

Thermodynamic Optimization of Turboshaft Engine using Multi-Objective Genetic Algorithm

S. Farahat, E. Khorasani Nejad, and S. M. Hoseini Sarvari

Abstract—In this paper multi-objective genetic algorithms are employed for Pareto approach optimization of ideal Turboshaft engines. In the multi-objective optimization a number of conflicting objective functions are to be optimized simultaneously. The important objective functions that have been considered for optimization are specific thrust (F / \dot{m}_0), specific fuel consumption (S_p), output shaft power ($\dot{W}_{shaft} / \dot{m}_0$) and overall efficiency (η_o). These objectives are usually conflicting with each other. The design variables consist of thermodynamic parameters (compressor pressure ratio, turbine temperature ratio and Mach number).

At the first stage single objective optimization has been investigated and the method of NSGA-II has been used for multi-objective optimization. Optimization procedures are performed for two and four objective functions and the results are compared for ideal Turboshaft engine. In order to investigate the optimal thermodynamic behavior of two objectives, different set, each including two objectives of output parameters, are considered individually. For each set Pareto front are depicted. The sets of selected decision variables based on this Pareto front, will cause the best possible combination of corresponding objective functions.

There is no superiority for the points on the Pareto front figure, but they are superior to any other point. In the case of four objective optimization the results are given in tables.

Keywords—Multi-objective, Genetic algorithm, Turboshaft Engine.

I. INTRODUCTION

IN most real-world problems, several goals must be satisfied simultaneously in order to obtain an optimal solution. The multiple objectives are typically conflicting and non-commensurable, and must be satisfied simultaneously. For example, we might want to be able to maximize the output shaft power of a turboshaft engine while minimizing the fuel consumption. Actually, multi-objective optimization is very

different than the single-objective optimization. In single objective optimization, one attempts to obtain the best design or decision, which usually the global minimum or the global maximum depending on the optimization problem is that of minimization or maximization. In multiple objective optimization, there may not exist one solution which is best (global minimum or maximum) with respect to all objectives. In multi-objective optimization problem, there exist a set of solutions which are superior to the rest of solution in the search space when all objectives are considered but are inferior to other solution in the space in one or more objectives. These solutions are known as Pareto-optimal solutions or nondominated solutions. Since none of the solution in the nondominated set is absolutely better than any other, any one of them is an acceptable solution [1-4].

There are many methods to solve multi-objective problems. In this paper we use the Non-dominated Sorting Genetic Algorithm (NSGA-II). NSGA-II proposed in Srinivas and Deb [5].

In this paper, an optimal set of design variables in turboshaft engines, namely, the input flight Mach number M_0 , the pressure ratio of the compressor π_c , and the Turbine temperature ratio τ_t are used by Pareto approach to multi-objective optimization. First, different pairs of conflicting objectives in an ideal turboshaft engine are selected for optimization. Then, a new diversity preserving algorithm called ϵ -elimination diversity algorithm is used for enhancing the performance of NSGA-II in terms of diversity of population and Pareto fronts. The modified algorithm has been used for multi-objective optimization with more than two objectives by Atashkari et.al [6]. Finally, four-objective optimization approaches of turboshaft engines is conducted considering η_o , F / \dot{m}_0 , $\dot{W}_{shaft} / \dot{m}_0$ and S_p as competing objectives. The superiority of the ϵ -elimination diversity preserving mechanism is shown, compared to that of NSGA-II.

II. MULTI-OBJECTIVE OPTIMIZATION

Multi-objective optimization, which is also called multicriteria optimization or vector optimization, is defined as finding a vector of decision variables satisfying constraints to give acceptable values to all objective functions [3,7]. In general, it can be mathematically defined as: find the vector

S. Farahat, is with the Department of Mechanical Engineering, University of Sistan & Baluchestan, Zahedan, Iran (e-mail: farahat@hamoon.usb.ac.ir).

E. Khorasani Nejad, is with the Department of Mechanical Engineering, University of Islamic Azad University Behbahan Branch, Iran (e-mail: ehsan.kh1002@gmail.com).

S. M. Hoseini Sarvari, is with the Department of Mechanical Engineering, University of Sistan & Baluchestan, Zahedan, Iran (e-mail: sarvari@hamoon.usb.ac.ir).

$$X^* = [x_1^*, x_2^*, \dots, x_n^*]^T \text{ to optimize}$$

$$F(X) = [f_1(x), f_2(x), \dots, f_k(x)]^T \quad (1)$$

subject to m inequality constraints

$$g_i(X) \leq 0, \quad i = 1, \dots, m \quad (2)$$

and p equality constraints

$$h_j(X) = 0, \quad i = 1, \dots, p \quad (3)$$

Where $X^* \in \mathfrak{R}^n$ is the vector of decision or design variables, and $F(X) \in \mathfrak{R}^k$ is the vector of objective functions, which must each be either minimized or maximized. However, without loss of generality, it is assumed that all objective functions are to be minimized. Such multi-objective minimization based on Pareto approach can be conducted using some definitions:

A. Pareto Dominance

A vector $U = [u_1, u_2, \dots, u_k] \in \mathfrak{R}^k$ is dominant to vector $V = [v_1, v_2, \dots, v_k] \in \mathfrak{R}^k$ (denoted by $U < V$) if and only if $\forall i \in \{1, 2, \dots, k\}, u_i \leq v_i \wedge \exists j \in \{1, 2, \dots, k\} : u_j < v_j$.

Thus, one can say there is at least one u_j which is smaller than v_j whilst the remaining u 's is either smaller or equal to corresponding v 's.

B. Pareto Optimality

A point $X^* \in \Omega$ is a feasible region in \mathfrak{R}^n satisfying Equations (2) and (3) is said to be Pareto optimal (minimal) with respect to all $X \in \Omega$ if and only if $F(X^*) < F(X)$. Alternatively, it can be readily restated as $\forall i \in \{1, 2, \dots, k\}, \forall X \in \Omega - \{X^*\} : f_i(X^*) \leq f_i(X) \wedge \exists j \in \{1, 2, \dots, k\} : f_j(X^*) < f_j(X)$.

In words, the solution X^* is said to be Pareto optimal (minimal) if no other solution can be found to dominate X^* using the definition of Pareto dominance.

C. Pareto Set

The Pareto set P^* is a set in the decision variable space consisting of all the Pareto optimal vectors $P^* = \{X \in \Omega \mid \nexists X' \in \Omega : F(X') < F(X)\}$.

Simply, there is no other X' as a vector of decision variables in Ω that dominates any $X \in P^*$.

D. Pareto Front

The Pareto front Pf^* is a set of vector of objective functions which are obtained using the vectors of decision variables in the Pareto set P^* , that is $Pf^* = \{(f_1(X), f_2(X), \dots, f_k(X)) : X \in P^*\}$.

So, one can say the Pareto front Pf^* is a set of the vectors of objective functions mapped from P^* .

Different algorithms have been widely used for multiobjective optimization because of their natural properties suited for these types of problems. The NSGA-II is one of these algorithms. In order to show this algorithm more clearly, some basics of NSGA-II are represented. In Fig. 1 demonstrated now selects individuals from the entire population R_t to construct the next parent population R_{t+1} . The entire population R_t is simply the current parent population P_t plus its offspring population Q_t which is created from the parent population P_t by using usual genetic operators. The selection is based on non-dominated sorting procedure which is used to classify the entire population R_t according to increasing order of dominance [6].

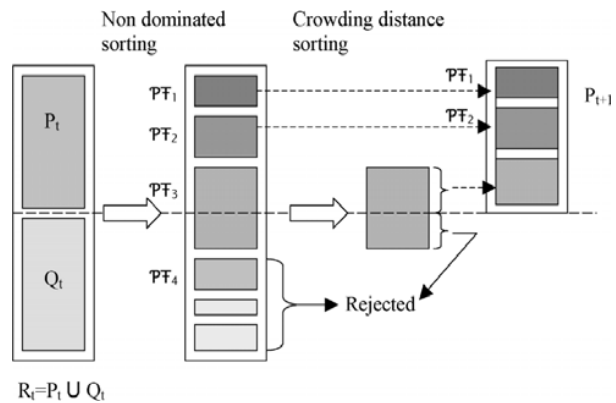


Fig. 1 Basics of NSGA-II procedure [6]

Thus, the best Pareto fronts from the top of the sorted list is chosen to create the new parent population P_{t+1} which is half the size of the entire population R_t . So, it should be noted that all the individuals of a certain front cannot be modified in the new parent population because of space, as shown in Fig. 1. To choose an exact number of individuals of that particular front, a crowded comparison operator is used in NSGA-II to find the best solutions to complete the new parent population. The crowded comparison procedure is based on density estimation of solutions surrounding a particular solution in a population or front. So, the solutions of a Pareto front are first sorted in each objective direction in the ascending order of that objective value. The crowding distance is then assigned equal to the half of the perimeter of the enclosing hyper box. Other objectives are sorted too and the overall crowding distance is calculated as the sum of the crowding distances from all objectives. The less crowded non-dominated individuals of that particular Pareto front are then selected to fill the new parent population. It is important to know that in a two-objective Pareto optimization, if the solutions of a Pareto front are sorted in a decreasing order of importance to one objective, these solutions are then automatically ordered in an increasing order of importance to the second objective. In other words, the hyper-boxes surrounding an individual solution remain unchanged in the objective-wise sorting procedure of the crowding distance of NSGA-II in the two-objective Pareto optimization problem. However, in multi-

objective Pareto optimization problem with more than two objectives, such sorting procedure of individuals based on each objective in this algorithm will cause different enclosing hyper boxes. Therefore, the overall crowding distance of an individual computed in this way may not exactly reflect the true measure of diversity or crowding property for the multi-objective Pareto optimization problems with more than two objectives.

In reference [6], a new method is presented which modifies NSGA-II so that it can be safely used for any number of objective functions (particularly for more than two objectives). The modified method is then used for a four objective thermodynamic optimization of turboshaft engines and the results are compared with those of the original NSGA-II.

III. THE ϵ -ELIMINATION DIVERSITY ALGORITHM [6]

In the ϵ -elimination diversity approach that is used to main loop in NSGA-II, all the clones and/or ϵ -similar individuals based on Euclidean norm of two vectors are recognized and simply eliminated from the current population. Therefore, based on a pre-defined value of ϵ as the elimination threshold ($\epsilon = 0.001$ has been used in this paper) all the individuals in a front within this limit of a particular individual are eliminated. It should be noted that such ϵ -similarity must exist both in the space of objectives and in the space of the associated design variables. This will ensure that very different individuals in the space of design variables having ϵ -similarity in the space of objectives will not be eliminated from the population. The pseudo-code of the ϵ -elimination approach is depicted in Fig. 2. Evidently, the clones or ϵ - similar individuals are replaced from the population with the same number of new randomly generated individuals.

```

Pseudo-code of  $\epsilon$ -elimination
 $\epsilon$ -elim= $\epsilon$ -elimination (pop) //pop includes design variables and
                                //objective functions.
define  $\epsilon$  //Define elimination threshold.
get k (k = 1 for the first front) //Front No.
i = 1
until i + 1 < pop_size
    j = i + 1
    until j < pop_size
        IF {||F(X(i)), F(X(j))|| <  $\epsilon$   $\wedge$  ||X(i), X(j)|| <  $\epsilon$ }
            F(X(i)), F(X(j))  $\in$   $\mathcal{P}_k^*$  X(i), X(j)  $\in$   $\mathcal{P}_k^*$ 
            THEN pop = pop \ pop(j) //Remove the  $\epsilon$ -similar individual.
        r_new_ind = make_new_random_individual
        //Generate new random individual.
        pop = pop  $\cup$  r_new_ind //Add the newly generated individual.
    end
end
    
```

Fig. 2 Pseudo-code of ϵ -elimination for preserving genetic diversity

IV. MULTI-OBJECTIVE THERMODYNAMIC OPTIMIZATION OF TURBOSHAFT

The Turboshaft engine is similar to the Turboprop except that power is supplied to a shaft rather than a propeller. The Turboshaft engine is used quite extensively for supplying

power for helicopters [8]. For analysis, we consider an ideal Turboshaft engine, whose exhausted gas develops thrust through a nozzle. That is shown in Figs. 3 and 4.

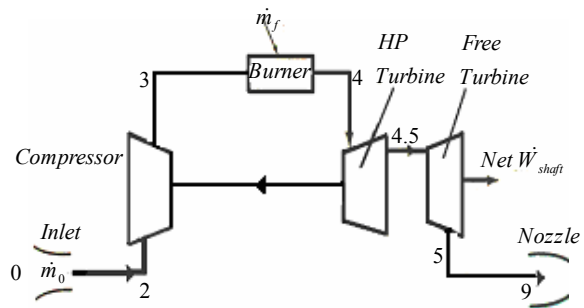


Fig. 3 Station numbering of Turboshaft engine

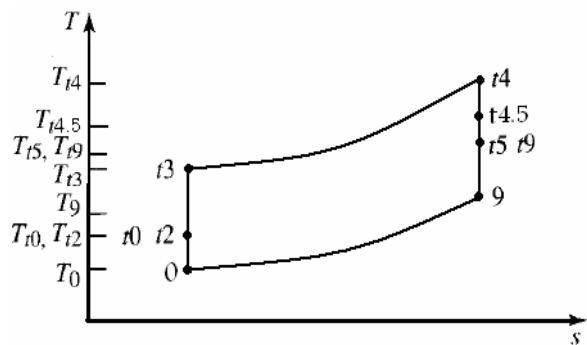


Fig. 4 The T-S diagram of ideal Turboshaft engine

The study of the thermodynamic cycle of a turboshaft engine involves different thermo-mechanical aspects such as specific output shaft power, specific thrust, overall efficiency, and specific fuel consumption [8]. A detailed description of the thermodynamic analysis and equations of ideal turboshaft engines is given in section V.

The input parameters in this thermodynamic analysis which assumed as an ideal turboshaft engine given in section V are flight Mach number (M_0), input air temperature (T_0), specific heat ratio (γ), heating value of fuel (h_{PR}), exit burner total temperature (T_{14}), turbine temperature ratio (τ_t) and compressor pressure ratio, (π_c). The output parameters in the thermodynamic analysis in the ideal turboshaft engine given in section V are, specific output shaft power ($\dot{W}_{shaft} / \dot{m}_0$), specific thrust (F / \dot{m}_0), fuel-to-air ratio (f), specific fuel consumption (S_p) and overall efficiency (η_o). In this study, some input parameters are already assumed as, $T_0 = 290$ K, $\gamma = 1.4$, $h_{pr} = 48000$ kJ.kg⁻¹, and $T_{14} = 1400$ K. The input flight Mach number $0.1 < M_0 < 0.5$, the turbine temperature ratio $0.2 < \tau_t < 0.9$ and the compressor pressure ratio $2 < \pi_c < 20$ are considered as design variables to be optimally found based on multi-objective optimization of 4 output parameters, namely, $\dot{W}_{shaft} / \dot{m}_0$, F / \dot{m}_0 , S_p , and η_o .

TABLE I
 NOMENCLATURE

Symbol	Quantity	Units
M_0	Flight Mach number	[-]
R	Gas constant	[kJ.kg ⁻¹ .K ⁻¹]
a	Speed of sound	[m.s]
g_c	Newton's constant	[-]
T_0	Inlet temperature	[K]
γ	Ratio of specific heats	[-]
C_p	Thermal conductivity	[kJ.kg ⁻¹ .K ⁻¹]
h_{PR}	Heating value	[kJ.kg ⁻¹]
T_{t4}	Burner exit total temperature	[K]
π_c	Compressor pressure ratio	[-]
τ_t	Turbine temperature ratio	[-]
$\dot{W}_{shaft} / \dot{m}_0$	Output shaft power	[kW.kg ⁻¹ .sec]
F / \dot{m}_0	Specific thrust	[N.kg ⁻¹ .s ⁻¹]
f	Fuel/air ratio	[-]
S_p	Specific fuel consumption	[mg.kW ⁻¹ .sec ⁻¹]
η_o	overall efficiency	[-]
C	Work output coefficient	[-]
$F(X)$	Vector of objective functions	[-]

$$\frac{V_9}{a_9} = \sqrt{\frac{2}{\gamma-1} \left(\tau_\lambda \tau_t - \frac{\tau_\lambda}{\tau_r \tau_c} \right)} \quad (12)$$

$$C_c = (\gamma-1) M_0 \left(\frac{V_9}{a_0} - M_0 \right) \quad (13)$$

$$C_{shaft} = \tau_\lambda \tau_{tH} (1 - \tau_{tL}) \quad (14)$$

$$C_{tot} = C_{shaft} + C_c \quad (15)$$

$$\frac{\dot{W}_{shaft}}{\dot{m}_0} = C_p T_0 C_{shaft} \quad (16)$$

$$\frac{F}{\dot{m}_0} = \frac{C_c C_p T_0}{V_0} \quad (17)$$

$$S_p = \frac{f}{C_{tot} C_p T_0} \quad (18)$$

$$\eta_o = \frac{C_{tot}}{\tau_\lambda - \tau_r \tau_c} \quad (19)$$

V. EQUATIONS

A. Assumptions

Inlet diffuser, compressor, turbine and exit nozzle, all operate isentropically.

No pressure loss in the burner. $f = (\text{fuel/air}) \ll 1$, P_e (turboshaft exit pressure) = P_o (ambient pressure).

B. Equation

$$R = \frac{\gamma-1}{\gamma} C_p \quad (4)$$

$$a_0 = \sqrt{\gamma R g_c T_0} \quad (5)$$

$$\tau_r = 1 + \frac{\gamma-1}{2} M_0^2 \quad (6)$$

$$\tau_\lambda = \frac{T_{t4}}{T_0} \quad (7)$$

$$\tau_c = (\pi_c)^{(\gamma-1)/\gamma} \quad (8)$$

$$f = \frac{C_p T_0}{h_{PR}} (\tau_\lambda - \tau_r \tau_c) \quad (9)$$

$$\tau_{tH} = 1 - \frac{\tau_r}{\tau_\lambda} (\tau_c - 1) \quad (10)$$

$$\tau_{tL} = \frac{\tau_t}{\tau_{tH}} \quad (11)$$

VI. RESULTS

To analysis the optimal thermodynamic behavior of turboshaft engines, at the first each objective function was optimized individually, then 5 different sets, each including two objectives of the output parameters, are considered. Such pairs of objectives to be optimized separately have been chosen as $(F/\dot{m}_0, \dot{W}_{shaft}/\dot{m}_0)$, $(\dot{W}_{shaft}/\dot{m}_0, \eta_o)$, $(\dot{W}_{shaft}/\dot{m}_0, S_p)$, $(F/\dot{m}_0, S_p)$ and $(F/\dot{m}_0, \eta_o)$. It can be observed that $F/\dot{m}_0, \dot{W}_{shaft}/\dot{m}_0, \eta_o$ are maximized whilst S_p is minimized in those sets of objective functions. Finally, all of objective functions have been optimized simultaneously. A population size of 40 has been chosen with crossover probability P_c and mutation probability P_m as 0.75 and 0.70, respectively for single-objective optimization and a population size of 120 has been chosen with crossover probability P_c and mutation probability P_m as 0.94 and 0.1 respectively for 2 and 4-objective optimization.

The results of the single-objective optimizations are summarized in Table II.

TABLE II
VALUES OF DECISION VARIABLES AND OBJECTIVE FUNCTIONS

$2 < \pi_c < 20$, $0.2 < \tau_t < 0.9$, $0.1 < M_0 < 0.5$	
$F / \dot{m}_0 = 597.2401$	$W_{shaft} / \dot{m}_0 = 1535.1$
$\pi_c = 7.4950$	$\pi_c = 13.4085$
$\tau_t = 0.7031$	$\tau_t = 0.4684$
$M_0 = 0.1$	$M_0 = 0.3938$
$S_p = 39.2849$	$\eta_o = 59.4746$
$\pi_c = 19.9997$	$\pi_c = 20$
$\tau_t = 0.4188$	$\tau_t = 0.4188$
$M_0 = 0.5$	$M_0 = 0.5$

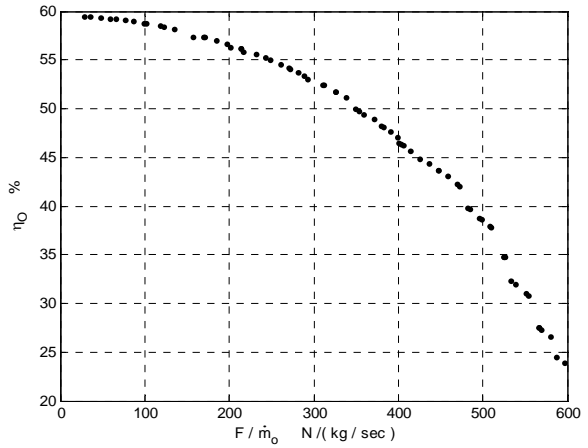


Fig. 7 Pareto front of specific thrust and overall efficiency in 2-objective optimization

Some Pareto fronts of each pair of two objectives have been shown through Figs. 5-9.

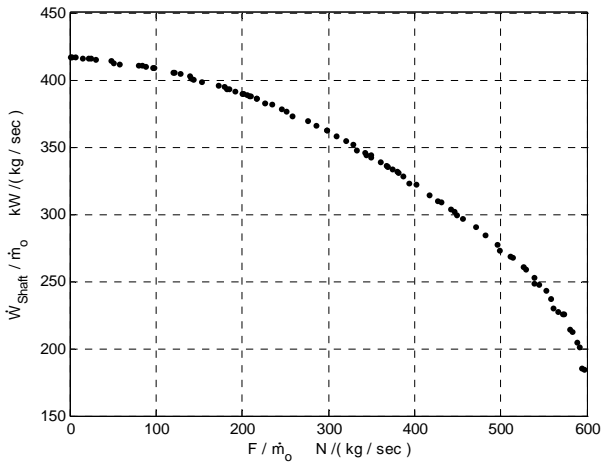


Fig. 5 Pareto front of specific thrust and specific output shaft power in 2-objective optimization

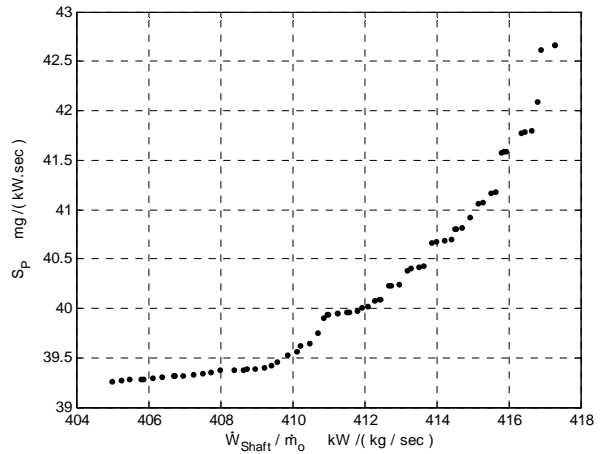


Fig. 8 Pareto front of specific output shaft power and specific fuel consumption 2-objective optimization

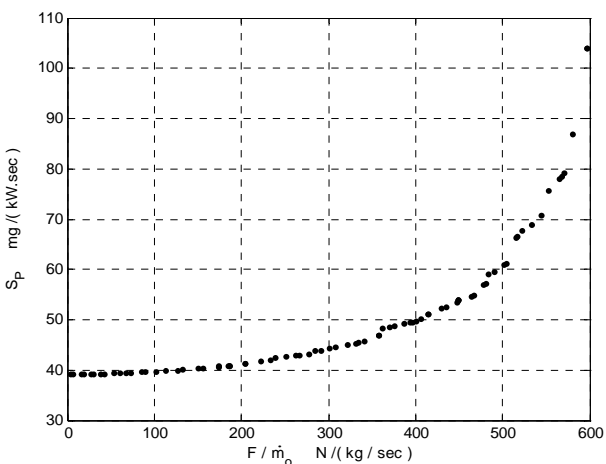


Fig. 6 Pareto front of specific thrust and specific fuel consumption 2-objective optimization

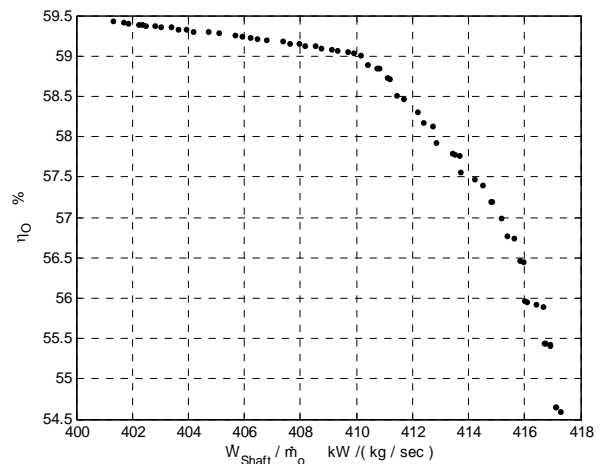


Fig. 9 Pareto front of overall efficiency and specific output shaft power 2-objective optimization

These figures and the associated values of the decision variables and the objective functions given in Table II simply cover all the 4 objectives studied in the two-objective Pareto optimization. The first and the end points of this diagrams that is explanatory extremum points at single-objective optimization are compared with the results given in Table II. The result of this comparison indicates the similar conformity.

Fig. 5 shows variation of specific thrust and specific output shaft power. Interval variations are (1.1382, 597.1973) and (184.4425, 417.197) for specific thrust and specific output shaft power, respectively. The initial and the end of values of this diagram are very similar to the optimal values of single-objective condition.

Fig. 6 shows variation of specific thrust and specific fuel consumption. Interval variations are (3.0331, 596.4545) and (39.2516, 104.1194) for specific thrust and specific fuel consumption, respectively. At this diagram by attention to characteristic problem designer can be determined optimal point. At single-objective condition (Table II) minimum point of specific fuel consumption and maximum point of specific thrust are 39.2849 and 597.2401, respectively, that this points is closer to the initial and the end points of this diagram [9].

Fig. 7 shows variation of specific thrust and overall efficiency. The initial and the end point to this diagram indicates maximum both functions, that accord with the result of the obtained single-objective optimization.

Fig. 8 shows variation of specific output shaft power and specific fuel consumption.

Fig. 9 shows variation of overall efficiency and specific output shaft power.

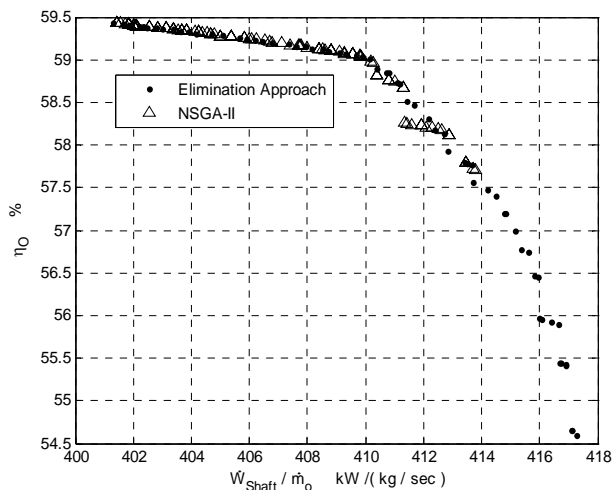


Fig. 10 Pareto front of overall efficiency and specific output shaft power in 2-objective optimization

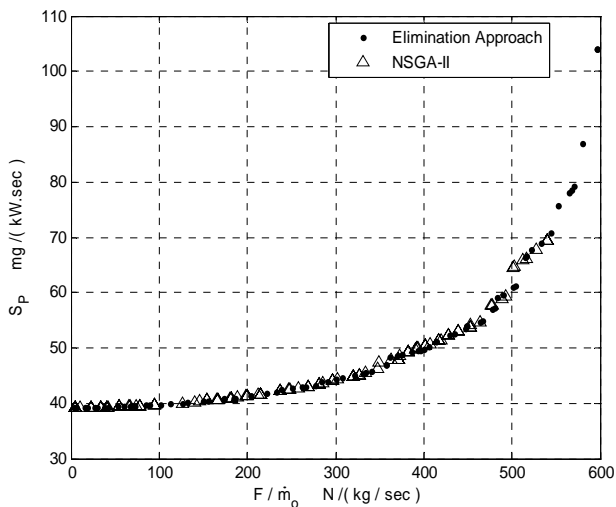


Fig. 11 Pareto front of specific thrust and specific fuel consumption in 2-objective optimization

Figs. 10 and 11, depicts comparison of approach NSGA-II with elimination approach. As seen elimination approach is smoother than other one.

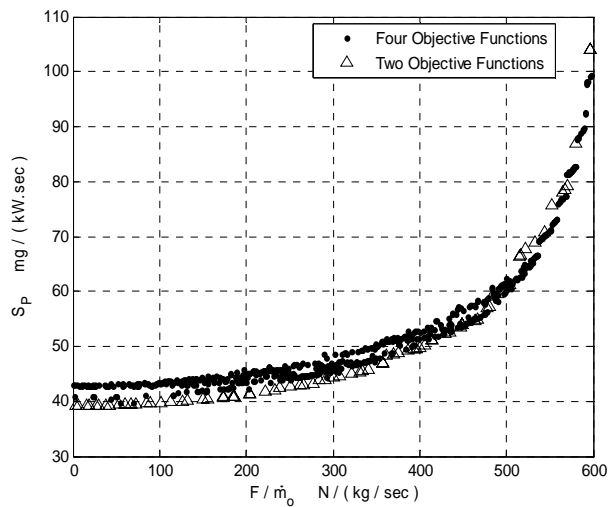


Fig. 12 Specific thrust variation with specific fuel consumption in both 4-objective & 2-objective optimization

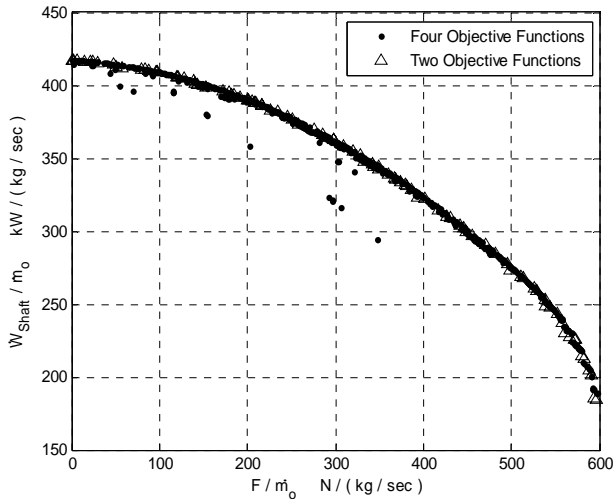


Fig. 13 Specific thrust variation with specific output shaft power in both 4-objective & 2-objective optimization

Fig. 12, demonstrate the non-dominated individuals in both 4-objective and previously obtained 2-objective optimization in the plane of $(F/\dot{m}_0, S_p)$. Such non-dominated individuals in both 4 and 2-objective optimization have alternatively been shown in the plane of $(F/\dot{m}_0, \dot{W}_{shaft}/\dot{m}_0)$ in Fig. 13. It should be noted that there is a single set of individuals as a result of 4-objective optimization of $F/\dot{m}_0, \dot{W}_{shaft}/\dot{m}_0, S_p$ and η_o that are shown in different planes together with the corresponding 2-objective optimization results. Therefore, there are some points in each plane that may dominate others in the same plane in the case of 4-objective optimization. However, these individuals are all non-dominated when considering all four objectives simultaneously. By careful investigation of the results of 4-objective optimization in each plane, the Pareto fronts of the corresponding two-objective optimization can now be observed in these figures. It can be readily observed that the results of such 4-objective optimization include the Pareto fronts of each 2-objective optimization and provide, therefore, more optimal choices for the designer.

VII. CONCLUSION

In the single objective optimization an objective function was investigated by changing several design variables, simultaneously. The correlation between the optimal point and the objective function and design variable are obtained. In the two-objective optimization, the comparison of the first and the end points of Pareto curvature with the result of single-objective show the compatibility with these diagrams.

Further, it has been shown that the results of 4-objective optimization include those of 2-objective optimization in terms of Pareto frontiers and provide, consequently, more choices for optimal design.

REFERENCES

- [1] N. Srinivas, K. Deb, "Multi-Objective Optimization using Nondominated Sorting in Genetic Algorithm," *Evolutionary Computation*, Vol. 2, No. 3, pp. 221-248, 1994.
- [2] C.M. Fonseca, P.J. Fleming, "Genetic Algorithm for Multi-Objective optimization: Formulation, discussion and generalization, in: S. Forrest(Ed.), *proc. Of the Fifth Int. Conf. On genetic Algorithm*," Morgan Kaufmann, San Mateo, CA, 1993, pp. 416-423.
- [3] C.A. Coello, A.D. Christiansen, "MultiObjective Optimization of Trusses using Genetic Algorithm, *Compute*," *Structures* 75, pp. 647-660, 2000.
- [4] C.A. Coello Coello, D.A. Van Veldhuizen, G.B. Lamont, "Evolutionary Algorithm for solving MultiObjective Problems," Kluwer Academic, Dordrecht, 2002.
- [5] Deb, K., Pratap, S., Agarward, S., "A Fast and Elitist Multi-Objective Genetic Algorithm," NSGAI, kangal report, 2001.
- [6] K. Atashkari, N. Nariman-zadeh, A. Pilchi, A. Jamali, "Thermodynamic Pareto Optimization of Turbojet Engine Using Multi-Objective Genetic Algorithm," *International Journal of Thermal Sciences*, 44, PP. 1061-1071, 2005.
- [7] A. Osyezka, "Multicriteria optimization for engineering design," in: J. S. Gero(Ed.), *Design Optimization*, Academic Press, New York, 1985, pp. 193-227.
- [8] Mattingly, J.P., "Elements of Gas Turbine Propulsion," *Mc Graw Hill*, 1996.
- [9] E. Khorasani Nejad, "Turboshaft Engine Performance Optimization using Multi-Objective Genetic Algorithm," M.Sc. Dissertation, The university of Sistan & Baluchestan, 2009.