

# The Performance Analysis of CSS-based Communication Systems in the Jamming Environment

Youngpo Lee, Sanghun Kim, Youngyoon Lee, and Seokho Yoon

**Abstract**—Due to its capability to resist jamming signals, chirp spread spectrum (CSS) technique has attracted much attention in the area of wireless communications. However, there has been little rigorous analysis for the performance of the CSS communication system in jamming environments. In this paper, we present analytic results on the performance of a CSS system by deriving symbol error rate (SER) expressions for a CSS  $M$ -ary phase shift keying (MPSK) system in the presence of broadband and tone jamming signals, respectively. The numerical results show that the empirical SER closely agrees with the analytic result.

**Keywords**—CSS, DM, jamming, broadband jamming, tone jamming.

## I. INTRODUCTION

**D**UE to its capability to resist jamming signals, recently, the chirp spread spectrum (CSS) technique has attracted much attention in the field of wireless communications [1] and has been adopted as a physical layer implementation of IEEE 802.15.4a and 802.15.4c, which are standards for low-rate wireless personal area network (WPAN) and WPAN within China, respectively [2], [3]. The CSS technique can be classified into two categories, depending on how to use chirp signals in the modulation process: binary orthogonal keying (BOK) and direct modulation (DM) [4]. The BOK scheme uses chirp signals for representing data: for example, bits '1' and '0' can be represented by chirp signals with positive and negative instantaneous frequency change rates, respectively. On the other hand, the DM scheme uses a chirp signal just as a spreading code and performs data modulation and demodulation separately and independently from the chirp processing. Thus, the DM scheme can be incorporated with various modulation techniques.

In this paper, we deal with the DM scheme with  $M$ -ary phase shift keying (DM-MPSK), which is a possible implementation in IEEE 802.15.4c [3]: specifically, the performance of DM-MPSK are analyzed in terms of symbol error rate (SER) in the presence of broadband and tone jamming signals, respectively.

The remainder of this paper is organized as follows. Section II describes the DM-MPSK system model. In Section III, jamming signal models are introduced and SER expressions for DM-MPSK are derived with the jamming signal models.

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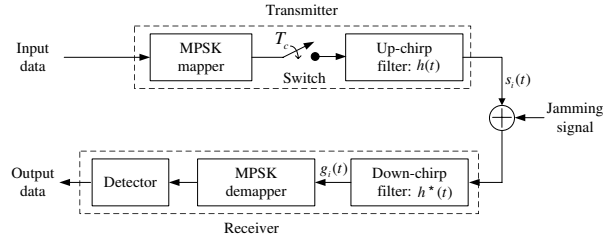


Fig. 1. A typical structure of the DM-MPSK system.

Section IV presents numerical results. Finally, Section V concludes this paper.

## II. SYSTEM MODEL

CSS is a spread spectrum technique that achieves high processing gain by compressing a chirp signal into a much narrower pulse-shaped signal. The lowpass equivalent  $c(t)$  of a chirp signal can be expressed as

$$c(t) = \sqrt{\frac{1}{T_c}} \exp(j\pi\mu t^2), \quad |t| < \frac{T_c}{2}, \quad (1)$$

where  $T_c$  and  $\mu$  ( $\neq 0$ ) denote the duration and instantaneous frequency change rate (chirp rate) of a chirp signal, respectively. When  $\mu > 0$  ( $\mu < 0$ ), the chirp signal is called up-chirp (down-chirp) signal, where instantaneous frequency increases (decreases).

Fig. 1 shows a typical structure of the DM-MPSK system. Input data is first mapped into MPSK constellation, and then, is modulated into a short intermediate frequency (IF) pulse every  $T_c$  seconds. By passing the phase-modulated IF pulse through an up-chirp filter with impulse response  $h(t) = \sqrt{\frac{1}{T_c}} \exp(j\pi\mu t^2)$ , where  $\mu > 0$  and  $|t| < \frac{T_c}{2}$ , subsequently, we can obtain the  $i$ -th DM-MPSK symbol expressed as, for  $i = 1, 2, 3, \dots$ ,

$$s_i(t) = \sqrt{E_s} c(t) e^{j\phi_i}, \quad |t| < \frac{T_c}{2}, \quad (2)$$

where  $E_s$  is the symbol energy and  $\phi_i$  represents the  $i$ -th data and takes a value among  $\{0, \frac{2\pi}{M}, \dots, (M-1)\frac{2\pi}{M}\}$  with equal probability. At the receiver, we take the received signal as the input of a down-chirp filter whose impulse response  $h^*(t) = \sqrt{\frac{1}{T_c}} \exp(j\pi\mu' t^2)$ , where  $\mu' = -\mu$  and  $|t| < \frac{T_c}{2}$ , which means that we compress the received chirp signal into

a much narrower sinc-like pulse-shaped signal to achieve high processing gain. Then, the  $i$ -th down-chirp filter output  $g_i(t)$  can be expressed as [5]

$$g_i(t) = s_i(t) \otimes h^*(t) = \sqrt{E_s} \frac{\sin\left\{\pi B t \left(1 - \frac{|t|}{T_c}\right)\right\}}{\pi B t} e^{j\theta_i}, \quad |t| < T_c, \quad (3)$$

where  $B (\triangleq \mu T_c)$  is the CSS bandwidth defined as the range of the instantaneous frequency of the chirp signal and ' $\otimes$ ' denotes the convolution operator. From (3), we can see that  $g_i(t)$  has its first zeros at  $t \approx \pm 1/B$ . Thus, the compression ratio or processing gain  $G_p$  (which is the ratio of chirp signal duration to the pulse width of  $g_i(t)$ ) is given by  $T_c/(1/B) = BT_c$ . Finally, the output data is determined by demodulating and detecting the down-chirp filter output sampled at  $t = 0$ , where the value of  $g_i(t)$  is maximized.

### III. PERFORMANCE IN THE JAMMING ENVIRONMENT

In this paper, we adopt the broadband and tone jamming models [6], [7]. The strategies of the former and latter models are to jam the entire bandwidth and a specific frequency (mainly, center frequency) of the transmitted signal, respectively.

#### A. Influence of Broadband Jamming Signals

In the broadband jamming model, a jamming signal can be modeled as a zero-mean Gaussian noise process with a flat power spectral density over the CSS bandwidth [6]. Thus, the SER of the DM-MPSK system in the presence of broadband jamming signals is identical to that of the DM-MPSK system over additive white Gaussian noise (AWGN) channels. In addition, when the chirp signal is used only as a spreading code as in the DM scheme and the channel is Gaussian, CSS has no effect on the error performance [4]. From the SER expression of MPSK over AWGN channels [6], thus, we can easily obtain the SER expression (denoted by  $P_e^B$ ) of DM-MPSK in the presence of broadband jamming signals as follows:

$$P_e^B = \begin{cases} Q\left(\sqrt{\frac{2E_b}{J_0}}\right), & \text{for } M = 2 \\ 2 \cdot Q\left(\sqrt{2k \frac{E_b}{J_0} \sin \frac{\pi}{M}}\right), & \text{for } M > 2, \end{cases} \quad (4)$$

where  $k (\triangleq \log_2 M)$  is the number of bits in a PSK symbol,  $E_b$  and  $J_0$  denote the bit energy of  $E_s/k$  and power spectral density of jamming signal, respectively, and  $Q(x) (= \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt)$  represents the Gaussian  $Q$  function [6].

#### B. Influence of Tone Jamming Signals

The tone jamming signal  $z_i(t)$  can be expressed as

$$z_i(t) = \sqrt{J} \cos(2\pi f_c t + \theta_i), \quad |t| < \frac{T_c}{2}, \quad (5)$$

where  $J$  and  $f_c$  denote the power and center frequency of the jamming signal, respectively, and  $\theta_i$  is a random variable

distributed uniformly over  $[0, 2\pi)$  [7]. Thus, the lowpass equivalent  $z_{i,l}(t)$  of the tone jamming signal is simply obtained as

$$z_{i,l}(t) = \sqrt{J} \exp(j\theta_i), \quad |t| < \frac{T_c}{2}. \quad (6)$$

Then, the down-chirp filter output  $g'_i(0)$  of the jammed signal in sampling time at  $t = 0$  can be expressed as

$$g'_i(0) = s_i(t) \otimes h^*(t)|_{t=0} + z_{i,l}(t) \otimes h^*(t)|_{t=0} = \sqrt{E_s} e^{j\theta_i} + z_{i,l}(t) \otimes h^*(t)|_{t=0}, \quad (7)$$

where the convolution of  $z_{i,l}(t)$  and  $h^*(t)$  can be expanded as

$$z_{i,l}(t) \otimes h^*(t) = \int_{-\infty}^{\infty} \sqrt{J} e^{j\theta_i} \cdot \sqrt{\frac{1}{T_c}} e^{j\pi\mu'(t-\tau)^2} d\tau, \quad (8)$$

$$= \sqrt{\frac{J}{T_c}} e^{j\theta_i} e^{-j\pi\mu t^2} \int_{\alpha}^{\beta} e^{j\pi\mu(2t\tau - \tau^2)} d\tau,$$

where  $\alpha = -T_c/2 + t$  and  $\beta = T_c/2$  for  $t \geq 0$ , and  $\alpha = -T_c/2$  and  $\beta = T_c/2 + t$  for  $t < 0$ . We observe that the integral equation in (8) is of the similar form to that of the Fresnel integral  $F(x)$  defined as [8]

$$F(x) = \int_0^x e^{j\pi\tau^2/2} d\tau = \int_0^x \cos\left(\frac{\pi}{2}\tau^2\right) d\tau + j \int_0^x \sin\left(\frac{\pi}{2}\tau^2\right) d\tau = C(x) + jS(x). \quad (9)$$

After some manipulations with (8) and (9), we obtain the following closed-form expression of  $z_{i,l}(t) \otimes h^*(t)$ .

$$z_{i,l}(t) \otimes h^*(t) = \sqrt{\frac{J_0}{2}} \left[ C\left(\sqrt{\frac{G_p}{2}}\right) - \sqrt{2\mu t} \right] + C\left(\sqrt{\frac{G_p}{2}}\right) - j \left\{ S\left(\sqrt{\frac{G_p}{2}}\right) + S\left(\sqrt{\frac{G_p}{2}} - \sqrt{2\mu t}\right) \right\} e^{j\theta_i}. \quad (10)$$

Thus, (7) can be rewritten as

$$g'_i(0) = \sqrt{E_s} e^{j\theta_i} + \sqrt{2J_0} \left\{ C\left(\sqrt{\frac{G_p}{2}}\right)^2 + S\left(\sqrt{\frac{G_p}{2}}\right)^2 \right\} \cdot \exp[j\theta_i + j\angle\{C(\sqrt{G_p/2}) - jS(\sqrt{G_p/2})\}] = \sqrt{E_s} e^{j\theta_i} + \sqrt{2J_0} \left\{ C\left(\sqrt{\frac{G_p}{2}}\right)^2 + S\left(\sqrt{\frac{G_p}{2}}\right)^2 \right\} e^{j\psi_i}, \quad (11)$$

where  $\psi_i = \theta_i + \angle\{C(\sqrt{G_p/2}) - jS(\sqrt{G_p/2})\}$  is a random variable distributed uniformly over  $[0, 2\pi)$ . From (11), we can obtain signal space representation of  $g'_i(0)$  as shown in Figs. 2, 3, 4, 5, and 6 drawn for  $\theta_i = 0$  as an example, where  $g'_i(0)$  is represented as a vector sum of symbol and jamming vectors, and the dotted line represents the trace of  $g'_i(0)$  due to the jamming vector.

Figs. 2 and 3 show signal space representations for  $\frac{E_b}{J_0} \leq 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$  and  $\frac{E_b}{J_0} > 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$ , respectively, when  $M = 2$  (DM-BPSK). In Fig.

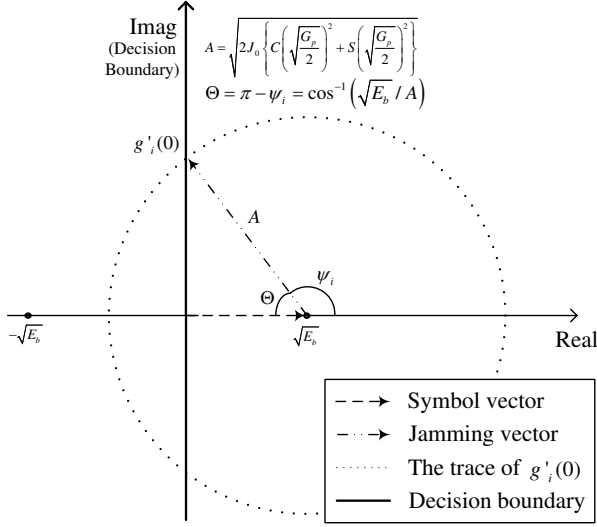


Fig. 2. A signal space representation of the down-chirp filter output  $g'_i(0)$  for DM-BPSK in the presence of tone jamming, when  $\frac{E_b}{J_0} \leq 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$  and  $\phi_i = 0$ .

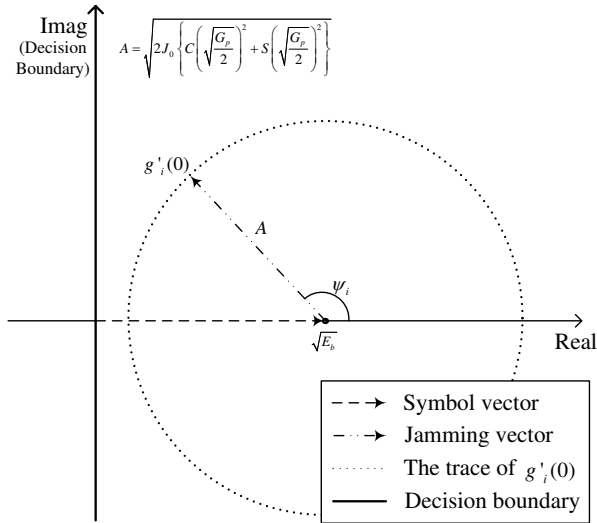


Fig. 3. A signal space representation of the down-chirp filter output  $g'_i(0)$  for DM-BPSK in the presence of tone jamming, when  $\frac{E_b}{J_0} > 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$  and  $\phi_i = 0$ .

2, the symbol error occurs when  $g'_i(0)$  falls on the left side of Imag-axis (decision boundary of BPSK). Thus, the SER  $P_e^{S, DM-BPSK}$  of DM-BPSK with tone jamming can be obtained

as, for  $\frac{E_b}{J_0} \leq 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$ ,

$$P_e^{S, DM-BPSK} = \frac{\Theta}{\pi} = \frac{1}{\pi} \cos^{-1} \left[ \sqrt{\frac{1}{2} \frac{E_b}{J_0} \frac{1}{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2}} \right]. \quad (12)$$

On the other hand, the position of the trace of  $g'_i(0)$  always remains on the right side of Imag-axis when  $\frac{E_b}{J_0} > 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$ , as shown in Fig. 3. As a consequence, the SER becomes zero. Thus, the SER  $P_e^{S, DM-BPSK}$  of DM-BPSK in the presence of tone jamming signal can be summarized as

$$P_e^{S, DM-BPSK} = \begin{cases} \frac{1}{\pi} \cos^{-1} \left[ \sqrt{\frac{1}{2} \frac{E_b}{J_0} \frac{1}{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2}} \right], \\ \text{for } \frac{E_b}{J_0} \leq 2\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\} \\ 0, \text{ otherwise.} \end{cases} \quad (13)$$

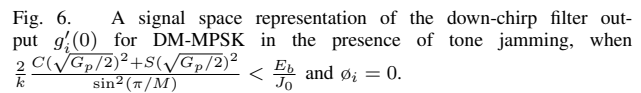
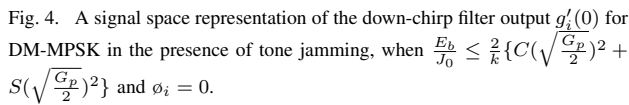
From Figs. 4, 5, and 6, similarly, we can derive the SER  $P_e^{S, DM-MPSK}$  of DM-MPSK ( $M > 2$ ) in the presence of tone jamming signals. Fig. 4 shows the trace of  $g'_i(0)$  when  $\frac{E_b}{J_0} \leq \frac{2}{k}\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$ . From the figure, we can easily see that the SER  $P_e^{S, DM-MPSK}$  is expressed as, for  $\frac{E_b}{J_0} \leq \frac{2}{k}\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}$ ,

$$P_e^{S, DM-MPSK} = \frac{(\Theta_1 + \Theta_2)}{\pi} = \frac{1}{\pi} \left( \frac{\pi}{2} - \frac{\pi}{M} \right) + \frac{1}{\pi} \cos^{-1} \left[ \frac{\sqrt{E_s} \sin(\pi/M)}{\sqrt{2J_0\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}}} \right] = \frac{M-2}{2M} + \frac{1}{\pi} \cos^{-1} \left[ \sqrt{\frac{k}{2} \frac{E_b}{J_0} \frac{1}{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2}} \sin \frac{\pi}{M} \right]. \quad (14)$$

On the other hand, from Fig. 5, where  $\frac{2}{k}\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\} < \frac{E_b}{J_0} \leq \frac{2}{k} \frac{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2}{\sin^2(\pi/M)}$ , it is seen that the SER is  $\frac{2\Theta}{\pi} = \frac{2}{\pi} \cos^{-1} \left[ \frac{\sqrt{E_s} \sin(\pi/M)}{\sqrt{2J_0\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}}} \right]$ . Finally, we can notice that the SER becomes zero when  $E_b/J_0$  is large enough that the whole circular trace stays within the decision boundary, i.e., when  $\sqrt{2J_0\{C(\sqrt{\frac{G_p}{2}})^2 + S(\sqrt{\frac{G_p}{2}})^2\}} < \sqrt{E_s} \sin(\pi/M)$ , as shown in Fig. 6.

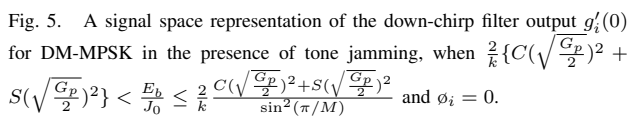
Thus, the SER expression of DM-MPSK ( $M > 2$ ) in the presence of tone jamming signal can be summarized as in (15) on the top of the next page.

(15)



In this section, we compare the theoretical SER of DM-MPSK derived in this paper with the empirical SER. The chirp signal is set to have the chirp duration ( $T_c$ ) of  $0.5\mu\text{s}$ , chirp rate ( $\mu$ ) of  $400\text{ MHz}/\mu\text{s}$ , processing gain ( $G_p$ ) of 100, and CSS bandwidth ( $B$ ) of 200 MHz. For data modulation, DM-BPSK ( $M = 2$ ), DM-QPSK ( $M = 4$ ), and DM-8PSK ( $M = 8$ ) systems are considered.

Fig. 8 shows the SER performance in the presence of tone jamming, where the theoretical SER curves were plotted using (13) and (15). In the figure, we can clearly see a close agreement between the theoretical SER derived in Section III.B and simulated SER. We can also observe that no symbol error occurs in the range of  $\frac{E_b}{J_0} > \frac{2}{k} \frac{C(\sqrt{G_p/2})^2 + S(\sqrt{G_p/2})^2}{\sin^2(\pi/M)}$ , as we expected. For  $G_p = 100$ , the threshold  $\frac{2}{k} \frac{C(\sqrt{G_p/2})^2 + S(\sqrt{G_p/2})^2}{\sin^2(\pi/M)}$  for zero SER is computed as  $10 \log(\frac{1.0946}{k \cdot \sin^2(\pi/M)})$  dB ( $= 0.392$  dB,  $0.392$  dB, and  $3.965$  dB for  $M = 2, 4$ , and  $8$ , respectively), which can be verified



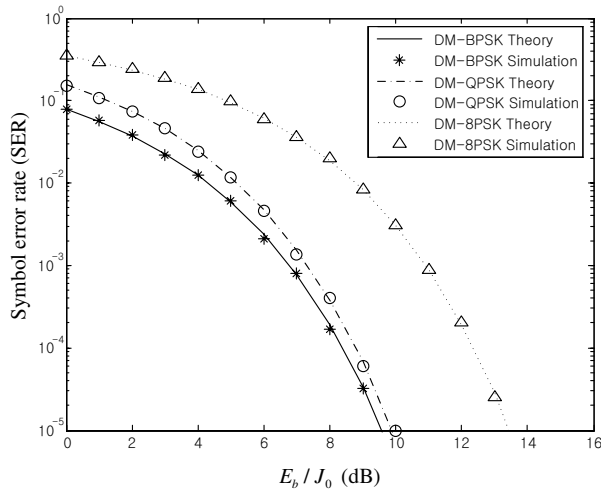


Fig. 7. SER performance of DM-MPSK in a broadband jamming environment ( $G_p = 100$ ).

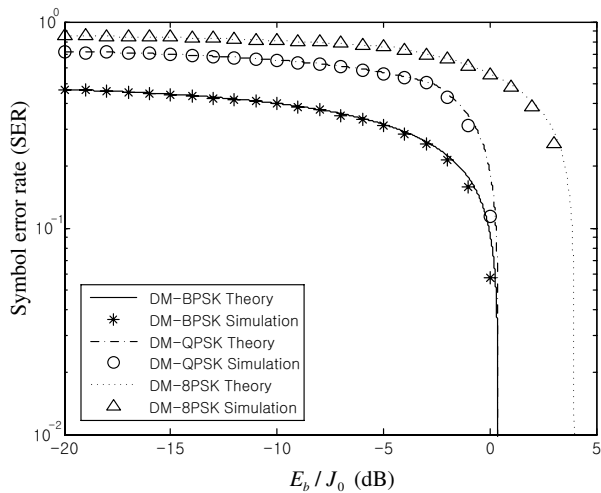


Fig. 8. SER performance of DM-MPSK in a tone jamming environment ( $G_p = 100$ ).

in Fig. 8.

## V. CONCLUSION

In this paper, we have analyzed the SER performance of DM-MPSK system in the presence of broadband and tone jamming signals. Modeling the broadband jamming as a Gaussian process, we have first obtained SER expressions of DM-MPSK in the presence of broadband jamming. Then, SER expressions of DM-MPSK in the presence of tone jamming have been derived based on geometric analysis of the down-chirp filter output. Finally, we have compared the derived SER with the empirical SER. From the numerical results, it has been observed that the analytic and empirical SERs closely agree with each other.

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