

The Role Played by Swift Change of the Stability Characteristic of Mean Flow in Bypass Transition

Dong Ming, Su Caihong

Abstract—The scenario of bypass transition is generally described as follows: the low-frequency disturbances in the free-stream may generate long stream-wise streaks in the boundary layer, which later may trigger secondary instability, leading to rapid increase of high-frequency disturbances. Then possibly turbulent spots emerge, and through their merging, lead to fully developed turbulence. This description, however, is insufficient in the sense that it does not provide the inherent mechanism of transition that during the transition, a large number of waves with different frequencies and wave numbers appear almost simultaneously, producing sufficiently large Reynolds stress, so the mean flow profile can change rapidly from laminar to turbulent. In this paper, such a mechanism will be figured out from analyzing DNS data of transition.

Keywords—boundary layer, breakdown, bypass transition, stability, streak.

I. INTRODUCTION

THE location of laminar-turbulent transition of a boundary layer depends heavily on the free stream turbulence (FST). At very low FST, e.g. less than 0.1%, natural transition is observed, in which it takes a long distance for the small amplitude T-S waves to evolve until the final breakdown; while at high FST levels, bypass transition is observed, in which one does not see the slow process of the evolution of T-S waves, and transition can take place even the Reynolds number is well below the critical Reynolds number.

In 1970s, under moderate FST, certain elongated stream-wise streaks in the boundary layers were observed in experiments, which later are named as Klebanoff mode[1]. Based on certain earlier theoretical works[2], [3], the so-called transient growth mechanism is proposed to explain the evolution of the streaks. Further, Luchini[4] derived the so-called optimal disturbances mathematically, which may have the maximum linear transient growth. After that, many experimental and numerical results showed that the streaks have similar forms of the optimal disturbance[5], [6]. The secondary stability of the streaks was studied experimentally and numerically[7]-[9], and sinuous mode was found to be more unstable than varicose mode. The whole bypass transition, from the introduction of the FST to the transition, including the emerging and merging of turbulence spots, were studied both by experiment[10] and DNS[11].

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So far, the whole scenario of bypass transition is described as consists of the generation of streaks by FST, the evolution of streaks, the secondary instability and the growing and merging of turbulent spots. However, there is no explanation in regard with how secondary instability leads to turbulent spot, and how turbulent spots merge, etc. In a word, there is no clear description about the inherent mechanism of breakdown. The aim of the present paper is to build this missing link in the process of bypass transition. Some previous studies for natural transition showed that the inherent mechanism of the breakdown in laminar-turbulent transitions was the quick change of the stability characteristics of the mean flow profile, induced by the existing disturbances (T-S waves)[12]. The change manifests itself in that the unstable zone in the frequency-wave number space enlarges rapidly, with the maximum amplification rate of the unstable T-S waves also increases significantly, so that many waves, which are stable under original laminar profile, become unstable, leading to the quick emerging of many more unstable waves, producing much bigger Reynolds stress, which in turn further modifies the mean flow profile. The whole process bears a nature of positive feedback, resulting in the catastrophic breakdown of the laminar flow. However, in bypass transition, there is no process of slow evolution of T-S waves, and there should be some different mechanism in comparison with natural transition. Nevertheless, the mean flow profile changes rapidly during the breakdown process both in natural transition and bypass transition, so our first goal is to verify if the stability characteristics of the mean flow profile has already been changed due to some means in the early stage of bypass transition. And we also intend to describe the detailed nature of breakdown in bypass transition. In this paper, we have four steps to achieve our goals, each step has some refinements in comparison with the previous one. The main tools of our investigation are direct numerical simulation of the transition and linear stability analysis.

II. 1ST STEP: BREAKDOWN TRIGGERED BY MEAN FLOW MODIFICATION + HIGH-FREQUENCY DISTURBANCES

A. Numerical Method in 1st Step

The simulation starts from somewhere that stream-wise streaks have already triggered by disturbances outside the boundary layer, and ends up until turbulent flow appears in the computational domain. The detailed method can be found in Reference[13], except for the following items:

1. Basic Flow and Inflow Condition

In general, the inflow condition of DNS should include a basic flow and some high-frequency disturbances. To verify our idea, the basic flow should include the effect of the already existing longitudinal streaks. Hence, a Blasius profile plus a certain mean flow modification, induced by the streaks are selected as the basic flow. The mean flow modification can be obtained from Ricco et al.'s results [14], namely, the mean flow modification induced by the longitudinal streaks, which are triggered by FST, just before the breakdown of laminar flow. Interesting enough that the mean flow profile modifications at different stages given by them were not only qualitatively, but also quantitatively close to what were observed in experiment[10]. Especially, before the breakdown, the mean flow profile exhibited inflection point somewhere. So our first step was to study the stability characteristic of the mean flow profile, to see how its unstable zone evolved. At first, we were not able to find unstable zone for the least stable T-S waves. Later, we found that there did exist unstable zone, but not for the least stable branch of T-S waves, but for another branch of T-S waves, which had appreciably larger phase speed (about 0.9). In our study, the location, where the Reynolds number Re_{δ^*} based on the displacement thickness of boundary layer δ^* is 634.6, is selected as the inlet of the computational domain, and the mean flow profile and the mean flow modification here, computed by Ricco et al, are shown in Fig. 1. As can be seen, there is an obvious inflection in the mean flow profile, implying a certain instability. And based on this basic flow, a group of T-S waves, including one unstable wave, whose eigenvalues are shown in Table I, were introduced at the entrance of the computational domain as the initial disturbances beyond the streaks.

TABLE I

EIGENVALUE OF THE HIGH-FREQUENCY DISTURBANCES INTRODUCED

No.	Amplitude	Stream-wise wave number	Stream-wise growth rate	Span-wise wave number	Frequency
I	0.05	0.0569	-0.00618	0.82	0.05
II	0.05	0.2695	-0.00276	0.82	0.25
III	0.05	0.3335	0.00578	0.00	0.30

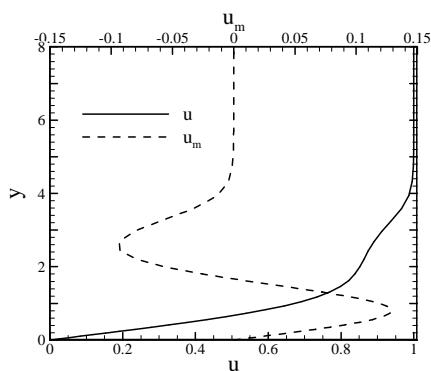


Fig. 1 Mean flow and modification at $Re_{\delta^*} = 634.6$

2. Body Force Term

When computing, we found the mean flow modification introduced at the basic flow cannot maintain itself, it decays rapidly as going downstream, while in the natural process, it is maintained by the external disturbances outside the boundary layer. To maintain the mean flow modification in our computation, a certain body force term is added to the governing equation.

3. Computational Domain and Grids

The computational domain is $850\delta^* \times 40\delta^* \times 7.66\delta^*$, and the number of grids is $1701 \times 56 \times 128$. Uniform meshes are used in both the stream-wise direction and the span-wise direction, non-uniform meshes are used in normal direction, the width of the mesh next to the wall surface is $0.03\delta^*$.

B. Numerical Results in 1st Step

The solid line in Fig. 2 shows the stream-wise distribution of wall friction coefficient (Cf curve). As we can see, although the Cf curve rises at about $x=200$, implying the breakdown process occurs, but its rising is not quick enough, resulting in a too long extent of the breakdown zone compared to the real process. Hence, some other factors must be missing.

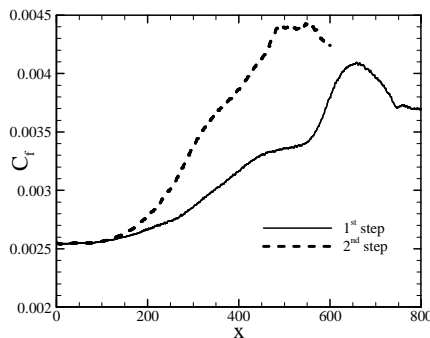


Fig. 2 C_f curves of 1st and 2nd step

III. 2ND STEP: BREAKDOWN TRIGGERED BY MEAN FLOW MODIFICATION + STREAKS + HIGH-FREQUENCY DISTURBANCES

A. Numerical Method in 2nd Step

Considering that the secondary instability considered by Ricco et al. was based on a basic flow including not only the mean modification, but also the flow representing longitudinal streaks themselves, which are ignored in the 1st step. So we refine our DNS as follows:

Theoretically, the Fourier mode of the profiles computed by Ricco et al. included waves $(0,0)$, $(0,n\beta)$, $(n\omega,0)$ etc. and the numbers in the parenthesis imply their frequency and span-wise wave number. It was claimed that components $(0,0)$, namely the mean flow modifications, and disturbances $(0,n\beta)$, namely longitudinal streaks, played the main role. In the 1st step, wave component $(0,0)$ has already been considered, which, together with the high-frequency disturbances, does not trigger transition as quick as we thought. Hence, we now add waves $(0, n\beta)$ as the basic flow, which, together with the high-frequency disturbances as in the 1st step, are introduced at the inlet of the computational domain. Body forces to maintain the waves $(0,$

$n\beta$) are also added into the equation. In this case, the computational domain is modified to be $600\delta^* \times 40\delta^* \times 7.66\delta^*$, which is shorter in stream-wise direction than previous case, and the number of grids is modified to be $1201 \times 56 \times 128$ correspondingly.

B. Numerical Results in 2nd Step

The dashed line in Fig. 2 shows the C_f curve in the 2nd step. Comparing with the solid line, the transition zone is appreciably shorter than those in the previous case, but the starting point of the breakdown stage has no appreciable change, which may imply that the mean flow modification plays the main role in determining the starting location of the breakdown, while the longitudinal streaks provide a certain catalytic effect in transition.

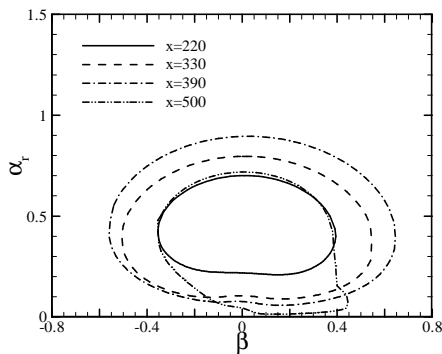


Fig. 3 Neutral curves in 2nd step

The Neutral curves at different locations in the 2nd step are shown in Fig. 3. It can be found that when breakdown are triggered, from about $x=220$, the unstable zones encircled by the neutral curves are enlarged, while at the late stage of breakdown, the unstable zones get smaller, and this scenario is qualitatively similar to those for the natural transition. The main difference is in the present case, the unstable zones do not enlarge so much as compared with those for natural transition. Also as we can see from Fig.4 that all the mean profiles at locations $x=0, 100, 200, 300, 400, 500, 600$, including those for the turbulent stage, exhibit inflection point, which is not in accord with the fact. Obviously, it is due to the body force we imposed. Hence, further refinement is necessary.

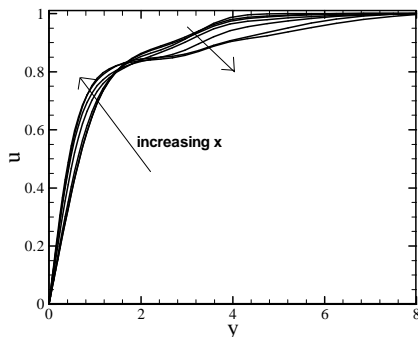


Fig. 4 Mean profiles in 2nd step

IV. 3RD STEP: BODY FORCE IMPOSED AT LAMINAR STAGE ONLY

A. Numerical Method in 3rd Step

Since the streaks can only be observed before the turbulent stage in experiments, the body force should not be imposed throughout the whole computational domain. Hence, for the refinement, we test two cases with different zones of body force. Case 1: the body force is imposed only from $x=0$ to $x=250$, Case 2: the body force is imposed only from $x=0$ to $x=200$. But in both cases, we allow the body force to diminish not abruptly, but gradually, so there is a buffer zone of 50δ for damping the body force to zero.

B. Numerical Results in 3rd Step

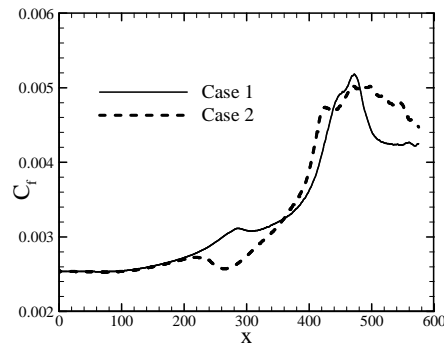


Fig. 5 C_f curves of Case 1 and Case 2 in 3rd step

The C_f curves of the two cases are shown in Fig. 5. Both curves show a depression of C_f where the body force vanishes, however, it increases again rapidly after a short distance, implying breakdown is triggered after the streaks vanishing. In comparing the two cases, we can see that the breakdown stage ends a little earlier in Case 2, but the difference is not essential.

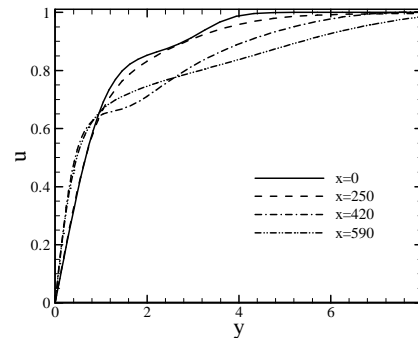


Fig. 6 Mean profiles of Case 2 in 3rd step

The mean profiles in Case 2 of the 3rd step, shown in Fig. 6, are significant different from those in 1st and 2nd step in locations where the body force no longer exist. At first, its inflection disappeared, for example, at $x=250$. However, in the later stage, when breakdown takes place, for example at $x=420$, a new inflection point appears, but its normal location is much lower than those for $x=0$, bringing about another group of unstable T-S waves. And at the full-developed turbulent stage, see $x=590$, no inflection point exists, and the mean flow profile

does have a log-law region as in the usual turbulent profile, but is not shown here to save space. At the streak-existing stage, the mean stream-wise velocity at the height of the inflection point of the mean flow profile is approximately 0.9, leading to the phase speed of the unstable waves to be also approximately 0.9, while at breakdown stage, the mean stream-wise velocity at the height of the inflection point is approximately 0.7, leading to another branch of unstable waves with a phase speed approximately equal to 0.7. Hence, the stability characteristic of the mean flow has also changed. The neutral curves of the first branch of unstable waves, at the location before the streaks vanish, with the phase speed of approximately 0.9, are shown in Fig. 7-(a), as can be found, the unstable zone is becoming smaller as traveling downstream, and finally vanish at the place of about $x=230$. It is different from the stability characteristic of the 2nd step shown in Fig. 3. However, after the location of about $x=400$, another branch of unstable wave emerges, as shown in Fig. 7-(b), the unstable zone enlarges rapidly in the downstream direction in-between $x=400$ and $x=440$ (the growth rates of the most unstable wave also increases rapidly, but not shown here), which is quite similar to the natural transition cases. After a short distance downstream, the mean flow profile becomes turbulent, and the unstable zone disappear correspondingly.

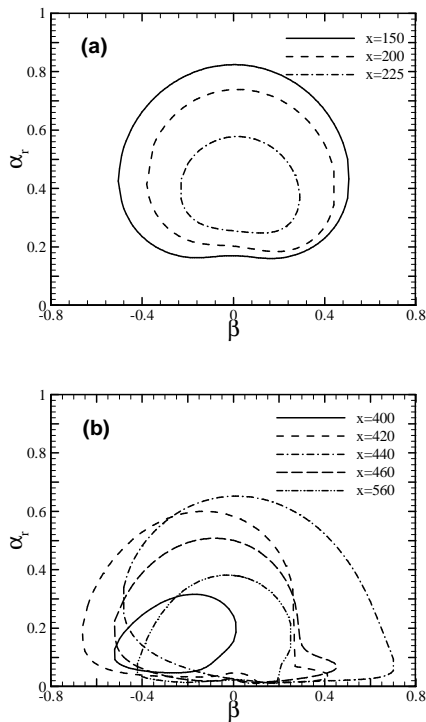


Fig. 7 Neutral curves in case 2 in 3rd step, (a) at $x=150,220,225$, (b) at $x=400,420,440,460,560$

V.4TH STEP: INTRODUCING DISTURBANCES WITH LOWER SPAN-WISE WAVE NUMBER BY BLOW AND SUCTION

A. Numerical Method in 4th Step

For the 3rd step, one should notice that the span-wise wave

numbers of the 3-D disturbances introduced is not located in the unstable zones of the neutral curves in Fig. 7. It is because the span-wise extent of our computational domain is too narrow, including only two stream-wise streaks, so now we enlarge the span-wise extent of our computational domain to 4 times of its original value. So we can introduce initial disturbances with smaller span-wise wave number. The added disturbances are introduced through blow and suction at the wall at the location from $x=95$ to 100, in the form as:

$$v(x, z, t) = 0.05 f_v(x) \sin(\omega t) \sin(\beta z) \quad (1)$$

Where $f_v(x) = 1 - 0.16(x - 97.5)^2$, $\omega = 0.05$, $\beta = 0.205$

. This blow and suction would generate T-S waves at about $200\delta^*$ downstream with amplitude approximately equal to 0.005.

B. Numerical Results in 4th Step

With these added disturbances, by DNS, it is found that the mean flow profile changes much quicker than those in the previous case, and at the location of $x=200$, the unstable zone of the neutral curves, shown in Fig. 8 is already similar as those of previous case at $x=440$, implying the breakdown would occur more upstream. Moreover, much more disturbances with different frequency and span-wise wave number are amplified as traveling downstream, for instance (0.1,0.21), whose stream-wise component's normal distribution at different locations are shown in Fig. 9, showing that the disturbance with frequency and span-wise wave number within the unstable zone is indeed amplifies rapidly during the breakdown stage.

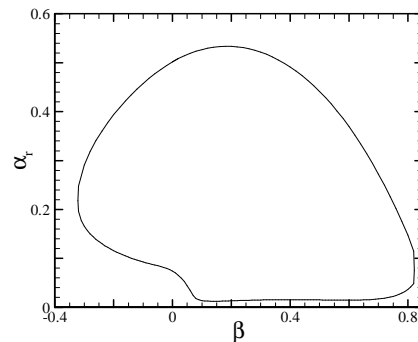


Fig. 8 Neutral curve of $x=200$ in 4th step

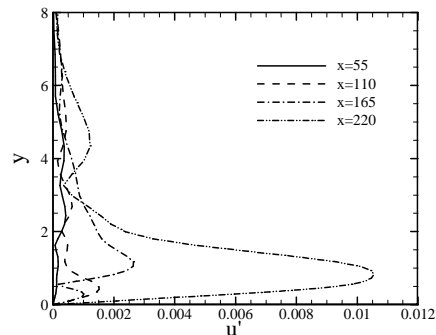


Fig. 9 The evolution of disturbance (0.1,0.21)

VI. CONCLUSIONS

In this paper, bypass transition is simulated via introducing certain mean flow modification and a group of unsteady disturbances, the results indicated that:

1) The presence of a proper mean flow modification is crucial for bypass transition, which makes the mean profile become unstable enough to trigger breakdown, while the disturbances ($0, n\beta$), namely the longitudinal streaks, do have a certain catalytic effects, making the breakdown stage to have catastrophic nature.

2) If the body force, maintaining the streaks in boundary layers, is imposed only at laminar stage, the breakdown process is more complex, which experiences a transition of the unstable mode. At the streak-existing stage, the phase speed of the unstable waves is about 0.9, corresponding to the mean velocity at the height of inflection point, while at the streak-absence stage, this branch of unstable waves would disappear, while another branch of unstable waves with a phase speed of about 0.7 is excited and amplified rapidly. As a consequence, the stability of the mean flow is changing rapidly throughout the transition process.

3) During the breakdown stage, a large number of waves with different frequencies and wave numbers are excited almost simultaneously, leading to the final breakdown of the laminar profile, just like what happens in the natural transition. Hence, the inherent mechanism is essentially the same for both natural and bypass transitions, the difference is what is the initial cause for the change of the mean flow profile.

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