

The Use of Minor Setups in an EPQ Model with Constrained Production Period Length

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Abstract—Extensive research has been devoted to economic production quantity (EPQ) problem. However, no attention has been paid to problems where production period length is constrained. In this paper, we address the problem of deciding the optimal production quantity and the number of minor setups within each cycle, in which, production period length is constrained but a minor setup is possible for pass the constraint. A mathematical model is developed and Iterated Local Search (ILS) is proposed to solve this problem. Finally, solution procedure illustrated with a numerical example and results are analyzed.

Keywords—EPQ, Inventory control, minor setup, ILS.

I. INTRODUCTION

THE economic production quantity (EPQ) model has been widely used in practice because of its simplicity. However, there are some drawbacks in the assumption of the original EPQ model and many researchers have tried to improve it with different viewpoints, and the assumption of the unconstrained production period length is one of these shortcomings.

The classic EPQ model assumes that production period length is unconstrained. However, in real production environment, this assumption does not accurately reflect the reality. Because, it can often be observed that the production period length is constrained due to some technical services reasons. Hence, the inventory policy determined by the conventional model might be inappropriate.

Khouja [1] reformulated some inventory models which allow for adjustments to the process within a production cycle to restore it to an "in control" state. These adjustments can be performed without interrupting the system or may require system stoppage and can be thought of as minor setups. He used the quality assumptions previously proposed by Porteus [2] and Rosenblatt and Lee [3] and showed that the incorporation of minor setups leads to an increased optimal lot size and improved yield.

In this paper, we reformulate the EPQ model in which production period length is constrained but a minor setup is possible for pass the constraint. A minor setup does not involve performing all activities of a full setup and incurs only a fraction of a full setup cost and time. We use ILS and

present a method to solve the economic production quantity model of minimizing the total annual cost subject to constrained production cycle length and minor setups. The paper is organized as follows: Section 2 describes the problem. Section 3 explains the basics of iterated local search (ILS). In section 4 we explain the steps of ILS algorithm to solve the problem. A numerical example and some results are represented in section 5. Finally, section 6 contains the conclusions.

II. PROBLEM DESCRIPTION

In this section, we derive a mathematical statement for the EPQ model with constrained production period length. The basic EPQ model is that of determining a production quantity of an item, subject to the following conditions related to the production facility and marketplace [4]:

- Demand rate and production rate are continuous, known and constant. Production rate is greater than or equal to demand rate.
- All demand must be met.
- Holding costs are determined by the value of the item.
- A single full setup is accomplished at beginning of each cycle.
- Setup time is assumed to zero.
- Unit production cost of product and setup cost are time and quantity invariant.
- There are no quantity constraints.
- No shortages are allowed.

Most of the assumptions in our mathematical model are the same as those in the conventional EPQ. Besides, we release some assumptions:

- A single full setup is accomplished at beginning of each cycle; also some minor setups are allowed equally spaced within the production cycle.
- The full setup at each cycle is accomplished at segment of previous cycle in which all production machinery are unemployment.
- Minor Setup time is assumed to be known and positive constant.

In classic EPQ model production period length is assumed to be unrestricted. Generally, in reality production machinery have a finite ability for continuous production due to fatigue of operators and equipments, necessity to service operations

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and etc. Also, production run length can influence the quality of produced goods. In this situation, at end of a long production cycle, produced items have lower quality. Therefore, some minor setups within a cycle may be justified to restore the production process to an "in control" state, in which produced goods quality is acceptable.

In order to state the problem mathematically, let

- D Annual demand rate of product
- P Annual production rate of product
- h Annual unit holding cost
- A The fixed cost of a full setup which involves all tasks required for preparing and adjusting the production process
- a The fixed cost of a minor setup which involves only the tasks required to pass the production length constraint
- s The fixed time of a minor setup
- τ The fixed constrained production period length after each full or minor setup
- c Unit production cost
- T Cycle length
- T_p Production period length in a cycle
- T_d Only-demand period length in a cycle
- I_1 Inventory level at beginning of the first minor setup within a cycle
- I_2 Inventory level at end of the first minor setup within a cycle
- I_{max} Maximum inventory level within a cycle
- Q Production quantity (real positive decision variable)
- m The number of minor setups within a cycle (integer non-negative decision variable)
- ATC Annual total cost (objective function)

The behavior of inventory level in classic EPQ model which minor setup is not allowed is illustrated in Fig.1.

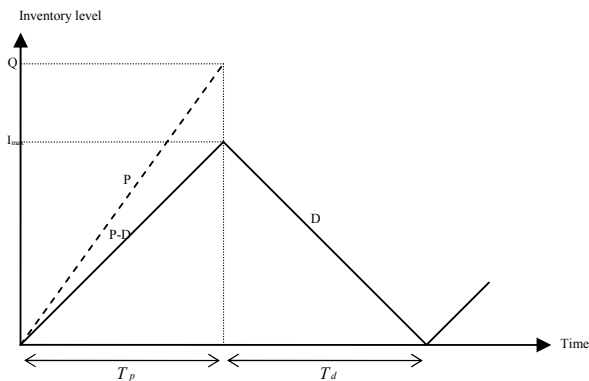


Fig.1. The relation between inventory level and time for one cycle without minor setup

From graphical representation of EPQ model in Fig.1, we have:

$$T_p = \frac{Q}{P} \tag{1}$$

$$T_d = \frac{Q}{D} - \frac{Q}{P} \tag{2}$$

The behavior of inventory level at a cycle with three minor setups is shown in fig. 2.

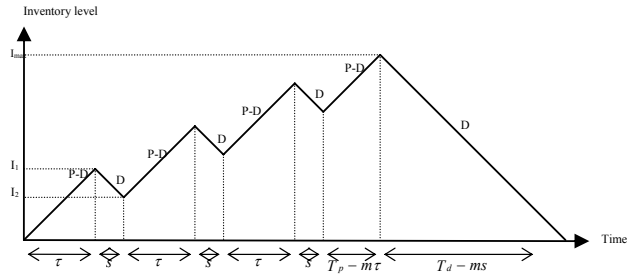


Fig. 2. The relation between inventory level and time for one cycle with three minor setups

From graphical representation of model in Fig.2, we have:

$$I_1 = \tau(P - D) \tag{3}$$

$$I_2 = \tau(P - D) - Ds \tag{4}$$

$$I_{max} = Q(1 - \frac{D}{P}) - Dms \tag{5}$$

$$m = \left\lfloor \frac{T_p}{\tau} \right\rfloor \tag{6}$$

$ATC =$ Annual holding cost + Annual setup cost + Annual production cost

$$ATC = \frac{h}{2} \left[\left((1 - \frac{D}{P}) - \frac{D}{Q} ms \right) (Q(1 - \frac{D}{P}) - msD) + (m\tau(P - D) + Q(1 - \frac{D}{P}) - 2msD) \left(\frac{D}{P} - \frac{D}{Q} m\tau \right) \right] + \frac{hD}{2Q} \left[\frac{m\tau(P - D)(\tau + s) + ms\tau(P - D) - ms^2D}{+ m(m - 1)(\tau + s)(\tau(P - D) - sD) + \frac{2A + 2ma}{h}} \right] + CD \tag{7}$$

Let relax the integrity of m temporarily.

The elements of Hessian matrix are:

$$\frac{\partial^2 ATC}{\partial Q^2} = \frac{h}{Q^3} \left[Dms\tau P(m + 1) + \frac{2D(A + ma)}{h} \right] \tag{8}$$

$$\frac{\partial^2 ATC}{\partial m \partial Q} = \frac{\partial^2 ATC}{\partial Q \partial m} = \frac{-hD}{2Q^2} \left[2ms\tau(P + D) + s\tau P + \frac{2a}{h} \right] \tag{9}$$

and

$$\frac{\partial^2 ATC}{\partial m^2} = \frac{hD}{Q} \left[D(s + \tau)^2 + (s + \tau) - \tau^2 P \right] \tag{10}$$

Evaluation of α_1 and α_2 shows that:

$$\alpha_1 = \frac{d^2 ATC}{dQ^2} = \frac{h}{Q^3} \left[Dms\tau P(m + 1) + \frac{2D(A + ma)}{h} \right] \geq 0 \quad \forall Q, m \tag{11}$$

and

$$\alpha_2 = \det \nabla^2 ATC(Q, m) = \det \begin{bmatrix} \frac{\partial^2 ATC}{\partial Q^2} & \frac{\partial^2 ATC}{\partial Q \partial m} \\ \frac{\partial^2 ATC}{\partial m \partial Q} & \frac{\partial^2 ATC}{\partial m^2} \end{bmatrix} = \quad (12)$$

$$\frac{\partial^2 ATC}{\partial Q^2} * \frac{\partial^2 ATC}{\partial m^2} - \frac{\partial^2 ATC}{\partial Q \partial m} * \frac{\partial^2 ATC}{\partial m \partial Q} = \frac{h}{Q^3} \left[Dms \tau P(m+1) + \frac{2D(A+ma)}{h} \right] * \frac{hD}{Q} \left[D(s+\tau)^2 + (s+\tau) - \tau^2 P \right] - \frac{h^2 D^2}{4Q^4} \left[2ms \tau (P+D) + s \tau P + \frac{2a}{h} \right]^2 \quad \text{free in sign}$$

Then *ATC* is a non-convex function. Therefore, taking the partial derivatives of *ATC* with respect to the *Q* and *m*, and solving equations $\frac{\partial}{\partial Q} ATC = 0$ and $\frac{\partial}{\partial m} ATC = 0$ don't

guarantee the necessary conditions for *Q* and *m* to be optimal.

For a given number of minor setups *m*, equation (7) gives an expression of *ATC* as a function of only *Q* which called *reduced-ATC*. Although *ATC* is a non-convex function, but it is obvious from expression (11) that reduced *ATC* is a convex function. Therefore, taking the first derivative of reduced *ATC* with respect to *Q* and setting $\frac{d}{dQ} \text{reduced } ATC = 0$ yields

the only positive solution at:

$$Q^* = \sqrt{\frac{Dms \tau P(m+1) + \frac{2D(A+ma)}{h}}{(1 - \frac{D}{P})}} \quad (13)$$

This problem poses a difficult computational task due to the integrity of *m* and non-convexity involved. Due the complexity to solve this optimization problem, we developed a method based on iterated local search (ILS) metaheuristic.

III. ITERATED LOCAL SEARCH

Iterated local search is a simple but powerful meta-heuristic algorithm [5]. It applies local search to an initial solution until it finds a local optimum; then it perturbs the solution and it restarts local search. The importance of the perturbation is obvious: too small a perturbation might not enable the system to escape from the basin of attraction of the local optimum just found. On the other side, too strong a perturbation would make the algorithm similar to a random restart local search.

A local search is effective if it is able to find good local optima, that is, if it can find the basin of attraction of those states. When the search space is wide and/or when the basin of attraction of good local optima is small, a simple multi-start algorithm is almost useless. An effective search could be designed as a trajectory only in the set of local optima s^* , instead of in the set *s* of all the states. Unfortunately, in most cases there is no feasible way of introducing a neighborhood structure for s^* . Therefore, a trajectory along local optima $s_1^*, s_2^*, \dots, s_i^*$ is performed, by applying the following scheme:

1) Execute local search from an initial state *s* until a local

optimum s^* is found.

2) Perturb s^* and obtain s' .

3) Execute local search from s' until a local optimum s'^* is reached.

4) On the basis of an *acceptance criterion* decide whether to set $s^* \leftarrow s'^*$.

5) Go to step 2.

The requirement on the perturbation of *s* is to produce a starting point for local search such that a local optimum different from *s* is reached. However, this new local optimum should be closer to *s* than a local optimum produced by a random restart. The *acceptance criterion* acts as a counter balance, as it filters and gives feedback to the perturbation action, depending on the characteristics of the new local optimum. A high level description of ILS steps is presented in Table I.

TABLE I
ITERATED LOCAL SEARCH (ILS) ALGORITHM

s_0	\leftarrow	Generate Initial Solution ()
s^*	\leftarrow	Apply Local Search(s_0)
		While termination condition not met do
s'	\leftarrow	Apply perturbation (s^*)
s'^*	\leftarrow	Apply Local Search (s')
s^*	\leftarrow	Apply acceptance criterion (s^*, s'^*)
		Memorize Best Found Solution
		End While

The design of ILS algorithms has several degrees of freedom in the choice of the initial solution, perturbation and acceptance criteria.

IV. APPLYING ILS ALGORITHM TO THE PROBLEM

To obtain the number of minor setups within each cycle and the economic production quantity of the above mentioned model we are to minimize *ATC*, using iterated local search. In this regard, the steps of this algorithm are briefly presented below where the following notation is used:

(m_0, Q_0)	Initial solution
(m, Q)	Current solution
(m^*, Q^*)	Local optimum solution
(m^*, Q^*)	Best solution
<i>ATC</i> (<i>m</i> , <i>Q</i>)	Value of the objective function at solution (<i>m</i> , <i>Q</i>)
ϵ	Maximum avoidable deviation
λ	Perturbation step-length parameter
<i>k</i>	Repetition counter
<i>L</i>	Number of repetitions allowed

Initialize the ILS control parameter (λ, ϵ, L)

select an initial solution (m_0, Q_0) by setting $m_0 = \left\lfloor \frac{A}{2a} \right\rfloor$ and

$$Q_0 = \sqrt{\frac{Dm_0s\tau P(m_0+1) + \frac{2D(A+m_0a)}{h}}{(1-\frac{D}{P})}}$$

set $m=m_0$ and $Q=Q_0$;

while $\left|\frac{\partial ATC}{\partial m}\right| > \varepsilon$ do

if $\left|\frac{\partial ATC}{\partial m}\right| < 0$

$m = m + 1$;

else

if $\left|\frac{\partial ATC}{\partial m}\right| > 0$

$m = m - 1$;

end

$$Q = \sqrt{\frac{Dms\tau P(m+1) + \frac{2D(A+ma)}{h}}{(1-\frac{D}{P})}}$$

end

set $m^* = m$;

set $m^* = m^*$;

set $Q^* = Q$;

set $Q^* = Q^*$;

set $k = 1$;

while $k < L$ do:

generate a binary random number $z \in \{0,1\}$

$$m = z \left[m^* + \lambda \left(\left\lfloor \frac{A}{a} \right\rfloor - m^* \right) \right] + (1-z) \left[(1-\lambda) m^* \right]$$

$$Q = \sqrt{\frac{Dms\tau P(m+1) + \frac{2D(A+ma)}{h}}{(1-\frac{D}{P})}}$$

while $\left|\frac{\partial ATC}{\partial m}\right| > \varepsilon$ do

if $\left|\frac{\partial ATC}{\partial m}\right| < 0$

$m = m + 1$;

else

if $\left|\frac{\partial ATC}{\partial m}\right| > 0$

$m = m - 1$;

end

$$Q = \sqrt{\frac{Dms\tau P(m+1) + \frac{2D(A+ma)}{h}}{(1-\frac{D}{P})}}$$

end

set $m^* = m$;

set $Q^* = Q$;

if $ATC(m^*, Q^*) \leq ATC(m^*, Q^*)$

set $m^* = m^*$;

set $Q^* = Q^*$;

end

$k = k + 1$;

reduce the Perturbation step-length parameter λ ;

end

The algorithm starts by initializing the so-called perturbation step-length parameter λ , the maximum acceptable deviation ε and number of repetitions allowed L . also; algorithm sets the initial solution (m_0, Q_0). It is logical that, minor setups are economic if the cost of a minor setup, a , be considerably less than the cost of a full setup, A . Else, it will be beneficial to restart the production system after elapsing each constrained period length, τ . At upper extreme case, if we suppose that annual unit holding cost, h , be almost zero, then maximum $\left\lfloor \frac{A}{a} \right\rfloor$ minor setup(s) can be economic.

Also, at lower extreme case, the number of minor setups cannot be negative. Therefore, we have $m^* \in \left[0, \left\lfloor \frac{A}{a} \right\rfloor \right]$. In

proposed model, the initial number of minor setups within each cycle, m_0 , is assumed to be rounded mean of the mentioned interval, namely, $m_0 = \left\lfloor \frac{A}{2a} \right\rfloor$. Initial production

quantity, Q_0 , is obtained with replacing m_0 in Eq. (13). To generate neighborhood solutions we use mathematical base of Eq. (7) as bellow: If the first derivative of ATC with respect to m at solution (m, Q) is negative, a neighborhood solution is generated by increasing of m at current solution by an amount of 1, and if the first derivative of ATC with respect to m at solution (m, Q) is positive, a neighborhood solution is generated by decreasing of m at current solution by an amount of 1, and new Q is obtained with replacing new m in Eq. (13). Then the generated solution replaces the current one. In proposed model, the values of m must be integer, so we select neighborhood step-length equal to 1. This procedure continues until a local optimum solution (m^*, Q^*) is reached, namely, the first derivative of Eq. (7) with respect to m at solution (m, Q) almost equals to zero. This first local optimum solution sets as best solution (m^*, Q^*). In the inner cycle of ILS, repeated while $k < L$, a perturbed solution of the current local optimum solution (m^*, Q^*) is generated as follows: with generating a binary random number $z \in \{0,1\}$, we select a direction for perturbation (increase or decrease). Perturbed solution is obtained by adding to or subtracting from current local optimum number of minor setups m^* , a dynamic amount, depending on the perturbation direction and λ , where $\lambda \in [0,1]$ is the perturbation step-length parameter playing an important role in our algorithm. Perturbed Q is obtained with replacing perturbed m in Eq. (13). The generated solution replaces the current one. Local search procedure is applied to the newly chosen solution. We suppose that after the local optimum is reached, it is always acceptable. Hereby, the most important feature of this algorithm, as a metaheuristic, is the possibility of accepting a worst solution, which can allow it to prevent falling into local optimum trap.

The choice of an appropriate λ is crucial for the performance of the algorithm. The value of parameter λ decreases during the search process, thus at the beginning of the search, diversification is high and as it gradually goes on its search path, intensification becomes intense. The terms diversification generally refers to the exploration of the search space, whereas the term intensification refers to the exploitation of the accumulated search experience. Hereby, with choice of an appropriate λ , a dynamic balance is given between diversification and intensification.

V. NUMERICAL EXAMPLE AND DISCUSSION

To illustrate the usefulness of the model developed in section 4 let us consider the inventory situation where a stock is replenished with Q units where production period length is constrained to 0.01 year. The parameters needed for analyzing the above inventory situation are given below:

Demand rate, $D = 12000$ units/year,

Production rate, $P = 20000$ units/year

Holding cost, $h = 30$ \$/unit/year

Full setup cost, $A = 500$ \$/cycle

Minor setup cost, $a = 100$ \$/minor setup

Minor setup time, $s = 0.005$ year/minor setup

Restricted production period length, $t = 0.01$ year

Unit production cost, $c = 25$ \$/unit

Using the above mentioned ILS procedure gives $Q^* = 2014.94$, $m^* = 9$ and $ATC^* = 307979.33$ whereas without permission for minor setups we have $Q^* = 1000$ and $ATC^* = 312000$. It is clear that using 9 minor setups within each cycle leads to larger lot sizes with lower ATC . In general, this reality is obvious from expression (13) that if at least one minor setup within each cycle has justified, optimal lot size will be increased. Also, the expression (13) reveals that optimal lot size has inverse relation with Minor setup cost, Minor setup time and restricted production period length.

VI. CONCLUSIONS

In this paper, we reformulated the economic production quantity (EPQ) model with constrained production period length which a minor setup is allowed for pass the constraint. A minor setup does not involve performing all activities of a full setup and incurs only a fraction of a full setup cost and time. The problem described with a mathematical model, and then the Iterated Local Search (ILS) algorithm proposed to solve it. The results showed that using minor setups within each cycle leads to larger lot sizes.

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