# The Fatigue Damage Accumulation on Systems of Concentrators

Alexander Urbach, Mukharbij Banov, Vladislav Turko

Abstract—Fatigue tests of specimen's with numerous holes are presented. The tests were made up till fatigue cracks have been created on both sides of the hole. Their extension was stopping with pressed plastic deformation at the mouth of the detected crack. It is shown that the moments of occurrence of cracks on holes are stochastically dependent. This dependence has positive and negative correlation relations. Shown that the positive correlation is formed across of the applied force, while negative one — along it. The negative relationship extends over a greater distance. The mathematical model of dependence area formation is represented as well as the estimating of model parameters. The positive correlation of fatigue cracks origination can be considered as an extension of one main crack. With negative correlation the first crack locates the place of its origin, leading to the appearance of multiple cracks; do not merge with each other.

**Keywords**—Correlation analysis, fatigue damage accumulation, local area, mathematical model.

#### I. INTRODUCTION

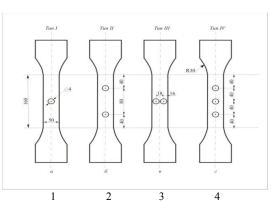
WHEN operating large-scale designs is difficult to undertake the whole checking of their technical condition due to high financial and time expenses. So it leads to choice some checking areas. For that reason it is required to investigate the processes of damage accumulation in these areas in order to determine their size and location.

As objects of study flat specimens with a different number and location and with the different holes diameters were chosen. Fatigue tests were conducted till the cracks appear in each of the holes on both sides of it. The moment of the crack appearance was fixed by wire crack's sensors, pasted along the both sides of the holes. The fixed length of the crack - 0.5 mm. Every crack was stopped due to the plastic deformation zone in front of the mouth of the crack occurred by poisson of 4 mm in diameter. The stresses on the sides of the hole for all types of specimens were identical and were 120 MPa –cycle maximum stress, 12 MPa - the cycle minimum stress, loading frequency -8.3 Hz. Two kinds of specimens were investigated – Type "A" and Type "B". (Fig.1).

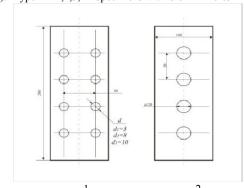
A.Urbach is with the Institute of Transport Vehicles Technologies, Faculty of Transport and Mechanical Engineering, Riga Technical University, Riga, LV 1658 Latvia (e-mail: Aleksandrs.Urbahs@rtu.lv).

M. Banov is with the Institute of Transport Vehicles Technologies, Faculty of Transport and Mechanical Engineering, Riga Technical University, Riga, LV 1658 Latvia (e-mail: muharbij@inbox.lv).

V. Turko is with the Research Center "Aviatest LNK", Ltd, Riga, LV 1073 Latvia, (corresponding author to provide phone: +371 29543164, e-mail: vladislav\_turko@mail.ru).



a).- Type "A-1,2,3,4" specimens with Ø 5 mm holes



b). -Type "B-1,2" specimens with Ø 3;8;10 mm (1) and Ø 20 mm (2)

Fig. 1 Two types of specimens to fatigue tests:

# II. TYPE "A" SPECIMENS TEST RESULTS

### A. The adoption of the cumulative distribution function

The fatigue life time  $\tau$  of holes is the time (cycle number N) point of crack's sensors pick-upping and is determinated as logarithm of cycle number N. It seems, that  $\tau = \lg N$  is random variable. It shown [1, 2], that the two-parametric biexponential cumulative distribution function (CDF) with logarithmic fatigue life time  $F(\tau)$  for reliable mechanical systems has the following form:

$$F(\tau) = \exp[-\exp(-\frac{\tau - \alpha}{\beta})] \tag{1}$$

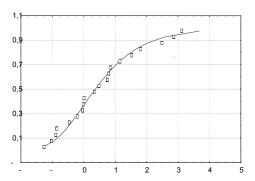
where  $\alpha$  – a location parameter,

 $\beta$  – a location parameter,

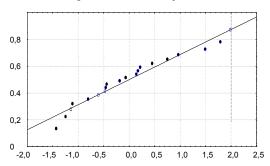
and best describes the life time, than others, including the lognormal CDF

$$F(\tau) = \Phi(\frac{\tau - a}{h}) \tag{2}$$

where a - a location parameter, Mathematical Expectation, b - a scale parameter,  $b^2$  - Variance.



 a).-two-parametric bi-exponential CDF with logarithmic fatigue life time, mean squared residual 0,02



b).-lognormal CDF, mean squared residual 0,04

Fig. 2 The experimental data matching with theoretical (Type "A-1" specimens)

The Fig.2 shows, that in the low-probability area the biexponential CDF (1) much better describes the fatigue life time, than lognormal CDF (2). In the future, in order to simplify the analysis and algebraic calculations, but without loss of generality of the findings, we accept a lognormal CDF for the distribution of fatigue life of the specimens' holes.

# B. Type"A-1" specimens test results.

Let denote the hole of 5 mm in diameter in the middle of the Type "A"-specimen as "inner hole", and holes that are closer to the grips of the test stand - as "outer hole».

It were tested 20 specimens of Type "A-1". The fatigue life time for a single hole can be seen as without any influence from the other holes. Sample estimates for the Expectation -  $a_{inner}$  and Variance -  $b_{inner}$  of the fatigue life time CDF (2) and coefficient of variation  $v_{inner}$  are equal to:

$$a_{inner} = 5,27233$$
  $b_{inner}^2 = 0,00582$   $v_{inner} = 1,5\%$  (3)

C. Type"A-2" specimens test results.

It were tested 14 specimens of Type "A-2". Put forward and test the statistical hypothesis of crack's appearance independence on the specimens of this type. If appearances of

cracks are independent, we have observations on the first  $F_{(1)}$  and second  $F_{(2)}$   $\{\tau\}$  order statistics of the fatigue life time CDF for outer holes [3]. In our case it has the following form:

$$F_{(1)}\{\tau\} = 1 - \left[1 - \Phi(\frac{\tau - a_s}{b_s})\right]^2 \tag{4}$$

$$F_{(2)}\{\tau\} = \Phi^2(\frac{\tau - a_s}{b_s}) \tag{5}$$

where  $a_s$  and  $b_s$  – location and scale parameters of fatigue life time CDF for outer holes. Sample estimates for the Expectations –  $a_{s(1,2)}$  and Variances –  $b_{s(1,2)}^2$  of the fatigue life time CDF (4), (5) and coefficient of variation  $v_{s(1,2)}$  are equal to:

$$a_{s(1)} = 5{,}12165$$
  $b_{s(1)}^2 = 0{,}00899$   $v_{s(1)} = 1{,}85\%$  (6)

$$a_{s(2)} = 5,23852$$
  $b_{s(2)}^2 = 0,00609$   $v_{s(2)} = 1,5\%$  (7)

It is also known [3] that the theoretical correlation coefficient  $\rho$  (1,2) between the first and the second order statistics values is equal to 0.467 for normal distribution. The sample estimate of this correlation coefficient is equal to 0,487. Put forward the null hypothesis of equality of well-known correlation coefficients against the alternative hypothesis of their inequality, and apply a test statistics T [4]. The test shows that there not reasons to reject the null hypothesis about equality of the sample correlation coefficient to theoretical. So we can assume that the appearance of cracks on the Type"A-2" specimens are independent and all 28 fatigue life time observations on 14 holes may be considered as one homogeneous sample with settings:

$$a_s = 5,18009$$
  $b_s^2 = 0,01080$   $v_s = 2,6\%$  (8)

Mathematical Expectations and Variances of the first and second order statistics CDF (4), (5) are related to the parameters of the initial distribution (2) by the following relations [3]:

$$a_{s(1,2)} = a_s + /-0.5642 b_s$$
 (9)

$$b_{s(1,2)}^2 = 0.6816b_s^2 \tag{10}$$

So the theoretical values of Mathematical Expectations and Variances of the first and second order statistics CDF are:

$$a_{\text{outer}(1)} = 5{,}12146 \qquad b_{\text{outer}(1)}^2 = 0{,}00736 \quad v_{\text{s}(1)} = 1{,}68\% \quad (11)$$

$$a_{\text{outer}(2)} = 5,23872$$
  $b_{\text{outer}(2)}^2 = 0,00736$   $v_{s(2)} = 1,64\%$  (12)

The comparison of sample parameters of order statistics CDF with theoretical confirms the independence of the fatigue damage on the specimens with two outer holes. This is well illustrated in the Fig.3.

However, the comparison of the inner hole's fatigue life time with the outer ones shows a significant reduction of the outer hole's fatigue life time. This may be due to the proximity of the testing machine grips or/and the *transition of power flow from one constructive structure to another*.

D. Type"A-3" specimens test results.

A similar analysis was carried out with 20 fatigue tests results of Type "A-3" specimens with two inner holes, located on the width of the specimen.

The sample estimate of the correlation coefficient between the first and the second order statistics is equal to 0,945 that statistically significantly more, than theoretical value of 0,467. So the appearances racks of are not independent on such type of specimens.

The sample parameters of the first and the second order statistics CDF:

$$a_{in(1)} = 5,24662$$
  $b_{in(1)}^2 = 0.00825$   $v_{in(1)} = 1,7\%$  (13)

$$a_{in(2)} = 5,30216$$
  $b_{in(2)}^2 = 0.00611$   $v_{in(2)} = 1,47\%$  (14)

and theoretical ones for (3):

$$a_{(1)} = 5,22929$$
  $b_{outer(1)}^2 = 0,00397$   $v_{s(1)} = 1,2\%$  (15)

$$a_{(2)} = 5{,}31537$$
  $b_{outer(2)}^2 = 0{,}00397$   $v_{s(2)} = 1{,}185\%$  (16)

The comparison of the parameters indicates that the Expectations value for inner holes remain virtually unchanged, while significantly increasing the Variances for the dependent appearance of cracks. So is shown on Fig.3.

This means that at holes, located on the width of the specimen, across the applied load, there is a positive correlation. Such a connection leads to a large scatter in the moments of the crack appearances, but does not reduce the longevity: the CDF of the first and the second crack appearance moments approaches to fatigue life time CDF of lonely inner hole, in limit ( $\square(1,2)=1,0$ ) matching with it, i.e. if there is a positive correlation of cracks appearances in the dependent fatigue damages accumulation zone it may be assumed a continuation of arising of the first crack.

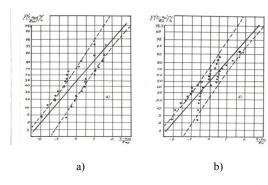


Fig. 3 The comparison of theoretical CDF of 1st and 2nd Order Statistics for fatigue life time of specimens with:

a). two outer holes

b). two inner holes

E. Type"A-4" specimens test results.

It were tested 17 specimens of Type "A-4". As before, assume the appearance of independence of fatigue cracks in the system of three concentrators - two outer holes and one inner hole. Then the sample Typ"A-4" can be represented as the union of two independent systems: one inner hole system with parameters (3) and of their two outer holes with parameters (8). Then the CDF of the order statistics are determined from [5]

$$F_{(1)}(\tau) = 1 - \left[1 - \Phi\left\{\frac{\tau - a_{in}}{b_{in}}\right\}\right] \left[1 - \Phi\left\{\frac{\tau - a_{s}}{b_{o}}\right\}\right]^{2}$$
(17)

$$F_{(1)}(\tau) = 1 - \left[1 - \Phi\left\{\frac{\tau - a_{in}}{b_{in}}\right\}\right] \left[1 - \Phi\left\{\frac{\tau - a_{s}}{b_{s}}\right\}\right]^{2}$$

$$F_{(2)}(\tau) = \Phi^{2}\left\{\frac{\tau - a_{s}}{b_{s}}\right\} + 2\Phi\left\{\frac{\tau - a_{s}}{b_{s}}\right\} \Phi\left\{\frac{\tau - a_{in}}{b_{in}}\right\} \left[1 - \Phi\left\{\frac{\tau - a_{s}}{b_{s}}\right\}\right]$$

$$F_{(3)}(\tau) = \Phi\left\{\frac{\tau - a_{in}}{b_{in}}\right\} \Phi^{2}\left\{\frac{\tau - a_{s}}{b_{s}}\right\}$$

$$(18)$$

$$F_{(3)}(\tau) = \Phi\{\frac{\tau - a_{in}}{b_{in}}\}\Phi^{2}\{\frac{\tau - a_{s}}{b_{s}}\}\tag{19}$$

The matching of the experimental and theoretical values are shown in Fig. 4

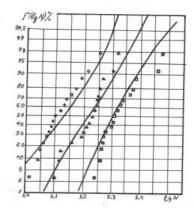


Fig.4. The CDF of the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> Order Statistics of fatigue life time on Type "A-4" specimen with experimental data.

So we see the systematic deviation of experimental data from the theoretical curves due to negative correlation relationship [5]. Compare the sample parameters (3) of fatigue life time CDF for the inner hole on the Type "A-1" specimen with the sample parameters of the fatigue life time CDF for inner hole on Type "A-4" specimen:

$$a_{inner4} = 5,30935$$
  $b_{inner4}^2 = 0,00625$   $v_{inner4} = 1,45\%$  (20)

The fatigue life time of inner hole on the system with two outer holes is significantly increased, which prove the interaction of outer and inner holes. Stochastic relationships on the Type "A-4" specimen are analyzing with paired correlation coefficients [6, 7] between fatigue life times of the first and the second order statistics of the cracks appearances on the outer holes and of fatigue life times on inner holes. The sample paired correlation  $\rho_{ii}$  coefficients are:

$$\rho_{(1)\text{inner}} = -0.152 \quad \rho_{(2)\text{inner}} = -0.483 \qquad \rho_{(1,2)} = 0.455$$
 (21)

 $\rho_{(1,2)}\,$  - correlation coefficient between the  $1^{st}\,$  and the  $2^{nd}$  order statistics of fatigue life time CDF on outer holes and shows the independence of the cracks appearances [3]. But  $\rho_{(2)inner} = -$ 0,483 is statistically significantly more, than theoretical. In shows, that the early the cracks are originated on sides of outer holes, the later the cracks are revealed on the inner hole. This fact is confirmed by the estimates of partial correlation coefficients [7]:

$$\rho_{(1)\text{in}/(2)} = -0.087$$
  $\rho_{(2)\text{in}/(1)} = -0.470$   $\rho_{(1,2)/\text{inn}} = 0.441$  (22)

ie cracks on the outer holes greatly increases the fatigue life time of the inner hole.

#### III. TYPE "B" SPECIMENS TEST RESULTS

#### A. Correlation depending on the diameter of the hole

It was tested 5 specimens with 8 holes on each (diameters 3 mm and 10 mm), and 4 specimens with a diameter of 8 mm (8 holes) and 20mm (for holes). See Fig.1.

As the main analytical tool, the multiple correlation analysis, the particular intraclass correlation coefficient (ICC) [8] has been selected. The use of ICC was due to the fact that in this case, the coupled moments of the crack's originations on the specimen are considered regardless of the order of their appearance, because otherwise it will be established correlation between the rank-order statistics of appearance of fatigue cracks, rather than when they occur [9].

The expression for the ICC is as follows:

$$R = \frac{1}{k - 1} \left( k \frac{b_z^2}{b^2} - 1 \right) \tag{23}$$

where R-intraclass correlation coefficient (ICC),

k – number of members in the intraclass, in our case the number of investigated holes (4 or 8),

 $b^2$  – total sample Variance on the overall Expectation,  $b^2_{\tau}$  – sample Variance of the class (holes) Expectations,

As for testing the hypothesis that the correlation coefficient is significantly different from zero, we apply a test statistics T [8]:

$$T = (z - z_0) \sqrt{\frac{2n - 3}{2}}$$
where  $Z = \frac{1}{2} \ln \frac{1+r}{1-r}, Z_0 = 0,$ 

n – number of observations,

r – sample intraclass correlation coefficient.

Significance level  $\alpha$  for testing the hypothesis assumed to be 0.05

TABLE I INTRACLASS CORRELATION COEFFICIENTS DEPENDS ON DIAMETERS OF HOLES

Holes Ø mm	Sampling R (ICC)	Sample size	Test statistic T (23)	Limit value of Test Statistic	
3	0,91	40	9,48	2,02	
8	0,76	32	5,50	2,04	
10	0,63	40	4.60	2,02	
20	0,28	16	1,10	2,13	

As seen from Table I, the values of the calculated estimates of ICC showed significant positive interdependence of the appearance of fatigue cracks on the sides of the holes, ie the first appearance of a fatigue crack on the side of the hole provoke second crack at the opposite side, but this dependence decreases with increasing hole diameter and virtually disappears at large diameter holes. From this we can draw two

conclusions: for small diameter holes (high pozitive values of CC) the second crack can be considered as a propagandation of the first, and for large diameter holes on both the appearance of cracks is independently and all cracks can be considered as one sample.

#### B. Correlation depending on the space between holes

To investigate the influence of the space between holes on the dependence of the appearance of fatigue cracks on them, an experiment was carried out on specimens with four 20 mm in diameter, located along the length of the specimen (Fig.1, B). As shown above, the moments of fatigue cracks origins can assume stochastic independence at the hole sides.

TABLE II
INTRACLASS CORRELATION COEFFICIENTS DEPENDS ON SPACE BETWEEN OF
HOLES

Space	Number of	-	Sample	Test	Limit
between	holes	Sampling	size	statistic	value of
the holes,	considered,	R (ICC)		T (23)	Test
mm	k (22)				Statistic
50	2	-0,80	24	4,99	2,07
100	3	-0,45	48	1,89	2,01
150	4	-0,29	32	1,13	2,04

As can be seen from Table II, with space increasing between the centers of the holes, the dependence of fatigue cracks origins is weakened. It should be noted that the ICC is not symmetric about zero - a negative relationship is more addictive in the same absolute values the correlation coefficient than for positive ones. As shown in [8], the range of ICC variation:

$$-\frac{1}{k-1} \le r \le 1,0 \tag{25}$$

Nevertheless, the negative correlation is obviously falling at a distance within 4 ... 5 diameters between centers of holes.

# ${ m IV.}$ THE MATHEMATICAL MODEL OF THE DEPENDENT FATIGUE DAMAGE ZONE

#### A. The mathematical model hypotheses

Let we consider some area  $V_0$  of the interested object. This area are formed by equal magnitude micro volumes  $\Delta Vi$ . There is interdependent accumulation of the fatigue damages in each of micro volumes under the action of the cyclic loads. Make the following suggestions:

- · the distance between the centres of micro volumes is  $\Delta l$ ,
- $\cdot$  for simplicity and without loss of generality, we consider the case of one dimension l,
- · the durability of whole zone  $V_0$  is determined by the longevity of its constituent micro volumes due to the loaded standby model [2].

Let suppose that fatigue damages are gradually accumulated in micro volumes along dimension l.

Assume that the correlation coefficient  $R(\bullet)$  of fatigue life time for two micro volumes at distance l from each other, is where R(l), (so as R(0) = 1), but the correlation coefficient  $\varrho(\bullet)$  of fatigue life time for two adjoining micro volumes at distance  $\Delta l$  from each other and at distance l from the first

damaged micro volume is dependent of number  $n = l/\Delta l$  micro volumes along dimension is where:

$$\rho(I + \Delta I) = 1 - \mu(I)\Delta(I) \tag{26}$$

where  $\mu(0) = 0$ 

Assume that this relation is correct [7]:

$$R(l + \Delta l) = \rho(l + \Delta l)R(l) \tag{27}$$

or inserting (26) to (27)

$$R(l + \Delta l) = R(l) - R(l)\mu(l)\Delta l \tag{28}$$

and transforming (28):

$$\frac{R(l+\Delta l)-R(l)}{\Delta l} = -R(l)\mu(l) \tag{29}$$

Letting  $\Delta l$  in (28) going to zero, we have

$$\frac{dR(l)}{dl} = -R(l)\mu(l) \tag{30}$$

Solving the resulting differential equation (30) for R(l), we have

$$R(l) = exp\left\{-\int_0^l \mu(x)dx\right\} = e^{-\varphi(l)}$$
 (31)

B The mathematical model checking

To determine the type of the function  $\varphi(l)$  in (31) let consider the experimental data out of Table I and Table 2. Causing the experimental values of ICC as a function of diameters at the charts you can get a good match for the logarithmic coordinates.

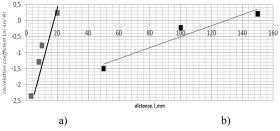


Fig. 4 The intraclass correlation coefficient changing versus the the distance between the places of fatigue cracks origins in logarithmic coordinates.

a). perpendicular to applied force (positive correlation relationship), Table I

b) along to applied force (negative correlation relationship), Table II The satisfied linearization of the data on the charts is thanks to the double logarithms turned the ICC values. So it may suppose the following:

$$Ln(-Ln(R(l))) = LnC + mLn(l)$$
(32)

where  $Ln(\bullet)$  – is the logarithm operation

C, m - parameters of the linear functions Fig.4.

*l* –diameters of holes (Fig.4,1.), the space

Comparing (32) and (31) get:

$$R(l) = \exp(-\varphi(l)) = \exp(-Cl^{m})$$
(33)

and thus determinate the function -  $\varphi(l)$  (See (31)):

$$\varphi(l) = Cl^m \tag{34}$$

Then the general form of the dependence of fatigue damage accumulation in a certain area has the following logarithmic regularity:

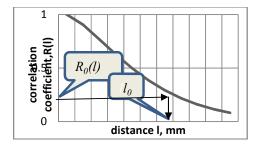


Fig. 5 Changing of the correlation coefficient depending on the distance between the fatigue damages according to the mathematical model (31).

 $R_0(l)$  - the correlation coefficient value statistically significant different from zero;

 $l_0$  – the dimension of local zone with dependent accumulation of fatigue damage.

Analyzing the (34), it seems that the parameter m describes the curvature of graph or how the correlation coefficient is reducing, but the parameter C – shows the power of the stochastic correlation (less than C - the relationship is expensioned over a greater distance).

# C The Mathematical Model Parameters Estimstion

Consider the (32) and determine the statistical estimation of parameters C and m for the linear regression model [10] with positive (Fig.4.a) and negative (Fig.4.b) correlation relationship. In this case if the correlation coefficient have negative sign we take absolute values of this correlation coefficien.

So we have the following statistical estimations for positive correlation relationship (perpendicular to applied force):

$$\hat{C} = 0.019 \qquad \hat{m} = 1.356 \tag{35}$$

for negative correlation relationship (along to applied force):

$$\hat{C} = 0.00047 \quad \acute{m} = 1.589$$
 (36)

where  $\hat{C}$  and  $\acute{m}$  - estimators of model (32), (34) parametrs.

Let assume the significance level  $\alpha = 0.1$  for testing hypothesis that the correlation coefficient is statistically significant different from zero. Then the critical value of the test statistics T = 3.42 (Student t-test criterion) for  $R_0(l)$ :

$$R_0(l) = 0.43$$
 (37)

Substitute  $R_0(l)$  from (37) and  $\hat{C}$  with  $\hat{m}$  from (35) to (33). Solving the resulting equation for l, we determinate the size  $l_0$  of the zone in the horizontal direction:

$$l_{0hor} = 16.8 \text{ mm}.$$
 (38)

Similarly, to determine the vertical size of the zone (substitute  $R_0(l)$  from (37) and  $\hat{C}$  with  $\hat{m}$  from (36) to (33). and solving the resulting equation for l):

$$l_{0vert} = 112,0 \text{ mm}.$$
 (39)

Inside this zone where are dependent accumulation of fatigue damage.

#### V.CONCLUSION

- A mathematical model for the dependent fatigue damage zone is represented.
- As analog of the zone with numerous fatigue damage the specimens with multi-hole concentrators nave been testing till fatigue cracks were originated on both side of concentrator. The test results showed a good agreement of experimental data with theoretical predictions.
- The estimation of model parameters and the determination of the zone sizes with the dependent accumulation of fatigue damages are executed. Shown that the dependence of fatigue damage accumulation is both positive (across the direction of applied force) and negative (along the direction of applied force) correlation relationship. Negative correlation relationship is more pronounced and extends over a greater distance.
- Positive correlation relationship implies that when a first crack is originated the next ones. Positive correlated fatigue cracks can be considered as the previous one's propagation, subsequently leading to one main crack (The airplane fatigue destruction model – one main crack propagation).
- Negative correlation relationship implies that the probability of an occurrence of new cracks in the dependent zone drops, thereby locating the position of first crack. But thus it is allowing the new cracks occur outside this zone. (The helicopter fatigue destruction model as the flow of failures, bearing failures for the appearance of numerous cracks).
- The corrective determination of the zone size with dependent accumulation of fatigue damages can concentrate the use of nondestructive testing in the most dangerous places of construction.

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Alexander Urbach is a 1981 undergraduate of the Faculty of Mechanical Engineering Riga Civil Aviation Engineering Institute and has been awarded the Dr.sc.ing. degree by the same faculty in 1986. In 1997 he was awarded the Dr.habil.sc.ing. degree by the Riga Aviation University. In 1989-1999 he is a Vice Dean and Dean at the Faculty of Mechanical Engineering of Riga Aviation University. Since 1999 he is a full time professor and Director at the Institute of Transport

Vehicles Technologies of the Riga Technical University. His field of scientific interest includes: Mechanical Engineering; Transport; Diagnostics of Machinery; Unmanned Land/Sea/Air Vehicles; Science of Aviation Materials; Aircraft Construction Mechanics; Structural Materials Processing – Surface Protection Technologies.



Mukharbiy Banov is a 1975 graduate of the Engineer Mechanics of Aircraft and Engines Maintenance of the Riga Civil Aviation Engineers Institute and has been awarded the Ph.D. degree by Engineering in 1979 of the Faculty of Aircraft Repair and Technology of the Riga Civil Aviation Engineers Institute.

Since 1981 to 1995 he was Assistant, Docent, Professor, Lecturer of Riga Civil Aviation Engineering Institute. Since 1995 to 1998 he was Assistant Docent Professor Lecturer of Riga

Assistant, Docent, Professor, Lecturer of Riga Aviation University. Since 1999 he is Assistant, Docent, Professor, Lecturer of Riga Technical University of Institute of Transport Vehicle Technologies. Since 2007 he is the Supreme State Engineering Courses of Improvement of Qualification on Non - Destructive Testing



Vladislav P. Turko has graduated the Mechanical Engineering Faculty of the Riga Civil Aviation Engineers Institute in 1974 and since 1979 to 1982 has been Graduated School in the Department of the Manufacturing & Repair of Aircrafts at the same Institute. After that he was activated in Riga Branch of State Scientific Research Institute of Civil Aviation as Researcher in management and execution of aircraft and its parts static and fatigue testing, experiments, data

analysis and mathematic modeling. Since 1997 he is the Deputy Director (Technical Director) of Riga Scientific Research Center "Aviatest LNK" Ltd. In 2000 he was awarded by Open Stock Company "Tupolev" - «the Medal of Academician A.N.Tupolev » - for execution of investigations on maintenance of safe operation of the airplane TU – 154.