

# The Modified Eigenface Method using Two Thresholds

Yan Ma, and ShunBao Li

**Abstract**—A new approach is adopted in this paper based on Turk and Pentland's eigenface method. It was found that the probability density function of the distance between the projection vector of the input face image and the average projection vector of the subject in the face database, follows Rayleigh distribution. In order to decrease the false acceptance rate and increase the recognition rate, the input face image has been recognized using two thresholds including the acceptance threshold and the rejection threshold. We also find out that the value of two thresholds will be close to each other as number of trials increases. During the training, in order to reduce the number of trials, the projection vectors for each subject has been averaged. The recognition experiments using the proposed algorithm show that the recognition rate achieves to 92.875% whilst the average number of judgment is only 2.56 times.

**Keywords**—Eigenface, Face Recognition, Threshold, Rayleigh Distribution, Feature Extraction

## I. INTRODUCTION

FACE recognition is a unique ability of human beings. But it is difficult for machine to automatically recognize human face. The main reason is: the features of face will change due to expression, age, hairstyle, illumination, distance and viewpoint. How to recognize face quickly and efficiently by computer is an active topic in the field of pattern recognition.

The eigenface method [1] proposed by Turk and Pentland transforms a set of face images into eigenvectors called as "Eigenfaces". The input image is projected onto the eigenface space in the recognition process. The eigenface method is simple and fast. But it has some drawback. Firstly, the recognition rate is not very high; Next, the recognition time taken for large face database is long. Because each image corresponds to a projection vector and each projection vector will be compared in each recognition; Thirdly, the recognition rate can not be controlled. If the recognition rate is high, the right rejection rate will be low; Otherwise, if the right rejection rate is high, the recognition rate will be low. Based on the eigenface method, it has already been noticed that the discrimination performance is not very well by choosing the eigenvectors corresponding to large eigenvalues.

Yan Ma is with the Computer Science Department, Shanghai Normal University, China (e-mail: mayan0208@hotmail.com).

ShunBao Li is with the Computer Science Department, Shanghai Normal University, China (e-mail: lsb@shnu.edu.cn).

Many feature chosen methods has been developed since then, such as enhanced fisher linear discriminant model [2], kernel discriminant analysis algorithm [3], second-order mixture-of-eigenfaces method [4], etc.

In this paper, we analyze the PDF (probability density function) of the distance between the projection vector of the input face image and the average projection vector of the subject in the face database, follows Rayleigh distribution. To improve the performance of the eigenface method, two thresholds have been used and the values of the acceptance and the rejection threshold are defined. A unique algorithm has been proposed, two thresholds will be close to each other as the number of trials increases. During training, the projection vectors for each subject are averaged. The average projection vector has been used to recognize in order to decrease the number of trials. The experiments on ORL standard face image database show that the recognition rate has been improved by 8% in comparison with that of the eigenface method and we have improved the recognition performance.

## II. THE EIGENFACE METHOD

We use normalized images as a set of training images. Taking the total scatter matrix of the training images as covariance matrix, that is:  $\Sigma = \frac{1}{M} \sum_{i=0}^{M-1} (x_i - \mu)(x_i - \mu)^T$ , where:

$x_i$  is the image vector of the  $i$  th training image,  $\mu$  is the average vector of training images,  $M$  is the total number of training images. We use the SVD dimensionality reduction [1] to calculate the eigenvalues  $\lambda_i$  of  $\Sigma$  and the eigenvectors  $u_i$ ,  $i = 0, 1, 2, \dots, M-1$ . Sort eigenvectors on eigenvalues in descending order:  $\lambda_0 \geq \lambda_1 \geq \dots \geq \lambda_{M-1}$ . Each face image in the training images is projected onto the eigenface space spanned with  $u_0, u_1, \dots, u_{M-1}$  to obtain the projection vector. The input image  $f$  is also projected onto the eigenface space and we obtain the projection vector  $Y$ . The distance between  $Y$  and each projection vector of the training images is calculated. If it is lower than a predefined threshold  $y_0$ , we can be sure that the input image  $f$  is among the training images, otherwise,  $f$  does not belong to the subject in the training images. This approach uses single threshold  $y_0$  which is same to different subject in the training images. Therefore, when the difference

is large between the input image and its own face image in the training sets, the input image cannot be correctly recognized. Thus, the eigenface method has great limitation.

III. EIGENFACE METHOD COMBINED WITH TWO THRESHOLDS

A. Single Threshold Case

Assume the subject  $I$  in the face database has  $N$  face images whose projection vector on the eigenface is  $w_i$  ( $i = 1, 2, \dots, N$ ), let  $\lambda = \sum_{i=1}^N p_i w_i$ , where  $p_i$  is the prior probability of the  $i$ th face image and  $\lambda$  is a constant for recognition process. Assume the input image  $X$  has  $n$  face images whose projection vector on the eigenface is  $\varphi_i$  ( $i = 1, 2, \dots, n$ ), let  $x_n = \frac{1}{n} \sum_{i=1}^n \varphi_i$ . Apparently,  $x_n$  follows Gaussian distribution.

There are two assumptions:

①  $H_0$ : Assume that the input image  $X$  and the subject  $I$  in the face database belong to the same person, denoted as recognition.

②  $H_1$ : Assume that the input image  $X$  and the subject  $I$  in the face database belong to the different person, denoted as rejection. Let  $t = x_n - \lambda$ .

Assume case ① is true, then the mean of  $t$  is  $Et = E[x_n - \lambda] = Ex_n - \lambda = O$  ( $O$  represents vector 0), the covariance of  $t$  is  $Dt = D[x_n - \lambda] = Dx_n = \Sigma$ . Therefore,  $t$  follows Gaussian distribution with 0 mean and  $\Sigma$  covariance and the PDF of  $t$  is represented as  $p(t|H_0)$ . Let  $y_n = |t|$ , then  $y_n$  is 1-D random variable and follows Rayleigh distribution (see Fig. 1).  $R_0$  is the recognition region in the Fig.1 and the PDF of  $y_n$  is represented as:

$$p_0(y_n) = p(y_n|H_0) = \frac{y_n}{\sigma^2} \exp\left[-\frac{y_n^2}{2\sigma^2}\right] \quad (1)$$

Assume the case ② is true, the mean of  $t$  is  $Et = E[x_n - \lambda] = Ex_n - \lambda \neq O$ , the covariance of  $t$  is  $Dt = D[x_n - \lambda] = Dx_n = \Sigma$ , so  $t$  also follows N-D Gaussian distribution. Similarly, let  $y_n = |t|$ ,  $y_n$  follows 1-D generalized Rayleigh distribution (see Fig. 1).  $R_1$  is the rejection region in the Fig.1, the PDF of  $y_n$  is represented as:

$$p_1(y_n) = p(y_n|H_1) = \frac{y_n}{\sigma^2} \exp\left[-\frac{1}{2\sigma^2}(y_n^2 + \mu^2)\right] I_0\left(\frac{\mu y_n}{\sigma^2}\right) \quad (2)$$

where  $I_0(x)$  is the 0-order revised Bessel function,  $\mu = |Ex_n - \lambda|$ ,  $\sigma^2 = |\Sigma|$ ,  $|\bullet|$  is determinant

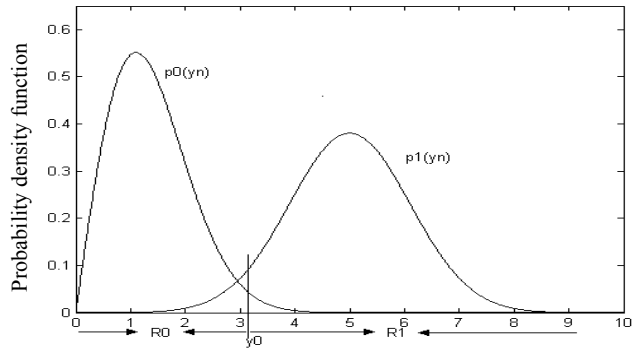


Fig. 1 single threshold case

Definition 1: Suppose  $y < y_0$ , the input image  $X$  and the subject  $I$  in the face database belong to the same person and they are recognized as the same person. This can be represented by  $P(D_0 | H_0)$  or  $P_D$ , the recognition rate then is,

$$P_D = P(D_0 | H_0) = \int_{R_0} p_0(y_n) dy_n = \int_0^{y_0} p_0(y_n) dy_n \quad (3)$$

Definition 2: Suppose  $y > y_0$ , the input image  $X$  and the subject  $I$  in the face database belong to the different person and they are recognized as different person. Represented by  $P(D_1 | H_1)$ , and the rejection rate is,

$$P(D_1 | H_1) = \int_{R_1} p_1(y_n) dy_n = \int_{y_0}^{+\infty} p_1(y_n) dy_n \quad (4)$$

Definition 3: The input image  $X$  and the subject  $I$  in the face database belong to the different person and they are recognized as the same person. Represented by  $P(D_0 | H_1)$  or  $P_f$ , the false acceptance then is,

$$P_f = P(D_0 | H_1) = \int_{R_0} p_1(y_n) dy_n = \int_0^{y_0} p_1(y_n) dy_n \quad (5)$$

Definition 4: The input image  $X$  and the subject  $I$  in the face database belong to the same person and they are recognized as the different person. Represented by  $P(D_1 | H_0)$ , the false rejection then is,

$$P(D_1 | H_0) = \int_{R_1} p_0(y_n) dy_n = \int_{y_0}^{+\infty} p_0(y_n) dy_n \quad (6)$$

B. Two Thresholds Case

Both of the false acceptance rate  $P_f$  and the recognition rate  $P_D$  will increase with the increase of the threshold  $y_0$  according to (3) and (5). In most cases, it is hoped that the false acceptance rate can be reduced and recognition rate can be increased. Using single threshold this requirement is not possible. Therefore, we set two thresholds  $y_{01}$  and  $y_{02}$  shown as Fig. 2:

$$P_f = \int_0^{y_{01}} p_1(y_n) dy_n \quad (7)$$

$$P_D = \int_{y_{01}}^{y_{02}} p_0(y_n) dy_n, \text{ where } y_{01} < y_{02} \quad (8)$$

In two thresholds case, the distance between the training subject and the input image will fall under any of the

following scenarios:

①  $y_n \geq y_{02}$ , in the  $R_1$  (see Fig. 2),  $H_1$  exists.

②  $y_n \leq y_{01}$ , in the  $R_0$  (see Fig. 2),  $H_0$  exists.

③  $y_{01} < y_n < y_{02}$ , in the  $R_2$  (see Fig. 2), no exact solution can be achieved. Assume that we know the recognition rate  $P_D$  and the false acceptance rate  $P_f$ , we can get values of  $y_{01}$  and  $y_{02}$  from (7) and (8):

$$y_{01} = \text{arcp}_1(P_f) \quad (9)$$

$$y_{02} = \text{arcp}_0(P_D) \quad (10)$$

where,  $\text{arcp}_1(\bullet)$  and  $\text{arcp}_0(\bullet)$  represents the inverse function of  $p_1(y_n)$  and  $p_0(y_n)$ , respectively.

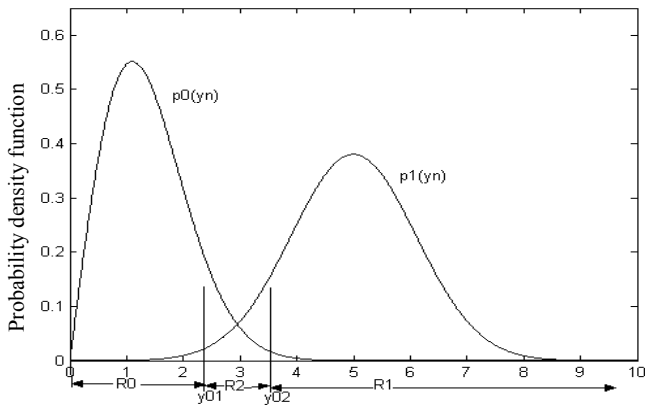


Fig. 2 two thresholds case

For scenarios ① or ②, the exact solution can be obtained so the recognition process can be terminated; if scenario ③ is true, we must arrive at the accumulated average to obtain the exact solution and it will not end till the scenario ① or ②

occurs. Let  $y_n = \frac{1}{n} \sum_{i=1}^n r_i$  represents the statistical result in the  $n$ th recognition for the input image, and  $y_1 = r_1$  represents the statistical result in the first recognition. If the false acceptance rate  $P_f$  and the recognition rate  $P_D$  are fixed, there exist two thresholds  $y_{01}(n)$  and  $y_{02}(n)$  in the first recognition, where  $n$  represents the  $n$ th recognition. If  $y_1 > y_{02}(1)$ ,  $H_1$  exists, known as rejection; If  $y_1 < y_{01}(1)$ ,  $H_0$  exists, known as recognition; If  $y_{01}(1) \leq y_1 < y_{02}(1)$ , a new trial will be done. With the increase in the number of recognition, the shape of  $p_0(y_n)$  and  $p_1(y_n)$  will be narrower until  $y_{01}(N) = y_{02}(N)$  (ignore cases beyond limitation) appears which leads to the end of recognition, which means the region of  $R_2$  does not exist (note:  $y_{01}(i)$  and  $y_{02}(i)$  represents the  $i$ th thresholds). That is to say, the judgment of accumulated average will end after  $N$  times and will not continue infinitely.

### C. The Algorithm

The proposed algorithm in this paper is as follows:

The first step: Get eigenface of the training set, the projection vector on the eigenface for each image and the average projection vector for each subject. The purpose is to reduce the number of the projection vectors and speed up the recognition process.

The second step: Acquire the mean and variance of each subject and calculate the conditioned PDF under the recognition and rejection instance.

The third step: Given the recognition rate  $P_D$  and the false acceptance rate  $P_f$ , calculate two thresholds  $y_{01}$  and  $y_{02}$  for each subject according to (9) and (10).

The fourth step: Acquire the input image and calculate its projection vector on the eigenface. Compare it with the average projection vector of each subject in the training set and calculate the distance. Recognize with two thresholds  $y_{01}$  and  $y_{02}$  and repeat it until the case ① or ② occurs.

## IV. EXPERIMENT

### A. The Preparation

In the experiments, we use ORL face database which includes 400 face images of 40 subjects and each subject consists of 10 different images. The ORL database is divided into three parts which are train database, in-train database and out-train database. The train database, which includes 100 face images for 20 fore-subjects with 5 fore-face images, is used as training subjects to obtain the eigenfaces; We use the in-train database to calculate the mean and variance of  $p_0(y_n)$ , which includes 100 face images for 20 fore-subjects with 5 latter-face images; We use out-train database to calculate the mean and variance of  $p_1(y_n)$ , which includes 200 face images for 20 latter-subjects with 10 face images.

### B. The Experiment with Two Thresholds

The projection vectors of each subject are averaged (Assume the probability of each face image is equal in the experiment), so the number of projection vectors decreases from 100 to 20. Assume both of the recognition rate  $P_D$  and the false acceptance rate  $P_f$  is same to each subject and two thresholds  $y_{01}$  and  $y_{02}$  of each subject can be calculated using (9) and (10).

We also use the eigenface method with single threshold to recognize and select the threshold as follows:

① represented as Eig1,

$y_0 = (\text{mean1} + \text{mean2})/2$ , where  $\text{mean1}$  is the average distance between the projection vectors in the in-train database and the train database, and  $\text{mean2}$  is the average distance between the projection vectors in the out-train database and the train database.

② represented as Eig2,

$y_0 = (N_1 * \text{mean1} + N_2 * \text{mean2}) / (N_1 + N_2)$ , where the meaning of  $\text{mean1}$  and  $\text{mean2}$  is the same as above.  $N_1$  and  $N_2$

represent the total number of face images of the in-train database and out-train database, respectively.

In the experiment by using the proposed algorithm, we change the order of 20 subjects with 10 face images constantly in order to simulate the camera by which takes a set of face images of each subject at different times and increase the number of experiment. 500 trials have been done. Table 1 lists the result of recognition rate by using single threshold and the proposed algorithm, where

$$\text{total right judgment rate} = \frac{(\text{the number of the right recognition} + \text{the number of the right rejection})}{\text{the total number of judgment}}$$

The total wrong judgment rate just likes above. Fig. 3 lists the average number of inputting images for same person.

TABLE I  
THE RESULT OF USING THE PROPOSED ALGORITHM AND THE EIGENFACE METHOD

	The proposed algorithm	Eig1	Eig2
right recognition rate (%)	97.5	75	82
wrong recognition rate (%)	2.5	25	18
right rejection rate (%)	88.25	88.5	78.5
wrong rejection rate (%)	11.75	11.5	21.5
total right judgment rate (%)	92.875	84	79.67
total wrong judgment rate (%)	7.125	16	20.33

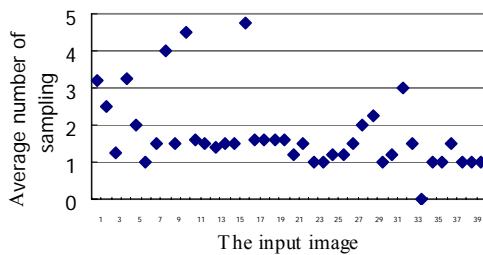


Fig. 3 the average number of inputting images for same person

It is clear that the recognition rate by using the proposed algorithm is much higher than that of the single threshold. The number of average judgment is only 2.56, that is to say, the total right judgment rate is about 8% higher than that of single threshold while the judgment time taken is a little higher but it is well worth it.

## V. SUMMARY

We modify the eigenface method proposed by Turk and Pentland. In the training process, the projection vectors of different images for each subject is averaged, which is taken as average projection vector and can reduce the number of judgment and increase the recognition speed. In addition, the total right judgment rate increases from 84% to 92.875% with the acceptance and rejection threshold for each subject while the average number of judgment is only 2.56 and increase the precision of recognition.

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