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The Evaluation of the Performance of Different Filtering Approaches in Tracking Problem and the Effect of Noise Variance

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***Abstract*—**Performance of different filtering approaches depends on modeling of dynamical system and algorithm structure. For modeling and smoothing the data the evaluation of posterior distribution in different filtering approach should be chosen carefully. In this paper different filtering approaches like filter KALMAN, EKF, UKF, EKS and smoother RTS is simulated in some trajectory tracking of path and accuracy and limitation of these approaches are explained. Then probability of model with different filters is compered and finally the effect of the noise variance to estimation is described with simulations results.

***Keywords*—**Gaussian approximation, KALMAN smoother, Parameter estimation.

I. Introduction

PREDICTION and estimation theories are fundamental concepts for signal processing in dynamical systems and many control applications. To implement these tools for a given noisy data, a time series model of the process should be generated. This article is concerned with filtering approach in non-linear state space models that measurements are obtained at discrete time instants from a non-linear measurement model and Gaussian noise is inputted to model. Generally, KALMAN filter is a linear filter for estimation in linear systems [1]. Simplicity and iterative structure of the derivation of the KALMAN filter makes it suitable for use in many practical engineering application and others majors too. However, many engineering systems, especially mechanical systems, are nonlinear in nature. And in many situations this filtering approach has limitation performance [2].

The extended KALMAN filter was developed to account for these nonlinearities in modeling. The EKF considers nonlinearities by linearizing the system in around the known distribution then updated KALMAN filter equations are applied to the linear system. Although EKF has been used successfully in many applications, it has some important limitations [2]. Higher order terms in dynamics of noise are ignored. This simplification in many condition leads to instability in simulation results [3], [4].

The unscented KALMAN filter tries to remove some of the disadvantages of the EKF model in the estimation of nonlinear dynamical system. Generally UKF is an extension of the KALMAN filter for the estimation of nonlinear systems and posterior mean value and covariance of the state of system are obtained from the transformed sigma points [5], [6].

During the last few decades, the speed of computers has increased, and due to that, numerical integration methods and other computational methods have developed rapidly. Thus more accurate approximations to the filtering equations are designed.in some methods we can model and integrate the whole data with some computational resource [7].

In the rest of the paper the method and different filtering approach such as EKF, UKF, EKS and RTS smoother is applied to simulation [8], [9]. In these methods the set of particles distribution is updated iteratively and additional resampling step is used for removing the noise. And then some challenging path is examined with modulating the noise in path for evaluating our filters methods and finally estimation of probability and change of variance is considered in experimental simulation.

II. Method and Approach

In this section the overall formulation of different filters and noticeable point about their algorithm is expressed. First the overall algorithm of discrete-time KALMAN smoother or (RTS) is described. This method is used for smoothing of distribution with the following form in (1):

$$\mathrm{P}\,(x_k \parallel y_{1:t}) = N(x_k \parallel m_k, p_k) \tag{1}$$

In (1) the mean and covariance m_k, p_k are computed with (2)-(6):

$$M_{k+1}^{-} = A_k M_K \tag{2}$$

$$P_{k+1}^{-} = A_k P_K A_K^T + Q_K \tag{3}$$

$$C_K^{-} = P_K A_K^T [P_{K+1}^{-}]^{\wedge} - 1 \tag{4}$$

$$M_K^{-} = M_K + C_K [M_{K+1}^{\wedge} - M_{K+1}^{-}] \tag{5}$$

$$P_K^{\wedge} = P_K + C_K [P_{K+1}^{\wedge} - P_{K+1}^{-}] C_K^T \tag{6}$$

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m_k, p_k are the filter estimates for the state mean and state covariance on time step k. C_K is the smoother gain on time step k which tell how much the smoothed estimate should be corrected and $M^{\wedge}_{K+1}$ and P^{-}_{K+1} are the predicted state mean and state covariance on time step k+1. This coefficient is the same as the Kalman filter. The difference between this method and Kalman filter is that the recursion in filter moves forward and in smoother mode to backward.

The other method of filtering such as EKF has simple iterative format and the filtering model used in the EKF is in format of (7) and (8) with process and measurement noise.

$$X(k) = f(x(k-1), k-1) + q(k-1) \tag{7}$$

$$Y(k) = h(x(k), k) + r(k) \tag{8}$$

Then, the first and second order extended KALMAN filters approximate the distribution of state x_k by giving the observations y_1 to y_k. Other algorithms that are used in this paper are well reported in [6], [7] and we don't go to the detail of this algorithm but the structure of total filter is very similar and they developed equations for better representation of higher order nonlinearity in equation.

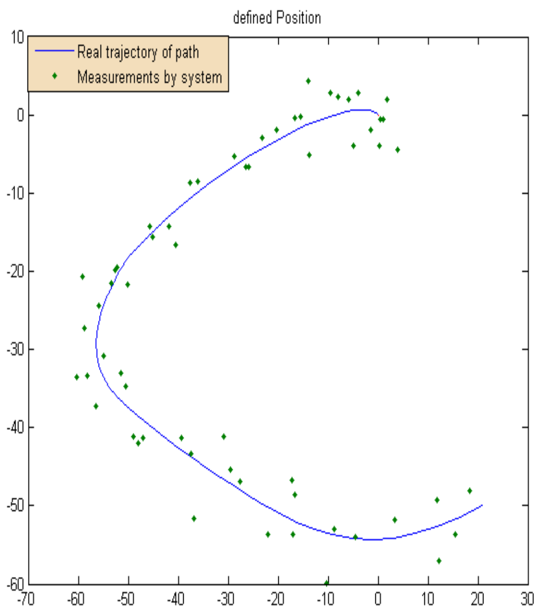

Fig. 1 The real position of the moving object and the simulated measurement

III. Simulation Results with KALMAN Filter and RTS Smoother

Firstly, in a simple case that an object is moved in 2D space with a sensor and their position on coordinates x and y are measured is simulated. Velocity and acceleration are the state of our dynamical system. In duration of motion a white noise process is inputted to object. So the acceleration of the object is perturbed and also the velocity is perturbed with a white noise disturbance. Its position from the sensor is observed and discrete-time state equation is applied to model and also the measurement has some variance. 80 time step is chosen for simulating the velocity and acceleration in this model. And also because of the variance in the measurement the difference between the KALMAN filter and smoother is clear in this simulation [10]-[12].

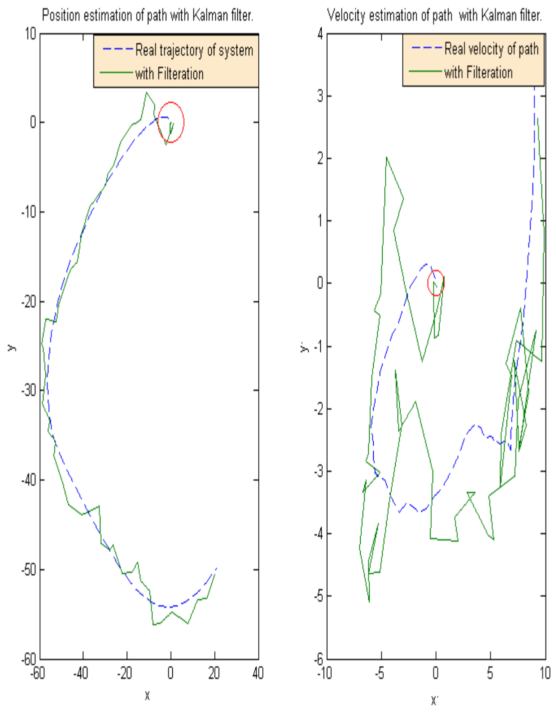

Fig. 2 The estimates for position and velocity of the moving object with KALMAN filter

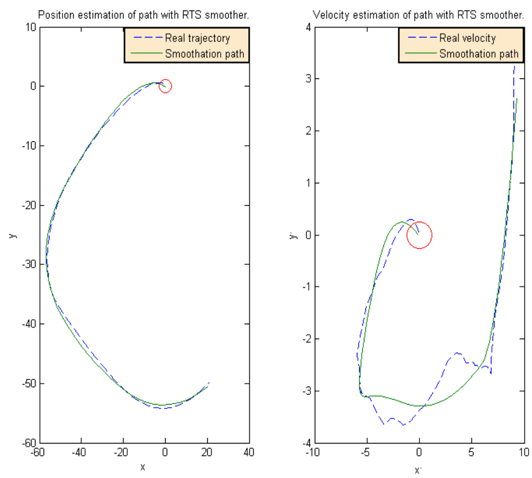

Fig. 3 The estimates for position and velocity of the moving object with RTS smoother

In Figs. 1 and 6 two different paths for object are plotted. Also, in Fig. 6 the variance of the measurement is higher from simulation one. Then the calculated estimated of position and velocity of object with KALMAN filter is plotted in Fig. 2.

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This case with applying RTS smoother is plotted in Fig. 3. Also, the second modeling is plotted in Figs. 7, 8.

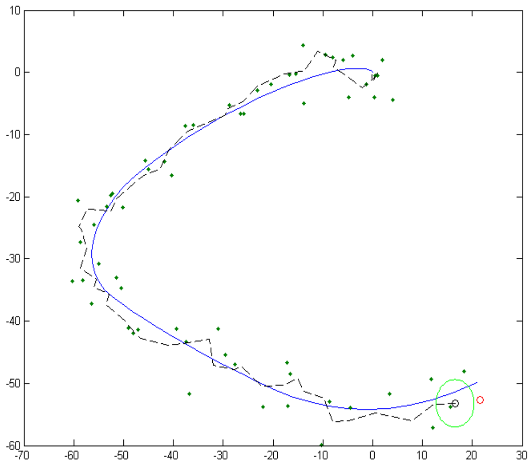

Fig. 4 Real trajectory tracking with filter KALMAN

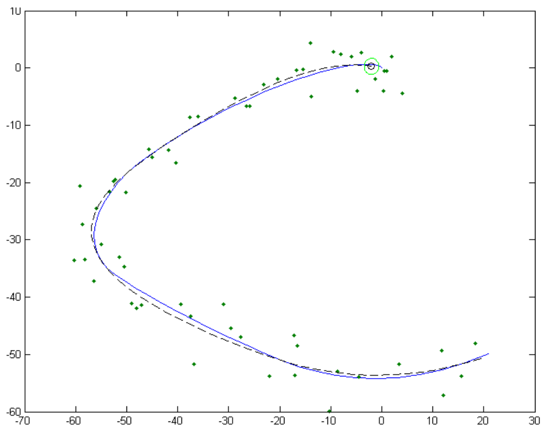

Fig. 5 Real trajectory tracking with RTS smoother

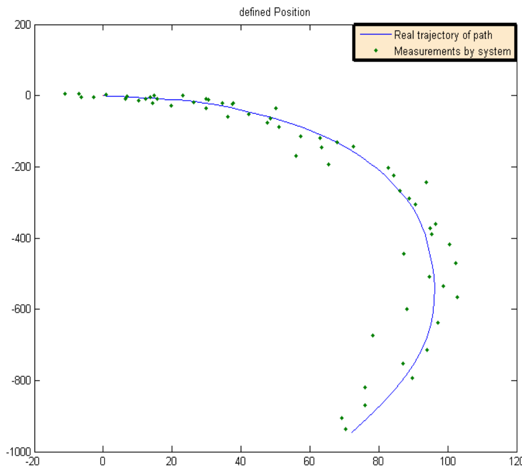

Fig. 6 The real position of the moving object and the simulated measurement with more noisy data

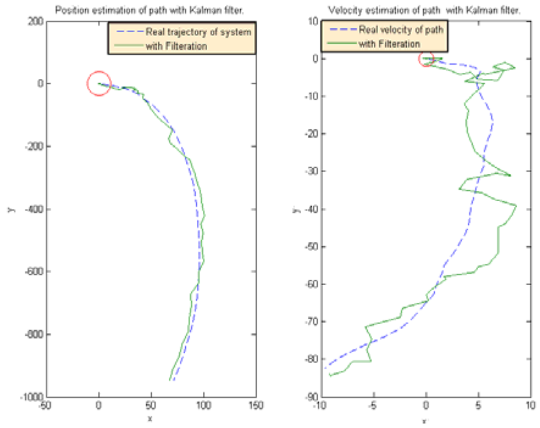

Fig. 7 The estimates for position and velocity of the moving object with KALMAN filter

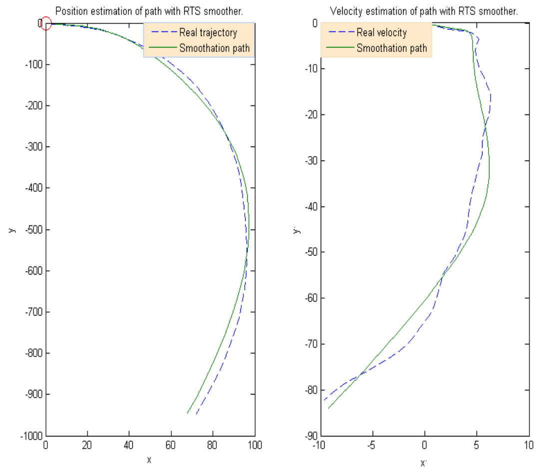

Fig. 8 The estimates for position and velocity of the moving object with RTS smoother

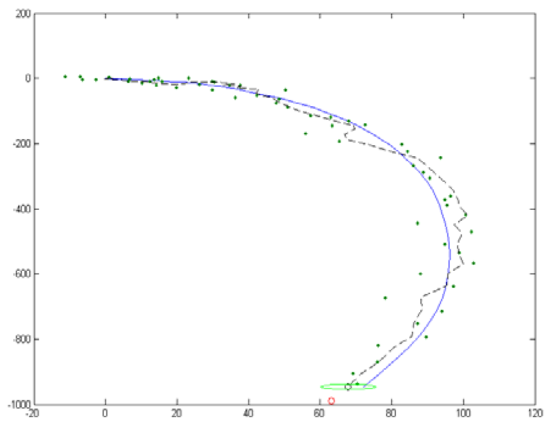

Fig. 9 Real trajectory tracking with filter KALMAN

It is evident that KALMAN filter has more variation along the path in both position and velocity but in velocity because of derivative and high variation it has more variation. Finally, real trajectory following in filter KALMAN is plotted in Figs.

4, 9. For first and second simulations and real trajectory for RTS smoother are plotted in Figs. 5 and 10, respectively.

An important note about this simulation is the shape of the path that has more effect from variance of measurement in variation of estimates of position and velocity. So, designing the path has important influence in designing the gains of filter.

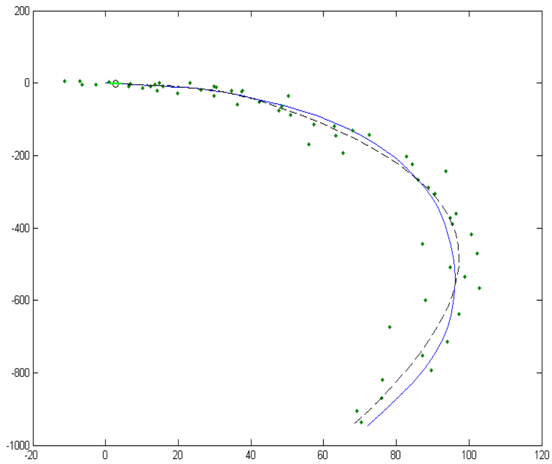

Fig. 10 Real trajectory tracking with RTS smoother

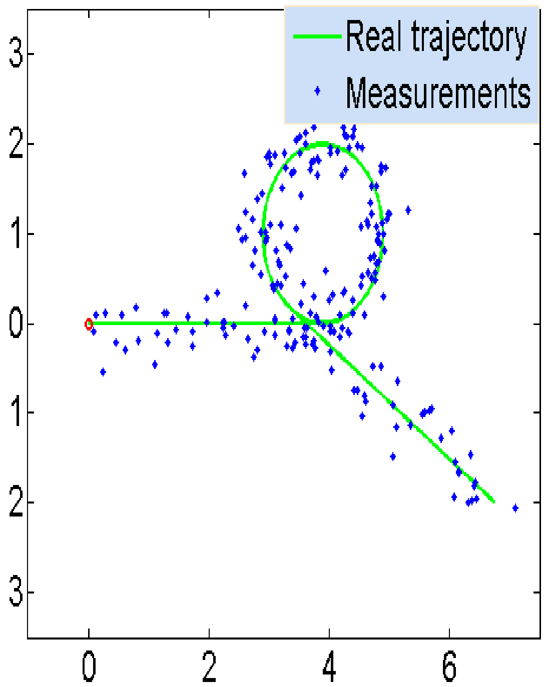

Fig. 11 Object trajectory and a sample of measurement

IV. Simulation Results and Comparison of Different Filtering Methods

In this section tracking a complex path with complex turning shape is simulated. As a simulation example the problem of tracking a target in two dimensional spaces executing a turn path with unknown and time-varying turn rate is considered. This is very typical setup in target tracking applications, and is useful for testing non-linear filters and smoothers [8]. An object that has sensor and move in 2D space is simulated then the first path is plotted in Fig. 1. Firstly, the performance of KALMAN filter for this tracking is simulated that it is plotted in Fig. 12. It is obvious that for achieving the best performance from KALMAN filter, both the dynamic model and stochastic measurement provided to filter must be accurate. And because the noise is inputted to measurement for this path and dynamics of path is difficult the tracking by filter KALMAN is not very well. And also the smoother in this condition cannot be applied such as RTS and this tracking with RTS and EKF is plotted in Fig. 13. On the other hand, in EKF filter because of ignoring the high order of nonlinearities in their modeling it is obvious form the figure it has the more variation in turning shape of the path. Next Position estimates with EKS and UKF are plotted. Like Taylor based approximation for EKF filter, UKF use Gaussian approximation to the joint distribution for random variables.

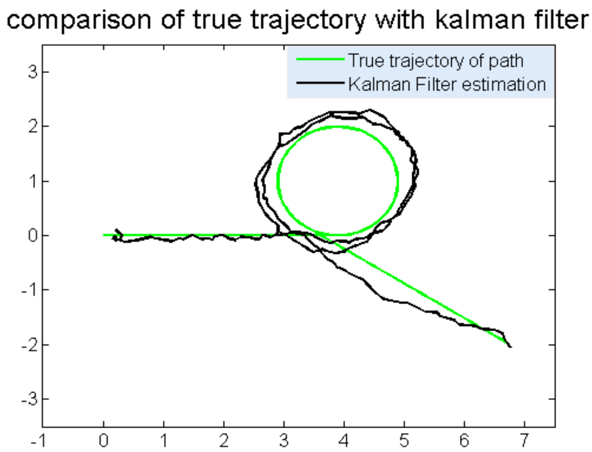

Fig. 12 Position estimates with Kalman filter

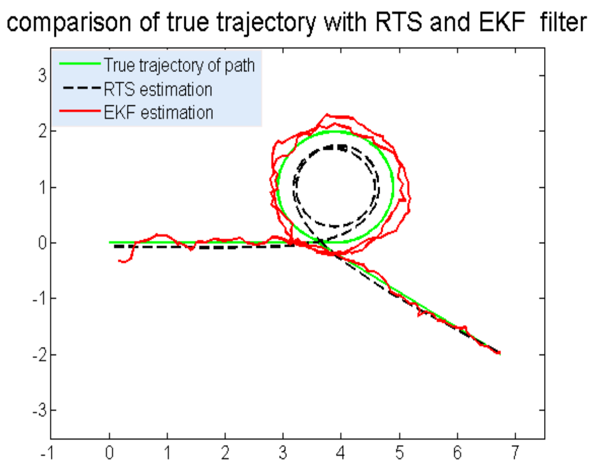

Fig. 13 Position estimates with RTS and EKF

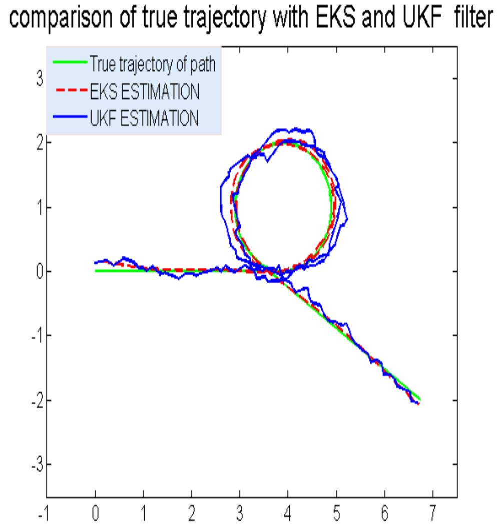

Fig. 14 Position estimates with EKS and UKF

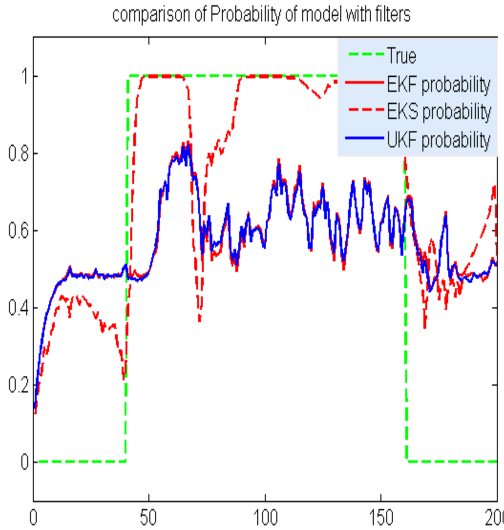

Fig. 15 Estimates for probability and comparison with different filters

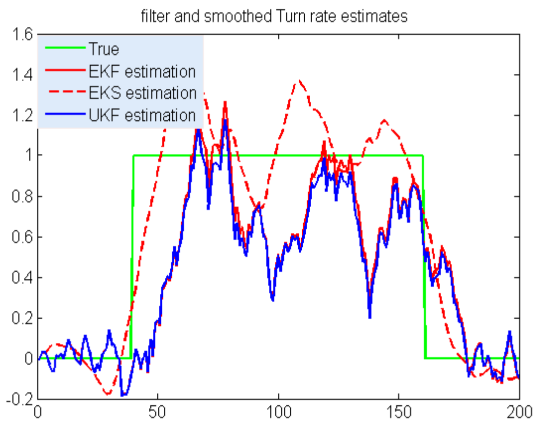

Fig. 16 Estimates of the turn rate parameter and comparison with different filters

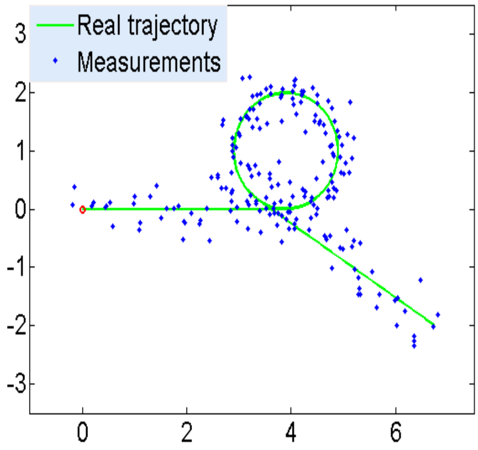

Fig. 17 Object trajectory and a sample of measurement

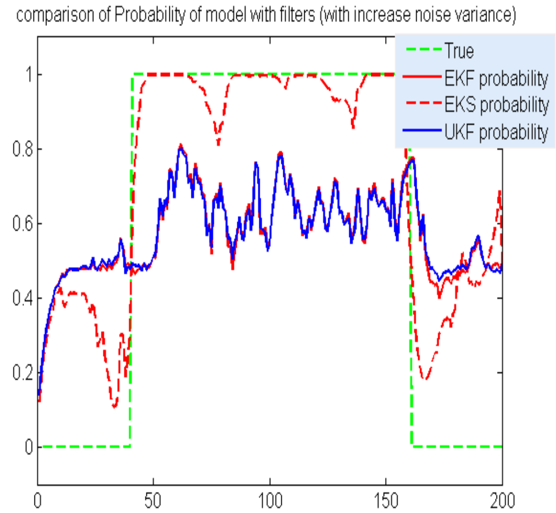

Fig. 18 Estimates for probability and comparison with different filters

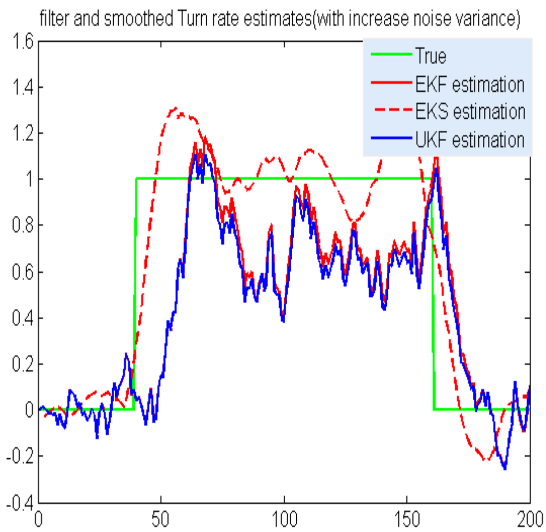

Fig. 19 Estimates of the turn rate parameter and comparison with different filters

In this method a fixed number of sigma points for evaluating the desired distribution of variables are chosen and then these sigma points for estimating transformed variable from them and advantage of this method from EKF is used that for better capturing higher order nonlinear terms. And in this figure EKS has best performance between the filters and also EKF has good application in many nonlinear applications and UKF capture first and second order terms of the nonlinear system in respect to the EKF.

In Figs. 15 and 16 estimation of probability and the turn rate parameter are plotted. In these figures the probability variation of true with different filters is shown. In these figures it is obvious that in turn rate probability the filters has more variation and with adding the curvature of path this variation is increased and also with adding the noise in measurement that the results is plotted in Fig. 17, and variation of probability of filters is getting so random that this simulation are plotted in Figs. 18 and 19. These results imply the limitation in each filtering method. EKF method has better fitting characteristics and these simulations prove the dynamics of model is more important from the measurement noise for influencing the performance of filters.

V. Conclusion

In this paper we have first shown how different algorithm for filtering and smoothing is compared in challenging path then we simulated result for smoother and finally the performance of different filtering and accuracy of those methods for tracking problem is explained and their performances are compared. This article proves that the variance in some path has more effect than other simple path in simulation results.

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