

# Transient Voltage Distribution on the Single Phase Transmission Line under Short Circuit Fault Effect

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**Abstract**—Single phase transmission lines are used to transfer data or energy between two users. Transient conditions such as switching operations and short circuit faults cause the generation of the fluctuation on the waveform to be transmitted. Spatial voltage distribution on the single phase transmission line may change owing to the position and duration of the short circuit fault in the system. In this paper, the state space representation of the single phase transmission line for short circuit fault and for various types of terminations is given. Since the transmission line is modeled in time domain using distributed parametric elements, the mathematical representation of the event is given in state space (time domain) differential equation form. It also makes easy to solve the problem because of the time and space dependent characteristics of the voltage variations on the distributed parametrically modeled transmission line.

**Keywords**—Energy transmission, transient effects, transmission line, transient voltage, RLC short circuit, single phase.

## I. INTRODUCTION

DUED to single or multi short circuit faults happening in different points on the transmission line with different durations, the waveform of the output is different than the transmitted waveform [1], [3]. The transmission line is decomposed as the interconnection of many time-invariant lumped parameter RLC sections. Any fault such as a short circuit between the transmitter end and load end of the transmission line divides the structure into two cascaded structures in between where the fault happens. The schematic view of the simulated structure is given in Fig. 1. Since the transmission line considered in this work is single phase, the wires have internal resistances, inductances (efficient for high frequencies) and capacitance between two wires. The duration of the short circuit fault affects the part of the circuit between source and fault and between the fault and load in a different way. Since, generally, the equivalent circuit values for transmission lines vary from manufacturer to manufacturer, and material to material. Considering the maximum transient frequency, the line length of a sufficiently high number of sections is chosen and the distributed nature of the transmission line can be simulated adequately [2]. The simulation is designed to be adaptive to variations in these values. The model is a continuous time invariant system model and for the solution, the conventional state space formulation and solution can be used.

Although the analytical solutions of the state space model is possible for only distortionless and lossless transmission lines

with simple termination, the numerical methods should be used for more general cases.

The distributed parametric form of the single phase transmission line given in Fig. 1 is simplified as a two stage circuit in T-form. Therefore, the line is as an open circuit at its receiving end (load end) and the last series resistance and inductance would not be included in the state space representation; hence, some part of the line would not be incorporated and this would reduce the accuracy of the lumped parameter representation. The model given here is not only valid for high voltage transmission, but also pulse transmission in the digital (computer) communication. In this study, we will simulate the power transmission and short duration pulse transmission giving some practical examples in the following section. In either case, the state space model is unique and may be formulated as:

$$dx(t)/dt = Ax(t) + Bu(t); x(0) = x(t_0) \quad (1a)$$

$$y(t) = Cx(t) + Du(t) \quad (1b)$$

In these equations, the state vector  $x$  contains some equivalent capacitor voltages and inductor current,  $u$  represents the input and  $y$  is the output vector.  $A$ ,  $B$ ,  $C$ , and  $D$  are the real constant matrices which depend on the lumped parameter values of the network, as given in (1.c)-(1.f).  $x(t_0)$  is the initial value of the state vector at the initial time  $t_0$ .

$$A = [-R_1/L - 1/L_1/C - 1/(C \cdot R_2)] \quad (1c)$$

$$B = [1/L_0] \quad (1d)$$

$$C = [0 \quad 1] \quad (1e)$$

$$D = [0] \quad (1f)$$

On the other hand, if pi-sections were used, this problem would not occur. However, then, the source parameters ( $R$  and  $L$ ) would not be combined with the line parameters and this would increase the total number of state variables.

### A. State Space Simulation

For the T-shape equivalence of the transmission line, the state space model of the system is given in (1a)-(1f). For some specific load termination, it will be convenient to use a specific lumped parameter transmission line model. For example, if the receiving end of the transmission line is short circuited, using L-sections for lumped parameter modeling removes the effect of the shunt capacitance and conductance at

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the last node [4]. A similar situation arises if pi-sections are used. Hence, L- or pi-sections are not preferred for the

representation of a transmission line with a short circuit end [5].

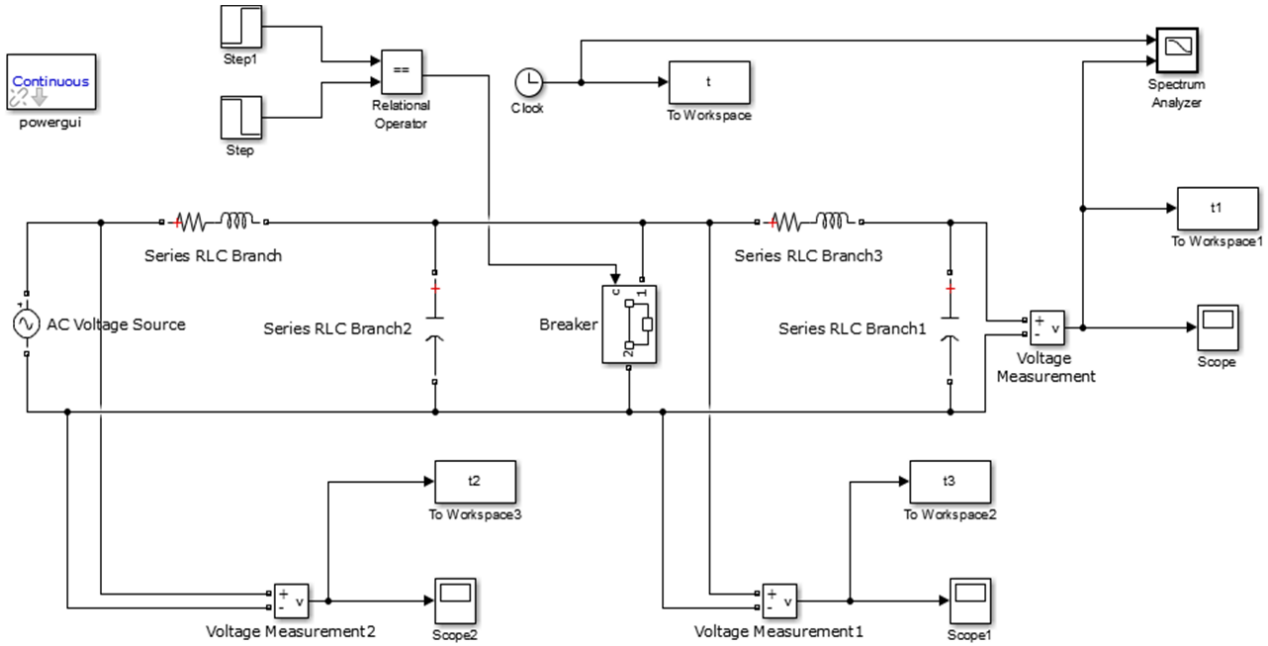


Fig. 1 RLC equivalent of transmission line modeled using Simulink

The short circuit fault model described above is used for lumped parameter representation of the line; hence the notation given above and the definitions of the state vector,  $x$ , as well as the coefficient matrices  $A$  and  $B$  given in (1.c) and (1.d). Hence, the state variables are defined as:

$$x = [i_1 \ v_1 \ i_2 \ v_2]^T \quad (2)$$

where  $i_i$  and  $v_i$ 's are the current and voltages of the inductors and capacitors in the circuit model, respectively.

The state equations can be solved using numerical integration method, but these methods are time consuming if the integration step is chosen to be small enough to increase the level of the accuracy. The solution of (1.a) for a sinusoidal excitation is:

$$x(t) = \text{Re}\{e^{A(t-t_0)}x_0 + (PI - A)^{-1} \times (e^{pt}BU - e^{A(t-t_0)}BUe^{pt_0})\} \quad (3)$$

where  $U$  is the peak value of the sinusoidal excitation. This equation gives the complete response of a lumped parameter system. The steady state solution, defined by limit time, goes to infinity in the equation and is written as:

$$x_{ss}(t) = \text{Re}\{(PI - A)^{-1}Be^{pt}U\} \quad (4)$$

#### B. Computer Simulation

As a first example, the values of the inductances, resistances and capacitors for each kilometer are given for 50 Hz. For higher frequency applications (data transmission for digital systems for example), the distributed values may be changed

easily as we will do in the next example.

The response of the system, under the short circuit fault instant and after, is obtained by using a closed form solution of the state space equations formulated from the lumped parameter equivalent of the distributed transmission system model and shown in Figs. 2-4. The model is simulated in Simulink and the equations are solved using MATLAB.

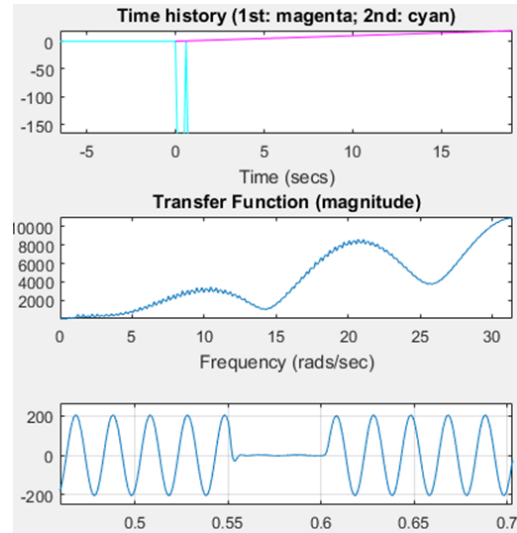


Fig. 2 Spectrum results of RLC equivalent with short circuit fault by Simulink

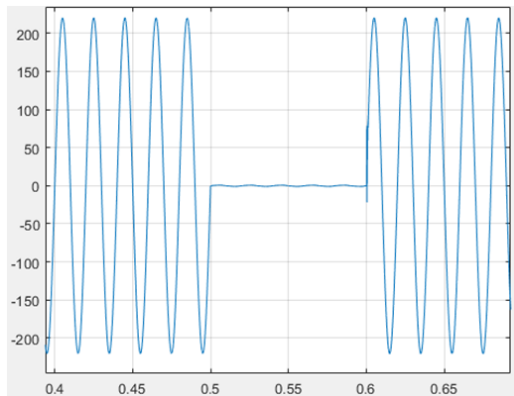


Fig. 3 Output result of RLC equivalent with fault by Simulink

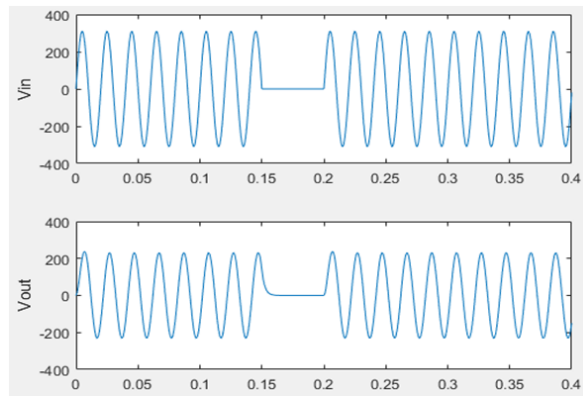


Fig. 4 The results of the RLC equivalent with fault by MATLAB

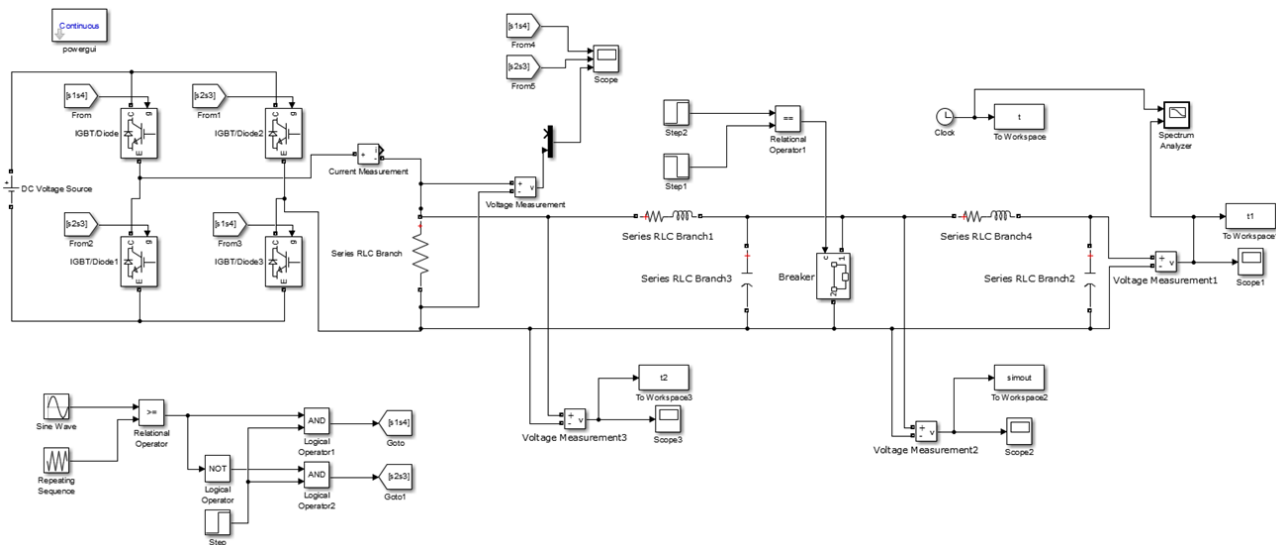


Fig. 5 The Simulink model of the line using pulse generation

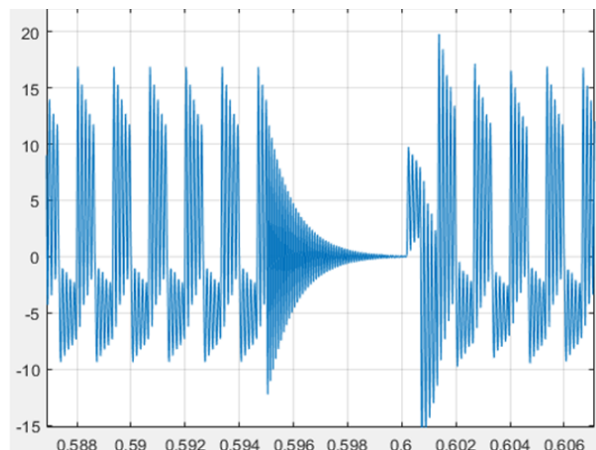
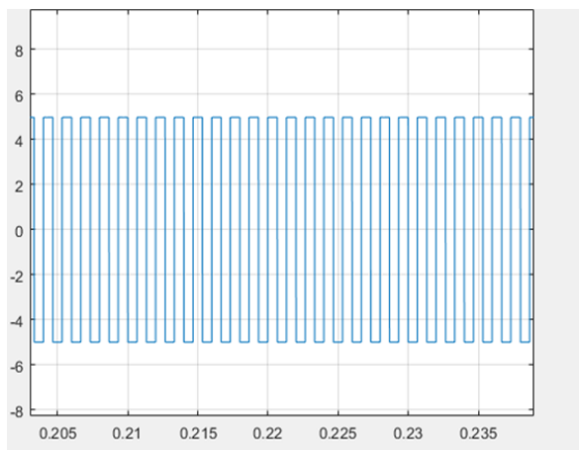


Fig. 6 Comparison between the input and output voltage signal in the Simulink model of the line using pulse generation

As a second example, bipolar binary pulses are applied to the transmission line. The line parameters are replaced by the real values used in the data transmission. The Simulink model of the line and pulse generation is given in Fig. 5 and the case

of a short circuit fault in the transmission line is generated and the waveforms at the input and output of the transmission line are given in Fig. 6.

### C. MATLAB Codes of the Solution

Equations (1a) and (1b) are used for a single-phase transmission line with parameters  $r = 0.03 \text{ ohm/km}$ ,  $L = 1 \text{ mH/km}$  and  $c = 10 \text{ nF/km}$ , and is computed using MATLAB that is shown in Fig. 7 With the knowledge that  $x=50 \text{ km}$  is the distance between the voltage source and short circuit.

```

1 - clc;
2 - clear all;
3 - close all;
4 - x=50;
5 - R1 = 0.03*x;
6 - R2 = 20;
7 - L = 1e-3*x;
8 - C = 10e-9*x;
9 - A = [-R1/L -1/L; 1/C -1/(C*R2)];
10 - B = [1/L; 0];
11 - C = [0 1];
12 - D = 0;
13 - [num den] = ss2tf(A,B,C,D);
14 - t = 0:0.001:0.4;
15 - H = tf(num,den);
16 - for i=1:length(t)
17 -     if t(i)>0.2 || t(i)<0.15
18 -         Vin(i) = 311*sin(314*t(i));
19 -     else
20 -         Vin(i) = 0;
21 -     end
22 - end
23 - subplot(2,1,1);
24 - plot(t,Vin);
25 - ylabel('Vin');
26 - [Vout u] = lsim(H,Vin,t);
27 - subplot(2,1,2);
28 - plot(t,Vout);
29 - ylabel('Vout');

```

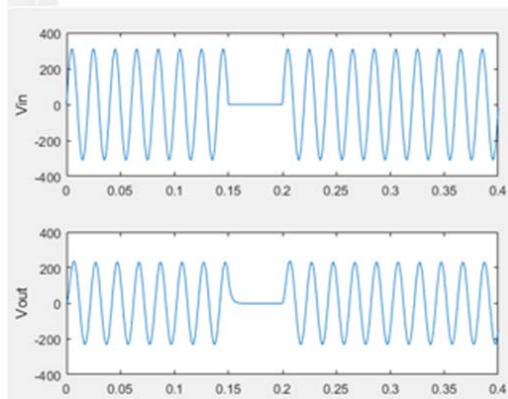


Fig. 7 MATLAB code of RLC equivalent with fault

### D. Conclusion

In this study, a transmission line is modeled as a lumped parameter circuit and the effect of the short circuit fault in between input and output is applied to see the change in the output waveform for sinusoidal input for energy transmission and pulse input for data transmission. The system is modeled using Simulink and solved by MATLAB.

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