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Thermophoresis Particle Precipitate on Heated Surfaces

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Abstract**—**This work deals with heat and mass transfer by steady laminar boundary layer flow of a Newtonian, viscous fluid over a vertical flat plate with variable surface heat flux embedded in a fluid saturated porous medium in the presence of thermophoresis particle deposition effect. The governing partial differential equations are transformed into no-similar form by using special transformation and solved numerically by using an implicit finite difference method. Many results are obtained and a representative set is displaced graphically to illustrate the influence of the various physical parameters on the wall thermophoresis deposition velocity and concentration profiles. It is found that the increasing of thermophoresis constant or temperature differences enhances heat transfer rates from vertical surfaces and increase wall thermophoresis velocities; this is due to favorable temperature gradients or buoyancy forces. It is also found that the effect of thermophoresis phenomena is more pronounced near pure natural convection heat transfer limit; because this phenomenon is directly a temperature gradient or buoyancy forces dependent. Comparisons with previously published work in the limits are performed and the results are found to be in excellent agreement.

Keywords**—**Thermophoresis, porous medium, variable surface heat flux.

I. INTRODUCTION

THERMOPHORESIS is a phenomenon, which causes small particles to be driven away from a hot surface and toward a cold one. Dust particles when suspended in a gas temperature gradient; experience a force in the direction opposite to the temperature gradient. This phenomenon has many practical applications in removing small particles from gas streams, in determining exhaust gas particles trajectories from combustion devices, and in studding the particulate material deposition on turbine blades.

Goren [1] studied the effect of thermophoresis on a viscous and incompressible fluid, the classical problem of flow over a flat plate is used to calculate deposition rates and it is found that the increasing of difference between the surface and free stream temperatures causes substantial changes in surface deposition. Gokoglu and Rosner [2] obtained a set of similarity solutions for the two dimensional laminar boundary layers, Park and Rosner [3] obtained a set of similarity solutions for the stagnation point flows. Chio [4] obtained the similarity solutions for the problem of a continuously moving surface in a stationary incompressible fluid, including the combined effects of convection, diffusion, wall velocity and thermophoresis. Grag and Jayaraj [5] discussed the thermophoresis of small particles in forced convection laminar flow over inclined plates embedded in a plain medium; Epstein et al. [6] have studied the thermophoresis transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid. Chiou [7] has considered the particle deposition from natural convection boundary layer flow onto an isothermal vertical cylinder. Convective flows in porous media have been extensively investigated during the last several decades, due to many practical applications, which can be modeled or approximated as transport phenomena in porous media. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop [8], Nield and Bejan [9].

In this work, the problem selected for study is the pure forced, pure natural and mixed convection heat and mass transfer problems from vertical surfaces embedded in a fluid saturated porous media with variable surface heat and mass fluxes. Full inclusion of the thermophoresis phenomena is done in the formulations.

II. MATHEMATICAL FORMULATION

Consider mixed convection from an impermeable vertical surface embedded in saturated porous medium. The analysis is carried out for the power-law variation of the surface heat flux $q_w(\mathrm{x}) = bx^m$ and the power law variation of the surface mass flux $q_0(\mathrm{x}) = Lx^n$, where b and L are constants and m and n are the exponents. The x coordinate is measured from the leading edge of the plate and the y coordinate is measured normal to the plate. The gravitational acceleration g is acting downward in the direction opposite to the x coordinate. The Darcy model which is valid under the conditions of low velocities and small pores of porous matrix is used in the analysis. Also the properties of the fluid are assumed to be constant and the porous medium is treated as isotropic. Allowing for both Brownian motion of particles and thermophoresis transport, the governing equations can be written as Lai and Kulacki [11]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = \frac{Kg}{\upsilon}\left(\beta_T (T - T_\infty) + \beta_c (C - C_\infty)\right) \tag{2}$$

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$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\frac{\partial (Cv_t)}{\partial y}=D\frac{\partial^2 C}{\partial y^2} \quad (4)$$

The first two terms on the left hand side of the mass transfer equation is the convective mass flux due to concentration differences and the third one is the thermophoresis mass flux due to temperature differences, while the right hand side is the conductive mass flux to concentration differences. The u and v are the Darcian velocity components in x and y directions respectively, T is the fluid temperature, C is the fluid concentration, K is the permeability of the porous medium, υ is the kinematic viscosity, D is the Brownian diffusion coefficient, α_m is the effective thermal diffusivity of the porous medium, and β_T and β_C are the thermal expansion coefficient of temperature and concentration, respectively. The effect of thermophoresis is usually prescribed by means of the average velocity, which a particle will acquire when exposed to a temperature gradient. Under boundary layer approximations the temperature and concentration gradients in the y-direction are very much larger than in the x-direction, and therefore only the thermophoresis velocity in the y-direction is considered. In consequence the thermophoresis velocity v_t can be expressed in the form:

$$v_t=-k_t\frac{\upsilon}{T}\frac{\partial T}{\partial y} \quad (5)$$

Here k is the thermophoresis coefficient. The boundary conditions that describe the governing (1)-(5) are:

$$\begin{aligned} &v=0,\ q_w(x)=bx^m,\ q_0(x)=Lx^n \qquad \text{at } y=0\\ &u=u_\infty,\ T=T_\infty,\ C=C_\infty \qquad \text{at } y\to\infty \end{aligned} \quad (6)$$

Note that *m*=0 corresponds to the case of constant wall heat flux and *n*=0 corresponds to the case of constant mass flux. Equations (1)-(6) can be transformed from the (*x*, *y*) coordinates to the dimensionless coordinate (ξ,η) by introducing the following non-dimensional variables:

$$\begin{aligned} &\eta=\frac{y}{x}Pe_x^{1/2}\xi^{-1}\ ,\ \xi=\frac{1}{[1+\left(Ra_x/Pe_x^{3/2}\right)^{1/3}]^{-1}}\\ &\psi=\alpha_m Pe_x^{1/2}f(\xi,\eta)\xi^{-1},\ \Theta(\xi,\eta)=\frac{(T-T_\infty)Pe_x^{1/2}}{q_w(x)x/k}\xi^{-1}\\ &\Phi(\xi,\eta)=\frac{(C-C_\infty)Pe_x^{1/2}}{q_0(x)x/k}\xi^{-1} \end{aligned} \quad (7)$$

In the equations above, the stream function ψ satisfied the continuity (1) with $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. Finally one can obtain the following system of dimensionless equations:

$$f''=(1-\xi)^3\left(\Theta'+N\Phi'\right) \quad (8)$$

$$\Theta''+\frac{1}{3}[(m+2)-(m+\frac{1}{2})\xi]f'\Theta-\frac{1}{3}[(2m+1)+(m+\frac{1}{2})\xi]f\Theta' =\frac{1}{3}(m+\frac{1}{2})\xi(1-\xi)\left(\Theta'\frac{\partial f}{\partial\xi}-f'\frac{\partial\Theta}{\partial\xi}\right) \quad (9)$$

$$\frac{1}{Le}\Phi''+\frac{1}{3}[(n+2)-(n+\frac{1}{2})\xi]f'\Phi-\frac{1}{3}[(2n+1)-(n+\frac{1}{2})\xi]f\Phi'+ \frac{k_t\Pr}{\Theta+N_t}[\Theta'\Phi'+\Phi\Theta''-\frac{\Phi}{\Theta+N_t}\Theta'^2] =\frac{1}{3}(n+\frac{1}{2})\xi(1-\xi)\left(\Phi'\frac{\partial f}{\partial\xi}-f'\frac{\partial\Phi}{\partial\xi}\right) \quad (10)$$

with the corresponding boundary conditions:

$$f(\xi,0)=0,$$
$$\Theta(\xi,0)=-1,\ \Phi(\xi,0)=1, f'(\xi,\infty)=\xi^2,\ \Theta(\xi,\infty)=0, \Phi(\xi,0)=0 \quad (11)$$

Here $Pe_x=u_\infty x/\alpha$, $Ra_x=g\beta_T q_w(x)Kx^2/k\upsilon\alpha$, $\Pr=\upsilon/\alpha_m$, $Le=\alpha_m/D$, $N=\beta_C q_0(x)/\beta_T q_w(x)$, $N_t=T_\infty/\left[q_w(x)x/k_t(Pe_x^{1/2}+Ra_{x)}^{1/3}\right]$, and the primes denotes partial differentiations with respect to η . In the system of dimensionless (8)-(11), the case of $\xi=0$ represents the pure natural convection heat transfer limit, the case of $\xi=1$ represents the pure forced convection limit. In this work the values of $\xi=0-1$ are included in order to cover the entire mixed convection regime.

Some of the physical quantities of practical interest include the velocity component *u* the local Nusselt number $Nu_x=hx/k$ and the dimensionless wall thermophoresis deposition velocity V_{tw}. They are given by:

$$u=u_\infty\xi^2 f'(\xi,\eta) \quad (12)$$

$$Nu_x(Pe_x^{1/2}+Ra_x^{1/3})^{-1}=1/\Theta(\xi,0) \quad (13)$$

$$V_{tw}(Pe_x^{1/2}+Ra_x^{1/3})^{-1}=\frac{k_t\Pr}{\Theta(\xi,0)+N_t} \quad (14)$$

The partial differential (8)-(10) under boundary conditions (11) are non-linear, coupled partial differential equations which posses no closed form solution. Therefore, they must be solved numerically by using an implicit iterative tridiagonal finite-difference method as described by Cebeci and Bradshaw [10].

III. Results and Discussion

The thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and to a cold one. The effect of thermophoresis is appeared in governing equations by the inclusion of N (buoyancy ratio), k_t (thermophoresis coefficient), N_t (heat flux ratio) and N_C (mass flux ratio).

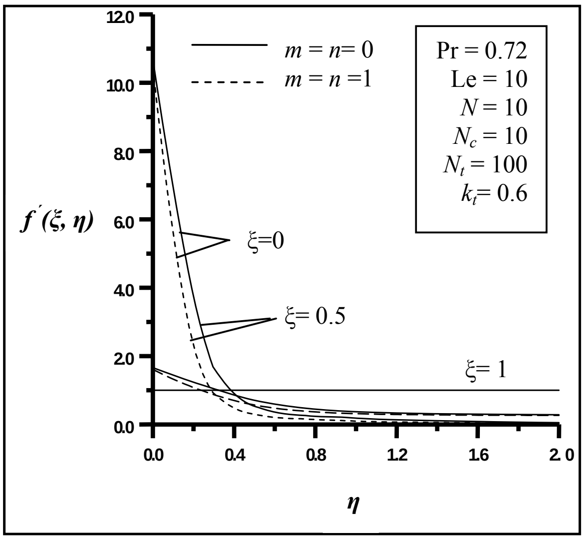

Fig. 1 Dimensionless velocity profiles for different values of mixed convection parameter

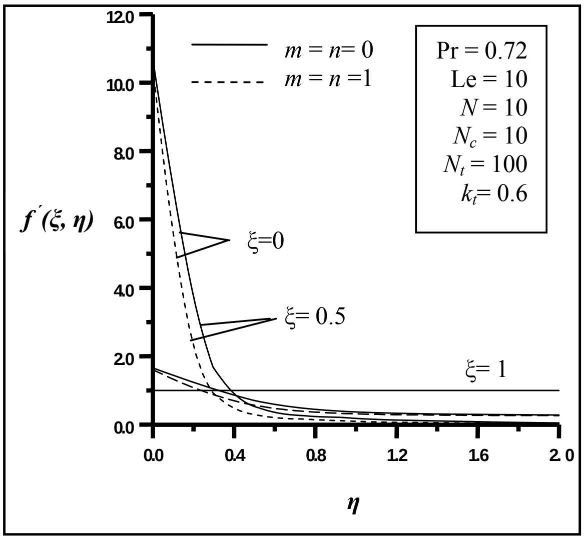

Fig. 2 Dimensionless temperature profiles for different values of mixed convection parameter

In Figs. 1-3 the velocity, temperature, and concentration profiles are drawn for Pr = 0.72, Le= 10, N= 10, Nc = 10, Nt = 100, kt = 0.6, n = m=0, n = m =1 and different mixed convection parameter $\xi = 0, 0.5, 1$. It is obvious as the mixed convection parameter is increased; the velocity inside boundary layer is increased due to favorable forced convection heat transfer effects and the temperature and concentration profiles are broadened; this leads to higher heat and mass transfer coefficients.

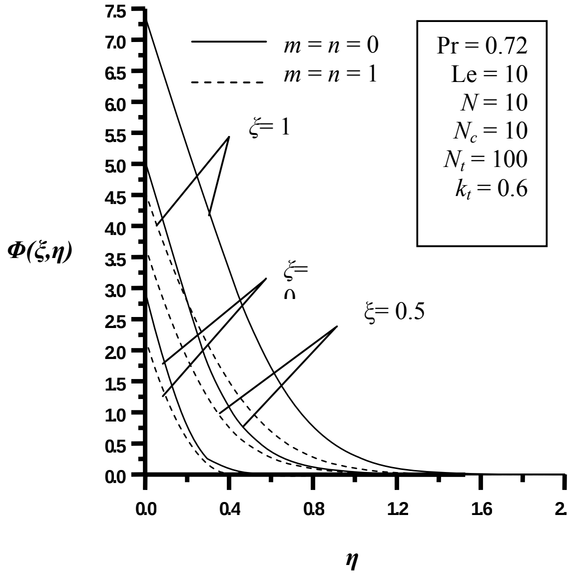

Fig. 3 Dimensionless concentration profiles for different values of mixed convection parameter

IV. Conclusions

Numerical solutions for heat and mass transfer by steady, laminar boundary layer of a Newtonian fluid over a vertical flat plate with variable surface heat and mass fluxes and embedded in a porous medium in the presence of thermophoresis particle deposition effect were studied. Based on the obtained graphical results, the following conclusions were deduced:

1) Both thermophoresis and local Nusselt number values are enhanced when heat and mass flux ratios between surface and free stream conditions are increased this is due to favorable buoyancy forces.
2) The effect of increasing power heating and mass index n, m is to enhance thermophoresis wall velocity and local Nusselt numbers; this is due to excessive heating and temperature differences.
3) When the buoyancy ratio parameter is decreased towards the zero, the thermophoresis parameter had no effect on both wall thermophoresis and local Nusselt numbers; this is due to small temperature differences between vertical surfaces and free stream condition.

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