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The Views of Elementary Mathematics Education Preservice Teachers on Proving

Belma Turker, Cigdem Alkas, Ebru Aylar, Ramazan Gurel, Oylum Akkus Ispir

***Abstract*—**This study has been prepared with the purpose to get the views of senior class Elementary Education Mathematics preservice teachers on proving. Data have been obtained via surveys and interviews carried out with 104 preservice teachers. According to the findings, although preservice teachers have positive views about using proving in mathematics teaching, it is seen that their experiences related to proving is limited to courses and they think proving is a work done only for the exams. Furthermore, they have expressed in the interviews that proving is difficult for them, and because of this reason they prefer memorizing instead of learning.

***Keywords*—**Belief on Proving, Mathematics Education, Proof.

I. INTRODUCTION

PROOF process, which we can define as a process followed in the name of reaching a correct judgment, consists of three stages that are different from each other but interrelated. These are introducing what is to be proved, designing the proof, and explaining it to other people [1]. Proof is given importance in mathematics education not only in the name of reaching correct information; but also in the name of knowing and doing mathematics, forming the basis of mathematical perception and understanding, using and improving mathematical knowledge [2], [3], [4].

Despite this attributed importance, proof is dealt with intensively in high school and university levels, in the process of mathematics education. In Turkey however, students encounter proof more or less, especially in geometry courses in the second stage of elementary education process, intensively in 9th grade in secondary education and also in further grades. Parallel to proof education condensating in secondary education level, a majority of studies done in this area deal with proof in elementary education level and some studies even advocate that proof is suitable to advanced secondary education level students in school mathematics [5].

As mentioned, proof in mathematics education is important in terms of understanding mathematical knowledge. Therefore, preventing memorizing in mathematics is critically valuable in mathematics education, in terms of realizing meaningful learning by building conceptual knowledge. In conclusion, mathematical proof must be used in elementary education level of education as well, in a way that will be suitable to the level of the student.

In the NCTM (National Council of Teachers of Mathematics) (2000) document, proof is not handled as a special activity that is done in specific times of specific topics of the curriculum. Proof and judgment, no matter on what topic, must be a part of the natural flow of the course teaching process [cited in 6]. This understanding, which argues to do proof more intensively, independent of the subject in the mathematics education, increases the importance of proof in mathematics teaching.

Even in the structuring approach, teaching process is not independent from the teacher. Either in teacher-centered or student-centered education, the knowledge, attitude and believes of the teacher create important effects on education. There is an exact correlation between the belief and attitude of a teacher on a specific topic and this correlation affects the teaching practice of the teacher directly [7], [8], [9]. In other words, the believes of the teacher towards mathematics and mathematics teaching are related to her/his content knowledge and pedagogical content knowledge. Besides, Thompson defines this relation between belief and knowledge as in [9], p.127, as "Teachers treat their believes as knowledge".

A similar situation is the case also for the preservice teachers who are in a process to form their belief and attitudes on mathematics and proof in mathematics during their education lives. This process will determine what kind of a teacher they will become, as well as shape their future teaching practices as a teacher [10]. Because of this reason, while indicating the importance of proof in mathematics education and taking this topic in the curriculum and the education process; views of preservice teachers on proof in mathematics should also be taken into account. The aim of

B. Turker is with Elementary Mathematics Education Division of Education Faculty, Aksaray University, Aksaray, Turkiye.
(e-mail: bturker@hacettepe.edu.tr)
C. Alkas is with Elementary Mathematics Education Division of Education Faculty, Pamukkale University, Denizli, Turkiye.
(e-mail: cigdemalkas@hacettepe.edu.tr)
E. Aylar is with Educational Science Department, Ankara University, Ankara, Turkiye.
(e-mail: ebruaylar@gmail.com).
R. Gürel is with Elementary Mathematics Education Division of Education Faculty, Mehmet Akif University, Burdur, Turkiye.
(e-mail: rgurel@mehmetakif.edu.tr)
O. Akkus Ispir is with elementary Mathematics Education Division of Education Faculty, Hacettepe University, Ankara, Turkiye
(e-mail: oyluma@hacettepe.edu.tr)

this study is to obtain the views of elementary education mathematics preservice teachers on proof in mathematics.

II. Method and Procedure

Study group consists of senior class elementary education mathematics preservice teachers. In order to get the views of this group on proof, Almeida's survey, as in [11] related to the topic has been revised and applied to 104 preservice teachers. According to the total points gathered from this application, depth interviews have been done with nine upper group (the ones that got the highest points from the survey) and seven lower groups (the ones that got the lowest points in the survey) participants. In these interviews 8th, 13th and 14th questions[*] in the survey have been re-asked; in addition to these questions several other questions have been asked to determine the preservice teachers' views regarding the place and importance of proof in mathematics education, types of proofs, the points that they find difficult in proving and their past experiences about proving.

Interviews have been made separately with each preservice teacher and each interview lasted nearly two hours. At first, preservice teacher has been given the survey that he/she has filled out before and his/her views on selected questions have been taken, then additional questions were asked. Each interview has been recorded with a tape recorder; their transcripts have been made and coded by three different researchers. Themes have been found from the common codification and in the light of these themes, findings part has been written.

III. Findings

When views about proof are analyzed, without the lower group-upper group separation, it is seen that all preservice teachers think that mathematics without proof cannot be thought about. Preservice teachers have listed the benefits proof provides to mathematics as follows; proof is effective in giving meaning to mathematics, saves students from memorizing, provides a larger vision to the students, increases permanence, enables meaningful learning, helps formalizing, enables the theory to be understood better. Preservice teachers have pointed out their views that proof has a very important role in mathematics education, by building a relationship between trusting the truth of a theorem and seeing its proof or doing its proof. One of the preservice teacher has made the comment below. (All the names are pseudonyms).

> *" When I see its proof, I look at the theorem more convinced."(Leman)*

The importance given to proof in mathematics education by preservice teachers has come out also in the emphasis they made during the interviews for the role of proving in mathematics education. Preservice teachers have used the following sentences for the place/role of proving in mathematics:

> *"Proof is the authority of acceptance".(Ali)*
> *"Proof is a tool for formalizing". (Asya)*
> *"I think proof is an indispensable element of mathematics". (Omer)*
> *"Proof shows realities in mathematics". (Mustafa)*
> *"Proof is more permanent knowledge". (Ozlem)*
> *"It is the source of mathematics. The place it was born from". (Ozge)*
> *"Mathematics is a science depending on proof". (Nejat)*

Despite these positive attitudes of the preservice teachers, their belief regarding 6-8 elementary education preservice teachers will understand and be able to do proof is low. This situation can be related to preservice teachers being inexperienced with proof in 6-8 elementary education level. Preservice teachers have mentioned that they have not learned proving completely until university level and they have not been informed about the importance of proof, so they could not internalize it in their lives as a student. But still almost all of them comment as "*proof should be used in classes if the students have the adequacy to understand*". A preservice teacher's comment on the topic is as follows:

> *"In their level (primary education) I honestly do not know if they can do proving, I mean can they do proving in that level. If they can, I would encourage them. They learn the formula or the theorem they proved better. I think that formula or theorem would be stable..."(Nehir)*

Moreover, it is seen that preservice teachers' experiences about proving is limited with the classes and that they think proof as a work for courses, exams.

The experiences of the preservice teachers about proving are limited with several courses (analysis, abstract algebra, linear algebra, analytical geometry, and elementary number theory) in their university years. They have also expressed that in these courses they have done proving generally by the guidance of the responsible faculty member (being told which methods to use while doing the proving) and they were not able to do proving on their owns. Preservice teachers have stated that they largely tend to memorize the proofs. Memorizing the proofs of the theorems in order to get high grades from the exams is one of the reasons why preservice teachers could not learn proof methods and they confuse the names of these proof methods. Because of these reasons, serious mistakes have been observed in the answers of the preservice teachers to the question of "how many different proof methods are there and what are these proving types called". Quotations below are examples for this situation.

> *"Proof-by-contradiction, direct, indirect, there was one more, I guess there were 4. Induction, deduction...I can not remember any other but I guess there should be." (Ferhan)*

[*] 8.. "If a result in mathematics is obviously true then there' s no point in proving it."
13.. "Although the theorem usually makes sense to me, I hardly understand the proof."
14.. "I can understand a proof only when the teacher shows it in the class."

"It has three types. Induction, proof-by-contradiction, direct proof but (...) but I could not remember that." (Ali)

Students' knowledge about proof methods's inadequateness have been cited at most of the studies [12]. Student usually can not apply the methods correctly because of this inadequateness. Same as those studies, preservice teachers in our study showed their inadequateness by giving different answers to the question of how many different proving types there are. The answers include four the most, then respectively five, three and two. To the question of what the proof types are, the answers given most by the preservice teachers are induction and proof-by-contradiction methods. These methods are followed by methods like direct proof, sampling in opposition, deduction, indirect proof, and geometrically proving.

The common mistake done by participants is putting deduction, proof-by-contradiction and direct proof methods into the same category and thinking sampling in opposition as a proof method. Besides, it has been observed in the interviews that although they know how to apply the proving methods, they do some mistakes in naming them. Proof-by-contradiction and sampling in opposition are the methods whose names are confused the most.

Preservice teachers have not been sure about their answers in the questions about the names of the methods they use, they have stayed in ambiguity. Quotations below are related to this situation.

"It may be induction but I think not. Maybe it can be called so…?"(Ferit)

"Was it sampling in opposition? I do not recall the names of the methods for disproving clearly actually." (Arzu)

"With one aspect it looks like induction, with another it looks like direct. But I do not remember their differences. Besides, induction was not a method we used frequently." (Arzu)

"Proof-by-contradiction, I think." (Asya)

Among the proving methods, direct proof and induction are the two methods that preservice teachers generally know the proving steps correctly and apply these steps to the questions.

The points that preservice teachers have difficulty in doing proving have been asked in the interviews. Like many studies have shown that students don't know how to begin proof [13], majority of the students stated that they have the most difficulty in the beginning stage, while doing proving. The points that create the most difficulty are designing the proof and choosing the proof type. The students have indicated the points they have difficulty in the beginning stage while doing proving as follows;

"…the beginning stage of proving is very difficult, so I think designing the proof in your head is burdensome. After starting first and writing something the rest comes as well, but that constructing stage is a difficult stage." (Ferit)

"I need to remember the knowledge first; I struggle when I do not remember. I need to remember the advance information. After that, after remembering, the rest comes. So, there is need for a spark." (Arzu)

The spark that is needed in order the preservice teachers to begin proving is formed difficultly because of a series of reasons. Among these reasons are deficiencies in advance learning, knowledge deficits related to proof methods and their prejudices for doing proving. Deficiencies in advance learning cause difficulties for students not only in the beginning part of the proving, but also in the further processes. After the beginning stage, students have stated that they secondly have difficulties in the construction process of the proof, which is in the middle. Additional theorems or mathematical knowledge, definitions that need to be included to the proof in the process of construction can not easily be seen by the students. The lack of understanding and using of mathematical definitions are one of the important difficulties that students face up to and were mentioned on some studies [14]. Preservice teachers, who cannot use the knowledge they gained in the prior stages of the education process while doing proving, have stated the difficulties they experience as follows:

"… There are not clear things. Then, I do not understand where some things come from, usually like in the middle parts." (Zeynep)

"I mean I have difficulty (...) in comparing the data we obtained with the ones gathered before. In some places you need a formula, I can not find them." (Mustafa)

Moreover, while the participants argue that proving should be given convenient to the level of the student in mathematics education; they relate the difficulties they experience to the reason that they struggle with proofs that are above their levels. A student has stated this situation as given below.

"But the proof should be given according to the level of the student. Some teachers are writing many many pages of proofs, that is very detailed and complex. Those proofs are really not understandable. The teacher is also writing it by looking at a paper. He/she is not writing that proof by totally knowing it. I think that those kinds of proofs are not necessary to be given. There is no way to keep it in mind." (Zeynep)

There is an important relationship between the points that preservice teachers have difficulty in proving and memorizing. Preservice teachers have mentioned that they give importance to proof against memorizing knowledge in mathematics, in order to achieve meaningful learning. On the

other hand, they have also stated that they tend to memorize the proof at the point they have difficulty in understanding in the proving process.

"Anyway until now, as we have not done anything with their proof until university years, many of our knowledge relies on memorizing." (Aysun)

The same preservice teacher has mentioned why "memorizing" is preferred in the further stages of the interview.

"As memorizing is easy, analyzing that question, proving it is difficult of course. We prefer to memorize." (Aysun)

Their dilemma as "Memorizing against memorizing" actually displays that proof is done without totally being understood. As a result of this, although they think proving is important, it becomes a mathematical expression that they have difficulty and forget quickly. This affects the preservice teachers' proof performances and attitudes against proof negatively.

"I mean the teacher explained it or we did not understand it, we have difficulty in understanding the proof as there has been a discontinuity at first, the person tends to memorize whatever. Because of that reason the proof of the theorem is forgotten." (Ege)

Preservice teachers, who tend to memorize the proof, argue that proving should be explained in the earliest convenient stage according to the student level, in order to resolve the memorizing problem. Students face proving in the university level intensively. They have indicated their thoughts as follows:

"Proof methods should be taught first. Maybe high level things can not be given in primary education level but what is what can be taught in general terms." (Sebnem)

" I:…Now like this, the main reason why we have difficulty (in proving) is that we did not get full education about proving before, I think the main reason is this.

R: Do you mean high school education?

I: Yes. If we had some education there, we would not have so many difficulties in the university now. As we face doing proof for the first time, and as we tend to memorize at first, it is normal that we cannot do a different proving. As the proofs given will be in the exams, they are being memorized. Because of that we have difficulty in proving." (Demir)

IV. CONCLUSION AND RECOMMENDATIONS

When the findings gathered from the views of preservice teachers on proving, it can be said that although students have positive views about proving, they prefer memorizing proofs instead of understanding them. The fact that they have not been introduced to proving until university level can be given as an example to this. It is seen that very few students have experiences about proving in secondary education years. Starting proving in the mathematics education in earlier grade levels can be recommended in order to make sure that students understand the proof instead of memorizing it. After the students are introduced with proving in the second stage of elementary education, a course including only proofs in the secondary education stage can be added to the program. Besides giving place to theoretical concepts and information about proving in the course content, they may be given opportunities to apply this knowledge.

While some of the preservice teachers have difficulty in selecting the proof type, some of them have stated that they have difficulty in completing the proof although they know the proof method. It may be commented that students who cannot chose the proof method have deficiencies in the theoretical and practical knowledge about proving. Because the preservice teachers have mentioned that they have forgotten, as they have not made much application about proofs. The fact that students cannot complete the proof although they know the proof method can be explained by deficiencies in their basic mathematics knowledge. Moreover, as the mathematics education of the preservice teachers always guides them to routine problems, students cannot use their high level thinking skills enough and develop them. As such, adding activities that might improve the high level thinking skills of the preservice teachers to the mathematics program can be recommended. As it is thought that it will develop preservice teachers' proof related skills and increase their attitudes, handling topics in the classes regarding mathematical proving by relying on cause-effect relationship especially in university level is recommended.

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Belma Türker is working as a research assistant at Department of Mathematics Education at Hacettepe University. Also she is a PHD student at Hacettepe University, Primary Education Program. She graduated from Department of Mathematics in Ankara University, in 2005.

Cigdem Alkas is working as a research assistant at Department of Mathematics Education at Hacettepe University. Also she is a PHD student at Hacettepe University. She is graduated from the Department of Mathematics Education in Hacettepe University, in 2007.

Ebru Aylar went to university at Ankara, at METU (Middle East Technical University). When she graduated from Departments of Mathematics, she started to study on economy and politics of education and mathematics education on graduate programs. Now she is a PHD student at Hacettepe University, Primary Education Program. Also, she is a research assistant at Educational Science Department of Ankara University.

Ramazan Gürel was graduated from the department of secondary school science and mathematics education integrated bs&ms program in teaching mathematics in Boğazici University., in 2007. Now he is working as a research assistant at Department of Mathematics Education at Mehmet Akif University. Also he is a MA student at Hacettepe University.

Oylum Akkus Ispir is currently working as an assistant professor at Hacettepe University. She is graduated from the department of mathematics education. Her research interest areas are elementary mathematics education, creative drama and mathematics, multiple representation usage in algebra.