# Time-Domain Analysis of Pulse Parameters Effects on Crosstalk (In High Speed Circuits) 

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#### Abstract

Crosstalk among interconnects and printed-circuit board (PCB) traces is a major limiting factor of signal quality in highspeed digital and communication equipments especially when fast data buses are involved. Such a bus is considered as a planar multiconductor transmission line. This paper will demonstrate how the finite difference time domain (FDTD) method provides an exact solution of the transmission-line equations to analyze the near end and the far end crosstalk. In addition, this study makes it possible to analyze the rise time effect on the near and far end voltages of the victim conductor. The paper also discusses a statistical analysis, based upon a set of several simulations. Such analysis leads to a better understanding of the phenomenon and yields useful information.


Keywords-Multiconductor transmission line, Crosstalk, Finite difference time domain (FDTD), printed-circuit board (PCB), Rise time, Statistical analysis.

## I. Introduction

DATA rate in digital electronics increases rapidly and thus, the influence of interconnects has to be considered at the design phase, too. Crosstalk may degrade the signal integrity and thus it is an important aspect in the design of printed circuit boards (PCBs). Crosstalk affects mainly and in a straight forward way the voltage of the victim signal, depending on the nature of the coupling between the aggressors and the victim. In this paper, we deal with data buses that are widely used in electronic systems to transfer data from one device to another or to multiple devices with a good level of precision. Such buses consist of a large number of coupled individual conductor traces implemented on a dielectric substrate within a printed circuit board (PCB). In industrial applications, such as embedded systems and motherboard of sophistical devices, the bus conductors can be excited with different voltage sources and also have different loads. In most cases, binary sequences propagate along the coupled traces by means of polar NRZ (Non Return to Zero) coding that enables a whole pulse for full duration of a bit. Logic "1" represents the signal at positive voltage and logic " 0 " represents the signal at negative voltage [2]. As far as many practical issues are concerned, we assume that the coupled conductors are excited randomly with unsynchronized pulses depending on the targeted performances. In order to handle the issue of source random occurrences, we carry out a time-domain statistical method that turns out very efficient in
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terms of determining crosstalk voltage variation at both the near and far ends of the victim trace.


Fig. 1 Example of a planar data bus
In this paper, the circuit model based on transmission line theory is introduced to analyze crosstalk problems and some factors affecting crosstalk. The analysis is based on numerical simulations using the finite difference time domain (FDTD). The FDTD is nowadays an established method in computational electromagnetic and is well suited for parasitic mode simulation [3]. The paper is organized as follows. In section II we describe the FDTD algorithm for the calculation of voltages and currents on the line especially at the input and output. Section III deals with the rise time effect on the near and far end voltages of the victim conductor by considering a simple case of 3-conductor transmission line. In section IV we propose a statistical analysis of crosstalk under random circumstances depending on the timing and probabilistic law of occurrences of the pulses. Finally in section V the paper is concluded.

## II. The FDTD Formulation

The transverse electromagnetic or TEM-mode model of a uniform $(\mathrm{n}+1)$-conductor, lossless transmission line is embodied in [4]:

$$
\begin{gather*}
\frac{\partial}{\partial \mathrm{z}} \mathbf{V}(\mathrm{z}, \mathrm{t})+\mathrm{L} \frac{\partial}{\partial \mathrm{t}} \mathbf{I}(\mathrm{z}, \mathrm{t})=0  \tag{1}\\
\frac{\partial}{\partial \mathrm{z}} \mathbf{I}(\mathrm{z}, \mathrm{t})+\mathrm{C} \frac{\partial}{\partial \mathrm{t}} \mathbf{V}(\mathrm{z}, \mathrm{t})=0 \tag{2}
\end{gather*}
$$

where V and I are $\mathrm{n} \times 1$ vectors of the line voltages (with respect to the reference conductor) and line currents respectively. The line cross-sectional dimensions are contained in the $\mathrm{n} \times \mathrm{n}$ per-unit-length parameter matrices of L (inductance) and C (capacitance). The position along the line is denoted as z and time is denoted as t . One of the important
approximate solution techniques is the Finite Difference TimeDomain method or FDTD [3]. In that method, the line axis $z$ is discretized in $\Delta z$ increments or spatial cells, the time variable $t$ is discretized in $\Delta \mathrm{t}$ increments or temporal cells, and the derivatives in the multiconductor transmission lines (MTL) equations are approximated by finite differences. The solution voltages and currents are obtained at these discrete points and represent an approximate solution of the MTL equations. In general, the accuracy of the solution depends on having sufficiently small spatial and temporal cell sizes [5].

In the case of MTL's, the boundary conditions are lumped loads at the two ends of the line, $\mathrm{z}=0$ and $\mathrm{z}=\mathrm{L}$ for a line of length L. Linear, resistive such terminations can be characterized by [6].

$$
\begin{gather*}
\mathbf{V}(0, \mathrm{t})=\mathrm{Vs}-R_{S} \mathbf{I}(0, \mathrm{t})  \tag{3}\\
\mathbf{V}(L, \mathrm{t})=R_{L} \mathbf{I}(L, \mathrm{t}) \tag{4}
\end{gather*}
$$

where Vs is the voltage sources
In order to insure stability in the FDTD solution, the discrete voltage and current solution points are not physically located at the same point but is staggered one-half cell apart [3].


Fig. 2 Spatial discretization of the line showing location of the interlaced voltage and current solution points

To apply this method, we divide the line into NDZ sections each of length $\Delta \mathrm{z}$ as shown in Fig. 2. Similarly, we divide the total solution time into segments of length $\Delta$ t. In order to insure stability of the discretization and to insure second-order accuracy we interlace the NDZ +1 voltage points, V1, V2, ... , VNDZ, VNDZ+1, and the NDZ current points, I1,12, ..,INDZ, as shown in Fig. 1. Each voltage and adjacent current solution points are separated by $\Delta z / 2$. In addition, the time points are also interlaced, and each voltage time point and adjacent current time point are separated by $\Delta \mathrm{t} / 2$.

Discretizing the derivatives in the MTL equations using the FDTD method we obtain:

$$
\begin{align*}
& \frac{V_{k+1}^{n+1}-V_{k}^{n+1}}{\Delta z}+\boldsymbol{L} \frac{L_{k}^{n+\frac{3}{2}}-I_{k}^{n+\frac{1}{2}}}{\Delta t}=0  \tag{5}\\
& \frac{I_{k}^{n+1 / 2}-I_{k-1}^{n+1 / 2}}{\Delta z}+\boldsymbol{C} \frac{V_{k}^{n+1}-V_{k}^{n}}{\Delta t}=0 \tag{6}
\end{align*}
$$

where we denote:

$$
\begin{align*}
& \boldsymbol{V}_{k}^{n}=V[(K-1) \Delta z, n \Delta t \quad]  \tag{7}\\
& \boldsymbol{I}_{k}^{n}=V[(K-1 / 2) \Delta z, n \Delta t] \tag{8}
\end{align*}
$$

Solving these gives the recursion relations at the source and the load:

$$
\begin{gather*}
\boldsymbol{V}_{1}^{n+1}=\boldsymbol{V}_{1}^{n}-\frac{2 \Delta t}{\Delta z} \boldsymbol{C}^{-1} \boldsymbol{I}_{1}^{n+1 / 2}+\frac{\Delta t}{\Delta z} \boldsymbol{C}^{-1}\left[\boldsymbol{I}_{s}^{n+1}+\boldsymbol{I}_{s}^{n}\right]  \tag{11}\\
\boldsymbol{V}_{N D Z+1}^{n+1}=\boldsymbol{V}_{N D Z+1}^{n}+\frac{2 \Delta t}{\Delta z} \boldsymbol{C}^{-1} \boldsymbol{I}_{N D Z}^{n+1 / 2}-\frac{\Delta t}{\Delta z} \boldsymbol{C}^{-1}\left[\boldsymbol{I}_{L}^{n+1}+\boldsymbol{I}_{L}^{n}\right] \tag{12}
\end{gather*}
$$

Using (3) and (4) we obtain:

$$
\begin{gathered}
\boldsymbol{V}_{1}^{n+1}=\left(\frac{\Delta z}{\Delta t} R_{s} \boldsymbol{C}+1\right)^{-1} \times\left[\left(\frac{\Delta z}{\Delta t} R_{s} \boldsymbol{C}^{-1}\right) \boldsymbol{V}_{1}^{n}-2 R_{s} \boldsymbol{I}_{1}^{n+\frac{1}{2}}+\left(\boldsymbol{V}_{s}^{n+1}+\boldsymbol{V}_{s}^{n}\right)\right](13) \\
\boldsymbol{V}_{N D Z+1}^{n+1}=\left(\frac{\Delta z}{\Delta t} R_{L} \boldsymbol{C}+1\right)^{-1} \times\left[\left(\frac{\Delta z}{\Delta t} R_{L} \boldsymbol{C}^{-1}\right) \boldsymbol{V}_{N D Z+1}^{n}-2 R_{L} \boldsymbol{I}_{N D Z}^{n+\frac{1}{2}}+\left(\boldsymbol{V}_{L}^{n+1}+\right.\right. \\
\left.\left.\boldsymbol{V}_{L}^{n}\right)\right]
\end{gathered}
$$

The voltages and currents are solved by iterating k for a fixed time and then iterating time. The initial conditions of zero voltage and current are used to start the iteration.

$$
\begin{gather*}
\boldsymbol{V}_{k}^{n+1}=\boldsymbol{V}_{k}^{n}-\frac{\Delta t}{\Delta z} \boldsymbol{C}^{-1}\left(\boldsymbol{I}_{K}^{n+\frac{1}{2}}-\boldsymbol{I}_{K-1}^{n+\frac{1}{2}}\right)  \tag{15}\\
\mathrm{k}=2, \ldots, \mathrm{NDZ} \\
\boldsymbol{I}_{K}^{n+3 / 2}=\boldsymbol{I}_{K}^{n+1 / 2}-\frac{\Delta t}{\Delta z} \boldsymbol{L}^{-1}\left(\boldsymbol{V}_{k+1}^{n+1}-\boldsymbol{V}_{k}^{n+1}\right)  \tag{16}\\
\mathrm{k}=1 \ldots \mathrm{NDZ}
\end{gather*}
$$

In the application of the conventional FDTD Yee algorithm for 1-D problems a necessary condition to insure stability of the solution is the Courant limit (CFL) [7]:

$$
\begin{equation*}
\Delta \mathrm{t} \leq \frac{\Delta \mathrm{z}}{\mathrm{Vp}} \tag{17}
\end{equation*}
$$

where $V p$ is the phase velocity of propagation of the waves.

## III. The Rise Time Effects on Crosstalk

In order to study the effects of the rise time upon the level of the near and far end crosstalk, the simulations are carried out with a three-conductor coplanar transmission lines shown in Fig. 3. The nominal parameters are $\mathrm{w}=20 \mu \mathrm{~m}, \mathrm{~s}=1.139 \mathrm{~mm}$, $\mathrm{h}=100 \mu \mathrm{~m}, \varepsilon \mathrm{r}=12$ and the total line length is 10 cm and is terminated at the near and far ends in $50 \Omega$ resistors. The source is a ramp function rising to a level of 1 V . The per-unitlength inductance and capacitance matrices were computed using a numerical method [1] giving:

$$
\begin{aligned}
& \mathbf{C}=\left[\begin{array}{cc}
134.693 & -67.3467 \\
-67.3467 & 1.07757
\end{array}\right] \mathrm{pF} / \mathrm{m} \\
& \mathbf{L}=\left[\begin{array}{cc}
0.805775 & 0.538783 \\
0.538783 & 97.6102
\end{array}\right] \mu \mathrm{H} / \mathrm{m}
\end{aligned}
$$

On one hand, the line is divided into section each of length
$\Delta \mathrm{z}$ o the other hand, the total time, which is 10 ns , is divided into interval each of length $\Delta t . \Delta t$ and $\Delta z$ are chose with respect to the stability condition with $\mathrm{Vm} 2=1.20476 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

The time domain simulations are performed for three values of rise time $\boldsymbol{\tau}$ that are $\boldsymbol{\tau}=1 \mathrm{~ns}, \boldsymbol{\tau}=2 \mathrm{~ns}, \boldsymbol{\tau}=3 \mathrm{~ns}$. The conductor (1) is the victim and the conductor (2) is the aggressor. Fig. 4 shows the near and far end voltages of the victim.


Fig. 3 Line under study. (a) The line dimensions and terminations. (b) The cross-sectional dimensions. (c) The source voltage wave shape

(a)

(b)

(c)

Fig. 4 (a) Near End and Far end crosstalk for $\tau=1$ ns. (b) Near End and Far end crosstalk for $\tau=2$ ns. (c) Near End and Far end crosstalk for $\tau=3 \mathrm{~ns}$

From the results discussed previously, it is observed that the near end crosstalk amplitude is decreased to 0.1 V when the rise time is increased from 1 ns to 3 ns . Whereas the far end crosstalk is slightly lowered to a magnitude of 0.05 V . As a result, the rise time influences the victim's voltage at both near end and far end. This conclusion must be taken into account when designing a fast system strength forward.

## IV. Transmission Line Excited by Shifted Pulses

As well known, the transfer of data along a transmission line causes inevitably electromagnetic coupling between conductors leading to crosstalk voltages. The transfer is generally performed by a code according to a random law.

In order to address this issue, for better understand, a simple configuration is proposed and analyzed in this section. We are limited to the case of transmitting two bits, each on different trace.

The simulations are carried out with a 4-conductor coplanar transmission lines with two aggressors and one victim as shown in Fig. 5. The nominal parameters are $\mathrm{w}=2.159 \mathrm{~mm}$, $\mathrm{s}=1.139 \mathrm{~mm}, \mathrm{~h}=1.20 \mathrm{~mm}, \varepsilon \mathrm{r}=4.7$ and the total line length is 10 cm and is terminated at the near and far ends in $50 \Omega$ resistors. Two lines are excited by two pulses whose rise time and fall
times are identical ( $\tau_{1}=200 \mathrm{ps}, \tau=1 \mathrm{~ns}$ ) rising to a level of 1 V . The coupling effect of these two active traces on the victim one surely takes place whatever the signal waveform. The crosstalk voltage level depends strongly on the random occurrence and the amplitude of the two pulses [8].

As an assumption, the amplitude is taken constant and deterministic. However, the two bits have random occurrences governed by the uniform law whether the pulses are synchronized or unsynchronized.

In most situations, unsynchronized pulses can have either a small time shift or large time shift between them. Both situations are analyzed by means of statistical study taking into account equiprobable case.


Fig. 5 Line under study: (a) The line dimensions and terminations (b) The cross-sectional dimensions (c) The source voltage wave shape

## A. Small Time Shift

We first consider the combinations of the synchronized and small time shifted pulses possibilities representing logic " 1 " and logic " 0 " as illustrated by Fig. 6. Excitation couples $\left(\mathrm{V}_{\mathrm{S} 1}, \mathrm{~V}_{\mathrm{S} 2}\right)$ are chosen randomly and independently with equiprobable occurrences according to the uniform law. Fig. 7 shows the computed results through the FDTD method, of the near end (NEXT) and far end (FEXT) crosstalk voltage running 3000 simulations.


Fig. 6 Possibilities of small time shifted pulses

(a)

(b)

Fig. 7 (a) FEXT crosstalk voltage for 3000 simulations. (b) NEXT crosstalk voltage for 3000 simulations

As well expected, there are several responses depending on the injection of excitation sources. It is interesting to note that crosstalk voltage reaches a maximum level that can be described as the worst-case, at a precise instant. Thus, FEXT and NEXT voltages present worst-case values at 400 ps and 200 ps, respectively. Figs. 8 and 9 show the histograms illustrating the number of occurrences of the worst-case values of FEXT and NEXT voltages.


Fig. 8 Histogram of NEXT crosstalk for 3000 simulations at $t=$ 200ps


Fig. 9 Histogram of the FEXT crosstalk for 3000 simulations at $t=$ 400ps

From Figs. 8 and 9, we notice that both NEXT and FEXT voltages present almost the same number of occurrences of the worst-case with different values. Ranging from -0.2 V to 0.2 V .

## B. Large time shift

In this section, we propose a further study to collect further information on the statistical behavior of the crosstalk response. This fact can be achieved by considering large time shifted pulses combined with synchronized ones as shown in Fig. 10. The time domain responses are identical to those obtained for the small time shift case. The excitation couples $\left(\mathrm{V}_{\mathrm{S} 1}, \mathrm{~V}_{\mathrm{S} 2}\right)$ are randomly and independently chosen with equiprobable occurrences according to the uniform law. The histograms, corresponding to the computed results at 200 ps and 400 ps , are displayed in Figs. 11 and 12.

According to the two histograms shown in Figs. 11 and 12, we can see that the maximum magnitude values of NEXT and FEXT crosstalk voltage do not have the greatest number of occurrences.


Fig. 10 Possibilities of large time shifted pulses


Fig. 11 Histogram of NEXT crosstalk for 3000 simulations at $\mathrm{t}=$ 200ps


Fig. 12 Histogram of FEXT crosstalk for 3000 simulations at $\mathrm{t}=$ 400ps

The main conclusion that can be drawn from the simulations is the fact that both cases of large time and small time shifted pulses combined with synchronized ones lead to almost the same variation in terms of the number of occurrences as far as equiprobable case are concerned separately. This aspect is observed when considering the maximum and minimum magnitudes of crosstalk at the far and near ends of the victim trace.

## V.CONCLUSION

A time domain method, based on the FDTD algorithm to
solve multiconductor transmission line equations, has been proposed to perform a statistical evaluation of the crosstalk on a data bus. A circuit that consists of three-conductor coplanar transmission lines one aggressor traces and one victim trace has been analyzed to study the effects of the rise time upon the level of the near and far end crosstalk. However, a simple configuration is proposed and analyzed with a 4-conductor coplanar transmission lines with two aggressors and one victim. The transmission line circuit under study has been excited randomly with two pulses according to the uniform law. The excitations, synchronized or unsynchronized, can be injected with either equiprobable occurrence. The statistical analysis has led to several histograms representing the number of occurrences of the crosstalk worst case level at both near and far ends of the victim trace. The crosstalk variation in the case of more than one aggressor has been handled producing great deal of useful information.

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