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The Use of the Limit Cycles of Dynamic Systems for Formation of Program Trajectories of Points Feet of the Anthropomorphous Robot

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***Abstract*—**The movement of points feet of the anthropomorphous robot in space occurs along some stable trajectory of a known form. A large number of modifications to the methods of control of biped robots indicate the fundamental complexity of the problem of stability of the program trajectory and, consequently, the stability of the control for the deviation for this trajectory. Existing gait generators use piecewise interpolation of program trajectories. This leads to jumps in the acceleration at the boundaries of sites. Another interpolation can be realized using differential equations with fractional derivatives. In work, the approach to synthesis of generators of program trajectories is considered. The resulting system of nonlinear differential equations describes a smooth trajectory of movement having rectilinear sites. The method is based on the theory of an asymptotic stability of invariant sets. The stability of such systems in the area of localization of oscillatory processes is investigated. The boundary of the area is a bounded closed surface. In the corresponding subspaces of the oscillatory circuits, the resulting stable limit cycles are curves having rectilinear sites. The solution of the problem is carried out by means of synthesis of a set of the continuous smooth controls with feedback. The necessary geometry of closed trajectories of movement is obtained due to the introduction of high-order nonlinearities in the control of stabilization systems. The offered method was used for the generation of trajectories of movement of point's feet of the anthropomorphous robot. The synthesis of the robot's program movement was carried out by means of the inverse method.

***Keywords*—**Control, limits cycle, robot, stability.

I. INTRODUCTION

THE modern dynamics of the controlled systems is characterized by nonlinearity, multidimensionality, complexity of the description of the purpose of control [1]. The problems of manipulating robots control are actual in the dynamics of mechanisms and machines. These control objects belong to the type of control systems with oscillatory modes. Since the system must have sufficient reserve of stability, the problem of synthesis of given modes connected with the stabilization.

In general, the problem of transferring the system either to a state of stable periodic movements or to an equilibrium position is solved. For improvement of qualitative and quantitative properties of such systems it is important to use methods of control oscillations with the given characteristics.

The significant progress in the direction of development of the theory of synthesis and stability of control systems with given invariant sets is reached in these works [2]-[5].

In this work, approach to the solution of a problem of synthesis of such control systems is offered. Sufficient conditions for the existence of asymptotic stability of the system are obtained. The solution method is based on the construction of Lyapunov's functions and is reduced to finding the set of the continuous smooth controls with feedback which stabilize the movement of the systems studied in the definitional domain of problem [6]. In other words, the parameters of the control functions are necessary for the required behavior of the total derivative of the Lyapunov's function in the layer, attached both outside and inside to a given surface. Stabilization of system in the neighborhood of given surfaces at such approach is based on Zubov's theorems of invariant asymptotically stable surfaces [7]. In this work, geometrically it corresponds to the creation in a phase space of oscillatory circuits of stable limit cycles with sites of near-rectilinear movement [8]. Results of researches are of interest in trajectory problems of robotics [9], [10].

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II. STATEMENT OF THE PROBLEM

It is required to construct the control stabilizing movements of a multi-channel control object in the neighborhood of one of the surfaces $\partial\mathbf{D}_l^{2n}=\left\{\mathbf{X}\in\mathbf{R}^{2n}\middle|c_l-\sum_{i=1}^{2n}\frac{x_i^{2m}}{a_i^{2m}}=0\right\}$, where surfaces $\mathbf{D}_l^{2n}\subset\mathbf{R}^{2n}$, $l=1,2$, under the condition $0<c_1<c_2$, form the layer $\mathbf{D}_1^{2n}\subset\mathbf{D}_2^{2n}$ in the state space of system.

Multiply connected system, whose phase space contains this layer, is considered. The upper and lower boundaries of the layer are invariant manifolds. Transfer of the controlled signals will be carried out according to the $2n$–channel scheme. In particular, in order to destabilize the system are intrasystem feedback controls. Also, we will enter intersystem feedback controls. Thus, the controlled system takes the form

$$\begin{cases}\dot{x}_{2i-1}=\alpha_{2i}x_{2i}^{2m-1}+\mathrm{U}_{2i-1}(x_{2i-1},x_{2i})+\mathrm{V}_{2i-1}(\mathbf{X}),\\ \dot{x}_{2i}=-\alpha_{2i-1}x_{2i-1}^{2m-1}+\mathrm{U}_{2i}(x_{2i-1},x_{2i})+\mathrm{V}_{2i}(\mathbf{X}),\end{cases}$$

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where $\mathbf{X}\in\mathbf{R}^{2n}$ is the state vector of the control object. The structure of the stabilizing controls is defined as:

1. Intrasystem feedback controls:

$$\mathrm{U}_{2i-1}(x_{2i-1},x_{2i})=\beta_{2i,2i}x_{2i}^{4m-1}+\beta_{2i,2i-1}x_{2i-1}^{2m}x_{2i}^{2m-1},$$
$$\mathrm{U}_{2i}(x_{2i-1},x_{2i})=\alpha'_{2i-1}x_{2i}+\beta_{2i-1,2i-1}x_{2i-1}^{4m-1}+\beta_{2i}x_{2i}^{2m+1}+\gamma_{2i}x_{2i}^{4m+1}+$$
$$\beta_{2i-1,2i}x_{2i}^{2m}x_{2i-1}^{2m-1}+\beta_{2i-1}x_{2i-1}^{2m}x_{2i}+\gamma_{2i-1}x_{2i-1}^{4m}x_{2i}+\gamma_{2i-1,2i}x_{2i}^{2m+1}x_{2i-1}^{2m},$$

2. Intersystem feedback controls:

$$\mathrm{V}_{2i-1}(\mathbf{X})=\sum_{\substack{j=1,\\ j\neq 2i-1,\, j\neq 2i}}^{2n}\beta_{2i,j}x_j^{2m}x_{2i}^{2m-1},$$

$$\mathrm{V}_{2i}(\mathbf{X})=\sum_{\substack{j=1,\\ j\neq 2i-1,\, j\neq 2i}}^{2n}(\beta_{2i-1,j}x_j^{2m}x_{2i-1}^{2m-1}+\beta_j x_j^{2m}x_{2i}+\gamma_j x_j^{4m}x_{2i})$$
$$+\sum_{\substack{j=1,\\ k\neq j}}^{2n}\sum_{\substack{k=1,\\ j,k\neq 2i-1,2i}}^{2n}\gamma_{j,k}(x_j x_k)^{2m}x_{2i},$$

where $i=1,2,\dots,n.$

III. Conclusion of the Parameters of Stabilizing Controls

When solving the problem of oscillation stabilization in the vicinity of one of the layer boundaries, the parameters of intrasystem and intersystem feedback controls are found. At such control one of surfaces of $\partial\mathbf{D}_1^{2n}$ or $\partial\mathbf{D}_2^{2n}$ is invariant, asymptotically stable for the controlled system. Following [6], [7], we will write down the invariance equation for boundaries:

$$\sum_{i=1}^{n}\left(\dot{x}_{2i-1}\frac{\partial\,\mathrm{F}(\mathbf{X})}{\partial x_{2i-1}}+\dot{x}_{2i}\frac{\partial\,\mathrm{F}(\mathbf{X})}{\partial x_{2i}}\right)=\sum_{i=1}^{n}P_{2i}x_{2i}^{2m}\left(c_1-\sum_{i=1}^{n}\frac{x_{2i-1}^{2m}}{a_{2i-1}^{2m}}-\sum_{i=1}^{n}\frac{x_{2i}^{2m}}{a_{2i}^{2m}}\right)$$
$$\left(c_2-\sum_{i=1}^{n}\frac{x_{2i-1}^{2m}}{a_{2i-1}^{2m}}-\sum_{i=1}^{n}\frac{x_{2i}^{2m}}{a_{2i}^{2m}}\right),$$

where the desired coefficients are $\alpha_{2i},\alpha_{2i-1},\alpha'_{2i-1},\beta_{2i,j},\beta_{2i-1,j},\beta_j,\gamma_j,\gamma_{j,k}$. We open the brackets, and group the summands with multipliers of $x_{2i}^{2m},x_{2i-1}^{2m-1},x_{2i}^{2m-1},x_j^{2m}x_{2i}^{2m-1},x_j^{2m}x_{2i-1}^{2m-1},x_j^{2m}x_{2i},x_j^{4m}x_{2i},(x_jx_k)^{2m}x_{2i},$ where $i=1,2,\dots,n;\ j,k=1,2,\dots,2n,\ j\neq k.$ Receive it:

$$\sum_{i=1}^{n}\left(x_{2i}^{2m}\left[\frac{2m\alpha'_{2i-1}}{a_{2i}^{2m}}-c_1c_2P_{2i}\right]+x_{2i}^{2m-1}x_{2i-1}^{2m-1}\left[\frac{2m\alpha_{2i}}{a_{2i-1}^{2m}}-\frac{2m\alpha_{2i-1}}{a_{2i}^{2m}}\right]+x_{2i-1}^{2m-1}x_{2i}^{4m-1}\right.$$
$$\left[\frac{2m\beta_{2i,2i}}{a_{2i-1}^{2m}}+\frac{2m\beta_{2i-1,2i}}{a_{2i}^{2m}}\right]+x_{2i}^{2m-1}x_{2i-1}^{4m-1}\left[\frac{2m\beta_{2i,2i-1}}{a_{2i-1}^{2m}}+\frac{2m\beta_{2i-1,2i-1}}{a_{2i}^{2m}}\right]+x_{2i}^{2m-1}x_{2i-1}^{2m-1}$$
$$\sum_{j=1}^{2n}x_j^{2m-1}\left[\frac{2m\beta_{2i,j}}{a_{2i-1}^{2m}}+\frac{2m\beta_{2i-1,j}}{a_{2i}^{2m}}\right]+x_{2i}^{2m}\sum_{j=1}^{2n}x_j^{2m}\left[\frac{2m\beta_j}{a_{2i}^{2m}}\pm\frac{2m(c_1+c_2)}{a_{2i}^{2m}a_j^{2m}}\right]+x_{2i}^{2m}$$
$$\sum_{j=1}^{2n}x_j^{4m}\left[\frac{2m\gamma_j}{a_{2i}^{2m}}\mp\frac{2m}{a_{2i}^{2m}a_j^{4m}}\right]+x_{2i}^{2m}\sum_{j=1}^{2n}\sum_{k=1}^{2n}x_j^{2m}x_k^{2m}\left[\frac{2m\gamma_{j,k}}{a_{2i}^{2m}}\mp\frac{4m}{a_{2i}^{2m}a_j^{2m}a_k^{2m}}\right]=0$$

Assuming the expressions in parentheses to be equal to zero, will receive relations on the control coefficients:

$$\alpha'_{2i-1}=\pm c_1c_2,\ \beta_{2i,j}=-\frac{4m}{a_{2i}^{2m}a_j^{2m}},\beta_{2i-1,j}=\frac{4m}{a_{2i-1}^{2m}a_j^{2m}},\beta_j=\mp\frac{c_1+c_2}{a_j^{2m}},$$
$$\gamma_j=\pm a_j^{-4m},\gamma_{j,k}=\pm 2(a_ja_k)^{-2m}.$$

on condition of $\dfrac{a_{2i-1}}{a_{2i}}=\sqrt[2m]{\dfrac{\alpha_{2i}}{\alpha_{2i-1}}}$, where $i=1,2,\dots,n;\ j,k=1,2,\dots,2n,\ j\neq k.$. The layer boundaries $\partial\mathbf{D}_l^{2n}$, $l=1,2$, are invariant for system trajectories with initial conditions from sets of $\left(\mathbf{D}_1^{2n}\setminus\{0\}\right)\cup\left(\mathbf{D}_2^{2n}\setminus\mathbf{D}_1^{2n}\right)$ and $\mathbf{D}_2^{2n}\setminus\mathbf{D}_1^{2n}=\left\{\mathbf{X}\in\mathbf{R}^{2n}:c_1<\sum_{i=1}^{2n}\frac{x_i^{2m}}{a_i^{2m}}<c_2\right\}$ respectively.

We investigate behavior of the controlled system on the following areas:

$$\mathbf{D}_1^{2n}\setminus\{\mathbf{0}\}=\left\{\mathbf{X}\in\mathbf{R}^{2n}:0<\sum_{i=1}^{2n}\frac{x_i^{2m}}{a_i^{2m}}<c_1\right\},$$
$$\mathbf{D}_2^{2n}\setminus\mathbf{D}_1^{2n}=\left\{\mathbf{X}\in\mathbf{R}^{2n}:c_1<\sum_{i=1}^{2n}\frac{x_i^{2m}}{a_i^{2m}}<c_2\right\},$$
$$\mathbf{R}^{2n}\setminus\mathbf{D}_2^{2n}=\left\{\mathbf{X}\in\mathbf{R}^{2n}:c_2<\sum_{i=1}^{2n}\frac{x_i^{2m}}{a_i^{2m}}\right\}.$$

If coefficients $\alpha'_{2i-1},\gamma_j,\gamma_{j,k}$ are positive and β_j is the negative, then the total derivative $\dfrac{d}{dt}(\mathbf{F}(\mathbf{X}))=\mathrm{P}(\mathbf{X})(c_1-\mathbf{F}(\mathbf{X}))$ will be definitely positive on a set $\left(\mathbf{D}_1^{2n}\setminus\{0\}\right)\cup\left(R^{2n}\setminus\mathbf{D}_2^{2n}\right)$ and the definitely negative on a set $\mathbf{D}_2^{2n}\setminus\mathbf{D}_1^{2n}$ where:

$$\mathrm{P}(\mathbf{X})=2m\sum_{i=1}^{n}\frac{x_{2i}^{2m}}{a_{2i}^{2m}}\left(c_2-\sum_{i=1}^{n}\frac{x_{2i-1}^{2m}}{a_{2i-1}^{2m}}-\sum_{i=1}^{n}\frac{x_{2i}^{2m}}{a_{2i}^{2m}}\right).$$

The system trajectories beginning in a set $\mathbf{D}_1^{2n}\setminus\{0\}$ are attracted to the boundary $\partial\mathbf{D}_1^{2n}$; because of the invariance of the surface $\partial\mathbf{D}_1^{2n}$ further trajectories do not cross it, remaining in area, bounded by the surface $\partial\mathbf{D}_1^{2n}$.

Trajectories are twisted from boundary $\partial\mathbf{D}_2^{2n}$ in $R^{2n}\setminus\mathbf{D}_2^{2n}$ and in inside layer with an attraction to border $\partial\mathbf{D}_1^{2n}$.

Thus, the boundary $\partial\mathbf{D}_1^{2n}$ is asymptotically stable for trajectories with initial conditions defined inside the manifold $\mathbf{D}_1^{2n}\setminus\{0\}$ and in some layer adjoining to the surface from the outside.

If coefficients $\alpha'_{2i-1},\gamma_j,\gamma_{j,k}$ are negative and β_j is positive, the system trajectories beginning in sets $\mathbf{D}_2^{2n}\setminus\mathbf{D}_1^{2n}$ and $R^{2n}\setminus\mathbf{D}_2^{2n}$, are attracted to boundary $\partial\mathbf{D}_2^{2n}$.

Consequently, under the control obtained, there exist regimes in which the outer boundary $\partial\mathbf{D}_2^{2n}$ of the layer has the property of attraction.

IV. Formation of Program Trajectories of Movement

In Figs. 1 and 2, we present the results of numerical model operation on the phase plane of the constructed control system. In subspace of $\mathbf{X}_1\times\mathbf{X}_2\subset\mathbf{R}_1^2$ there is a stable limit cycle with sites of near-rectilinear movement. The assignment of control functions in the form of polynomials of high degrees allowed to receive the required form of the trajectories of the movement of the system [8], [11].

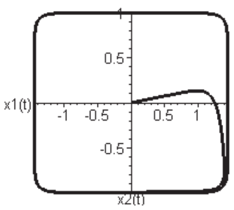

Fig. 1 Stable limit cycle

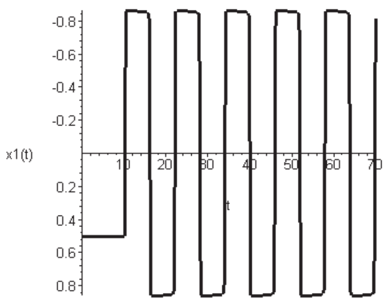

Fig. 2 Vertical movement of points feet

The proposed method of synthesis can be used to solve the trajectory problems of robotics. In Fig. 2, the curve of vertical movement of points feet of the drive of the walking robot is shown [9], [12]. The effort in each drive can be set by means of the proportional-differential regulator:

$$u(t)=k_1\Delta+k_2\dot{\Delta}, \tag{1}$$

where k_1 and k_2 are intensification coefficients of feed-backs on mismatches of the relative movements and speeds of the drive $\Delta,\ \dot{\Delta}$.

The drive mismatch on the relative movement is equal to a difference between the measured $q_c(t)$ and the program $q_p(\mathbf{x}_p)$ value $q_p(\mathbf{x}_p)$

$$\Delta=q_c(\mathbf{x})-q_p(\mathbf{x}_p)$$

The program value of the relative displacement $q_p(\mathbf{x}_p)$ is found from the solution of the equation of movement of a walking robot of the form [13]-[15]:

$$\begin{cases}\mathbf{M}\ddot{\mathbf{x}}-\mathbf{D}^{\mathrm{T}}\mathbf{p}-\mathbf{D}_w^T\mathbf{p}_w=\mathbf{f}(\dot{\mathbf{x}},\mathbf{x},t),\\ \mathbf{D}\ddot{\mathbf{x}}=\mathbf{h}(\dot{\mathbf{x}},\mathbf{x}),\\ \mathbf{D}_w\ddot{\mathbf{x}}=\ddot{\mathbf{w}}(t).\end{cases} \tag{2}$$

here $\mathbf{D}_w$ is the matrix of variable coefficients of equations of connections for points feet, $\ddot{\mathbf{w}}(t)$ is the vector of accelerations of the specified points, $\mathbf{p}_w$ is the vector of Lagrange multipliers, corresponding to connections with the set program trajectories. The solution of the system for non-trivial cases is possible only in a numerical form. For such a numerical solution, special programs for modeling the dynamics of coupled systems of bodies are used.

Use of control in a form (1) does not guarantee stability to system (2) as functions of control depend from significantly - non-linear expressions $q_c(\mathbf{x})$. In addition, the application of the control in the form (1) assumes an empirical piecewise non-linear task of functions describing the trajectories of points feet of the drive of the walking robot which will lead to abrupt changes of forces in the drives.

The vector $\mathbf{w}(t)$ contains components describing the trajectories of the robot's body and points feet of the propellers.

In the program movement of the propeller point the trajectory consists of near-rectilinear sites. The received trajectory of movement is piecewise-analytic, i.e., composed of various functions analytic at each site. In the connection points of sites functions have a derivative discontinuity.

The method offered in this work will provide a continuity of the functions describing a program trajectory with sites of near-rectilinear and the asymptotic stability of movement of a point on the given limit cycle (Fig. 1).

The gait generation unit of the anthropomorphous robot AR600 (Fig. 3) consists of the trajectory generator of the points foot, the center of mass trajectory generator and the additional link generator. The control system of self-oscillatory type constructed in this work was realized as a generator of program trajectories of movement of points feet of the robot and showed its effectiveness.

V. Conclusion

In robotics, often it is required to receive cyclic movements of various forms and to provide the stability of movement as a particular link and the system as a whole. Usually in this case, robastny PID – regulators, which cannot always provide stability of all system, are used. Therefore, development of methods for the synthesis of stable modes of the movement in multidimensional mechanical systems remains relevant.

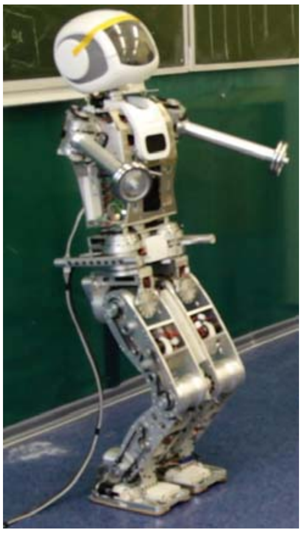

Fig. 3 The anthropomorphous robot AR600

REFERENCES

[1] Andrievskii B. R., Guzenko P. Yu., Fradkov A. L., "Control of Nonlinear Oscillation in Mechanic Systems by the Steepest Gradient Method", Avtomat. i Telemekh., 1996, no. 4, 4–17

[2] Gorobtsov A.S., Grigoryeva O.E., Ryzhov E.N. Attracting ellipsoids and synthesis of oscillatory regimes. Automation and Remote Control, 2009, no. 8, p. 40 – 48.

[3] Gorobtsov A. Lame - manifolds in problems of synthesis of nonlinear oscillatory modes / Gorobtsov A., Ryzhov E., Churzina A. // JVE. Journal of Vibroengineering, Vol. 10, Issue 4, 2008, p. 456–459.

[4] Gorobtsov A. Principals of Multilinked Nonlinear Stabilization and Lame-Manifolds in Dynamic Systems / Gorobtsov A., Ryzhov E., Churzina A. // Rare Attractors and Rare Phenomena in Nonlinear Dynamics: mater. of the Int. Symposium RA08, Riga–Jurmala (Latvia), 8–12 September 2008 / Riga Techn. Univ., Inst. of Mechanics RTU (etc.). – Riga, 2008, p. 29-32.

[5] Gorobtsov A.S., Ryzhov E.N. Analytical synthesis of generators of self-oscillations on two oscillatory links//Automatics and telemechanics. − 2007. − № 6. p.35–44.

[6] Lyapunov A.M. The General problem of movement stability. – M.: Gostekhizdat, 1950 − 472 p.

[7] Zubov V.I. Stability of movement− M: The Higher school, 1973 − 272 p.

[8] Gorobtsov A.S., Ryzhov E. N., Churzina A.S. Detecting of Oscillations Close to Explosive. Biomedical Radioelectronics, 2009, no. 8, p. 32-34.

[9] Gorobtsov A. About formation of the stable modes of the movement of multilink mechanical systems / Gorobtsov A., Ryzhov E., Polyanina A. // Vibroengineering Procedia. Vol. 8: proc. of 22nd International Conference on Vibroengineering (Moscow, Russia, 4-7 October 2016) / Publisher JVE International Ltd. - Kaunas (Lithuania), 2016. - P. 522-526.

[10] Polyanina A.S. Problem of synthesis of the self-oscillatory modes in multivariate dynamic systems. International journal of applied and fundamental research, 2015, no. 12-4, p. 618-621.

[11] Gorobtsov A.S., Ryzhov E.N., Polyanina A.S. Calculation of the parameters of synthesis of oscillations with multi-channel stabilization (Proc. Xth all-Russian scientific conference of Yu. I. Neymark "Nonlinear oscillations of mechanical systems"), Nizhny Novgorod, 2016, p. 268-273.

[12] Gorobtsov A. Theoretically Investigations of the Control Movement of the CLAWAR at Statically Unstable Regimes. Walking and Climbing robots. Vienna, Austria, 2007, p. 95 – 106.

[13] Fumagalli, A., Gaias, G., Masarati, P.: A simple approach to kinematic inversion of redundant mechanisms. In: IDETC/CIE 2007 ASME 2007 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, Las Vegas, NE, USA, 4–7 September 2007

[14] Shabana, A.A.: Dynamics of Multibody Systems, 3rd edn. Cambridge University, New York (2005)

[15] Cristina P. Santos, Nuno Alves, Juan C. Moreno.: Biped Locomotion Control through a Biomimetic CPG-based Controller. Nonlinear Dyn (2010) 62: 27–37.