequency parameter Ω also increases with the increasing values of thickness parameter γ_1. The present analysis will be of great use to the design engineers in obtaining the desired frequency by varying one or more plate parameters considered here.

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