

The Estimate Rate of Permanent Flow of a Liquid Simulating Blood by Doppler Effect

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Abstract—To improve the characterization of blood flows, we propose a method which makes it possible to use the spectral analysis of the Doppler signals. Our calculation induces a reasonable approximation, the error made on estimated speed reflects the fact that speed depends on the flow conditions as well as on measurement parameters like the bore and the volume flow rate. The estimate of the Doppler signal frequency enables us to determine the maximum Doppler frequency F_{dmax} as well as the maximum flow speed. The results show that the difference between the estimated frequencies (F_{de}) and the Doppler frequencies (F_d) is small, this variation tends to zero for important θ angles and it is proportional to the diameter D . The description of the speed of friction and the coefficient of friction justify the error rate obtained.

Keywords—Doppler frequency, Doppler spectrum, estimate speed, permanent flow.

I. INTRODUCTION

THE content of frequency of the Doppler spectrum determined by several factors [1] the characteristics of blood dispersion, the range of speeds present in the measurement volume and the spectral evaluation method used. The spectrum of the signal obtained in Doppler ultrasonic echocardiography is a true image of the local distribution rates of the blood flow; it represents a major element in the medical diagnosis of cardiovascular pathologies [2] such as the lesions caused by arteriosclerosis [3].

II. EXPERIMENT AND METHOD

A. The model of the Doppler signal [4], [5]

The Doppler signal is the sum of the contribution of all the diffusers passing by the volume of measurement [6]; the exact movement of each particle of the fluid is unknown and non measurable; it is impossible to simulate the Doppler signal without simplifying the problem. To reduce the complexity of the problem, we make the following assumptions [6]:

- the speed vector of each retro

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- diffuser is parallel to the walls of the blood vessels.
- the blood vessel is a rigid cylindrical tube, an assumption which eliminates the vibrations of the walls M.D.

- the flow is permanent, which involves that the amplitude of the Doppler signal as well as the speed are constant while the particle passes through the volume of measurement.

The designed model simulates blood circulation by means of latex pipes of different diameters (4, 6, 8) mm, the circulating fluid is an aqueous solution containing micro - spheres with a diameter of $10\mu m$, which is close to the size of the blood cells. The density of the solution is 1 g/cm^3 and is measured at ambient temperature. A flow meter was used to measure the flow and to check the states of steady flow (regular). An adapted positioning system was used to fix the probe plunged in water and to change with precision the θ angle.

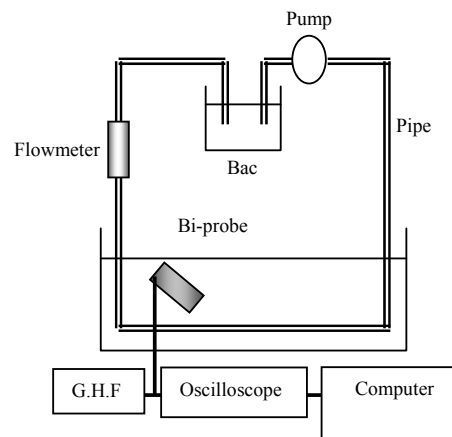


Fig. 1 Diagram of the simulation experiment

This bi-probe has a $Z=38$ impedance for a frequency of emission $F_e=6.5\text{MHz}$. The digital oscilloscope makes it possible to visualize the reflected signal and to sample it; the spectra of amplitude are calculated by means of the FFT (Fast Fourier Transform).

B. Characterization of the modes of flow

The dispersion of the fluid is directly related to its hydrodynamic properties like viscosity, the density and the nature of the flow [21]. This last characteristic is very important: our study is based on the knowledge of the nature of the flow which enables us to determine the circulatory speed of blood. The modes of flow are clearly distinguished

according to the Reynolds number, Re , which makes it possible to define the laminar or turbulent character of a flow. For our application, we supposed that blood is incompressible and viscous; thus the blood flow is considered to be laminar for values of $de\ Re < 2500$ and turbulent for $Re > 2500$.

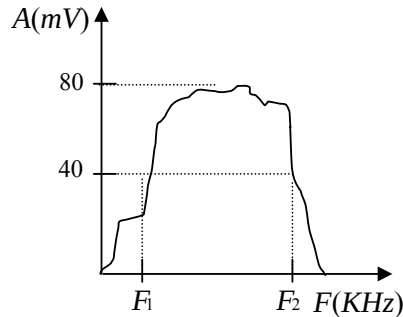


Fig. 2 Spectrum of Doppler amplitude in laminar mode

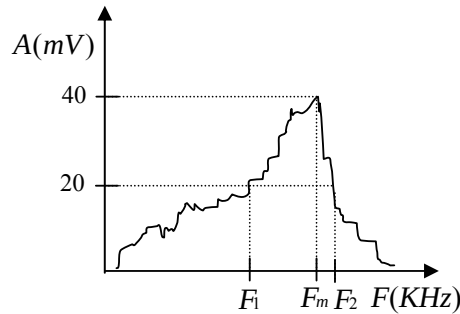


Fig. 3 Spectrum of Doppler amplitude in turbulent mode

The laminar flow is characterized by a more or less wide flat profile spectrum the higher part of the spectrum is a stiff side. The spectrum of a turbulent flow is recognizable by its profile which is situated towards the higher part, which indicates maximum spectral amplitude. The higher part of the spectrum is more or less regular and it goes upwards with the increase of Re . We can infer from it that even in the absence of a measuring device, only the shape of the spectrum, when it is not ambiguous, is sufficient to recognize the mode of the flow.

III. ESTIMATE OF THE DOPPLER FREQUENCY

The bi-probe used allows the emission and the reception of the ultrasonic beam, hence the Doppler relation:

$$F_d = \frac{2F_e V \cos(\theta)}{C} \quad (1)$$

where F_d is the Doppler frequency, F_e is the frequency of emission, $V \cos(\theta)$ is the apparent speed of the liquid, θ is the angle formed between the ultrasonic beam and the direction of flow, C is the speed of the considered environment.

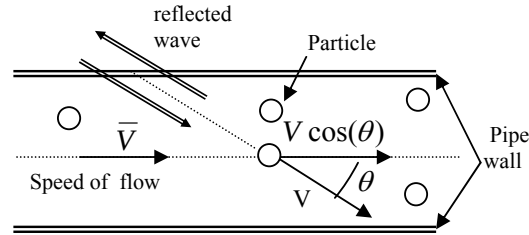


Fig. 4 Retro-diffusion of the ultrasonic wave and movement of the particle

The Doppler frequency is directly related to the apparent speed of the fluid $V \cos(\theta)$; and as the sum of the spectrum gives us the variation of the signal amplitude in the frequency field, we proceed with the estimate of the Doppler frequency.

The frequencies characterizing the spectrum are:

F_m which corresponds to the maximum amplitude (A_m) of the spectrum, F_1 and F_2 correspond to $\frac{A_m}{2}$ and $F_2 > F_1$.

The throughput volume is given by the relation:

$$Q_v = \bar{V} S \quad (2)$$

where Q_v is the throughput volume, $\bar{V}$ is the flow speed and S the section of the pipe.

$$\bar{V} = \frac{Re\mu}{D} \quad (3)$$

where μ is the kinematic viscosity of the pipe and D is pipe diameter.

A. The laminar flow

For a laminar flow in a pipe, the speed profile in a cross section and a longitudinal plan passing by the axis of the pipe has a parabolic form [7]

$$V(r) = V_{\max} \left(1 - \frac{r^2}{R^2}\right) \quad (4)$$

r : ray of the speed profile and R the ray of the pipe.

$V_{\max}$ the maximum speed of the flow.

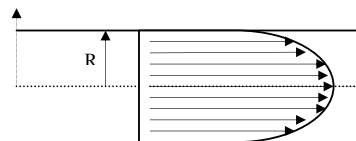


Fig. 5 Profile laminar speed mode

The mean velocity [7]:

$$Q_v(r) = S(r) V(r) : \text{the volume throughput.}$$

$$S(r) = 2\pi r : \text{the variable section of the pipe.}$$

$$Q_v(r) = (2\pi r) \left(V_{\max} \left(1 - \frac{r^2}{R^2}\right) \right)$$

$$Q_v = \int_0^R 2\pi r V_{\max} \left(1 - \frac{r^2}{R^2}\right) dr = \frac{1}{2} \pi V_{\max} R^2$$

According to the relation (2)

$$\bar{V} = \frac{Q_v}{S} = \frac{\frac{1}{2} \pi V_{\max} R^2}{\pi R^2}$$

$$\bar{V} = \frac{V_{\max}}{2}$$

B. The turbulent flow

In the turbulent flow, the profile takes a much more flattened form than in the laminar flow [7].

$$V(r) = V_{\max} \left(1 - \frac{r}{R}\right)^n$$

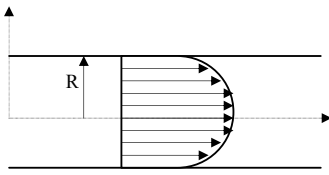


Fig. 6 Profile turbulent speed mode

The mean velocity [1]:

$$\bar{V} = \frac{2n^2}{(n+1)(2n+1)} V_{\max} \quad (7)$$

In this precise case $n=6$

$$\bar{V} = \frac{V_{\max}}{1.26} \quad (8)$$

$$V_{\max} = 2\bar{V} \quad ; \text{ in laminar flow.}$$

$$V_{\max} = 1.26\bar{V} \quad ; \text{ in turbulent flow}$$

C. Constant angle and variable flow

$$\theta = 60^\circ$$

We consider $\bar{V} = V \cos(\theta)$, and according to (1)

$$F_d = \frac{2F_e}{C} \bar{V} \quad (9)$$

and according to (2), we have :

$$F_d = \frac{2F_e}{SC} Q_v \quad (10)$$

Expression which we can write: $F_d = KQ_v$

where K is a constant of manipulation expressed in $\text{Hz} \cdot \text{min} \cdot \text{cm}^{-3}$

We suppose that : $F_2 = F_d$ then

$$F_2 = K_{L1} \cdot Q_{vl} \quad ; \text{ in laminar flow.}$$

$$F_2 = K_{T1} \cdot Q_{vt} \quad ; \text{ in turbulent flow}$$

K_{L1} and K_{T1} are calculated starting from the mean of the ratios $\frac{F_{2i}}{Q_{vi}}$ of the experimental points.

(5) The experimental values of the fluid speed $V \cos(\theta)$ are deduced from (1)

$$V \cos(\theta) = \frac{C}{2F_e} F_{di} \quad (11)$$

(6) F_{di} being the Doppler frequency estimated for each spectrum having an index i .

For the whole data bank, we determine the Doppler frequency F_d (9) and the F_2 frequency (Fig.2).

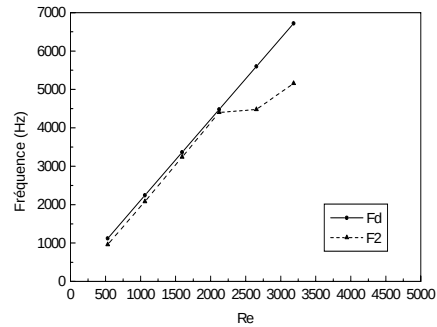


Fig.7 Curves of the frequencies (F_2, F_d) constant angle and variable flow, $D = 4\text{mm}$

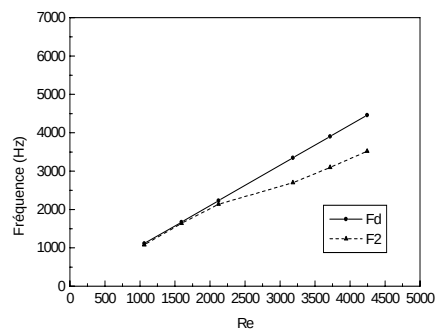


Fig.8 Curves of the frequencies (F_2, F_d) constant angle and variable flow, $D = 8\text{mm}$.

Thus, according to figures 7, and 8, the variance between F_2 and F_d is small in laminar flow ($Re < 2500$). In turbulent flow, this variance is more important and it reaches an average of about 8.7 %.

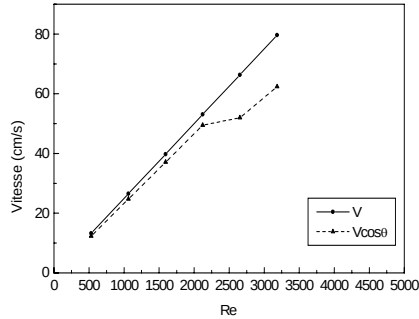


Fig.9 Curves speeds ($\bar{V}$, $V \cos \theta$) with constant angle and variable flow, $D=4\text{mm}$

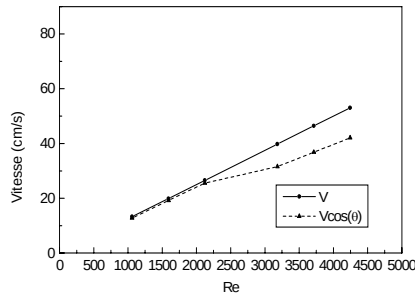


Fig.10 Curves speeds ($\bar{V}$, $V \cos \theta$) with constant angle and variable flow, $D=8\text{mm}$

Knowing the angle (θ), the figure 9 and 10 show that apparent speed varies parallel to the flow speed keeping a more or less constant variance. Our calculation induces a reasonable approximation, the error made on estimated speed reflects the fact that speed depends on of the flow conditions as well as on measurement parameters like the bore and the volume flow rate.

D. Constant flow and variable angle

If we admit that the particles which circulate with a maximum speed $V_{\max}$ give a frequency $F_{d\max}$, then :

$$F_{d\max} = \frac{2F_e V_{\max} \cos(\theta)}{C} \quad (12)$$

According to the relations (4) et (5), we can write :

$$\text{-Laminar flow : } F_{d\max} = \frac{4F_e \bar{V}}{C} \cos(\theta) \quad (13)$$

$$\text{-Turbulent flow : } F_{d\max} = \frac{2.52F_e \bar{V}}{C} \cos(\theta) \quad (14)$$

formulas which we can express :

$$F_{d\max} = K_{L2} \cos(\theta) \quad \text{and} \quad F_{d\max} = K_{T2} \cos(\theta)$$

where K_{L2} and K_{T2} are two constants of manipulation expressed in Hz.

We suppose that :

$$F_2 = F_{d\max} \quad \text{in laminar flow.}$$

$$F_m = F_{d\max} \quad \text{in turbulent flow.}$$

Then :

$$F_2 = K_{L2} \cos(\theta) \quad \text{and} \quad F_m = K_{T2} \cos(\theta)$$

K_{L2} and K_{T2} are calculated starting from the mean of the ratios $\frac{F_{2i}}{\cos(\theta_i)}$ and $\frac{F_{mi}}{\cos(\theta_i)}$, the experimental points for the laminar flow and turbulent flow respectively.

The experimental values of the speed are deduced from (1)

$$V \cos(\theta) = \frac{C}{2F_e} F_{di} \quad (15)$$

F_{di} being the Doppler frequency estimated for each spectrum having an i index. The concept of maximum speed is introduced given its importance for the medical diagnosis where the pulsation of the flow plays a very important part; in cardiology, knowing the degree of arterial stenosis is required to determine the need for the carotid artery surgery. The degree of stenosis is directly related to the maximum speed of blood, the latter being usually estimated starting from measurements of the Doppler spectrum [4].

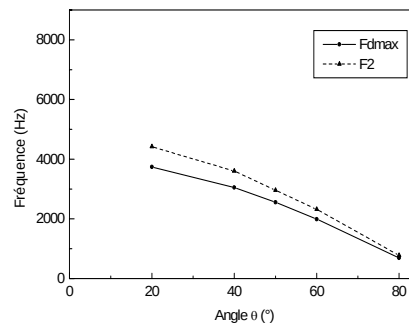


Fig.11 Curves of the frequencies laminaire ($F_{d\max}$, F_m) in constant flow and variable angle, $Re=1415$, $D=6\text{mm}$.

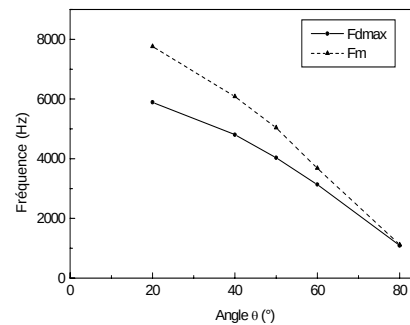


Fig.12 Curves of the turbulent frequencies ($F_{d\max}$, F_m) with constant flow and variable angle, $Re=3537$, $D=6\text{mm}$

According to figure 11, the F_2 value is close to $F_{d\max}$ in laminar flow. The error is of about 5.6%. For a turbulent flow (fig. 12), $F_{d\max}$ approaches F_m as θ increases.

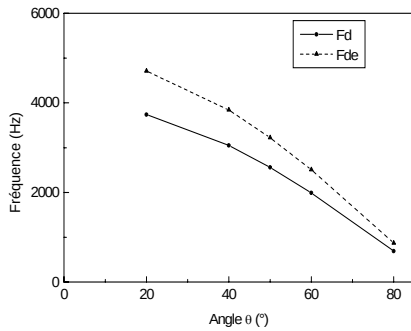


Fig.13 Curves of the laminar frequencies (F_d , F_{de}) with constant flow and variable angle, $Re=2829$, $D=6mm$

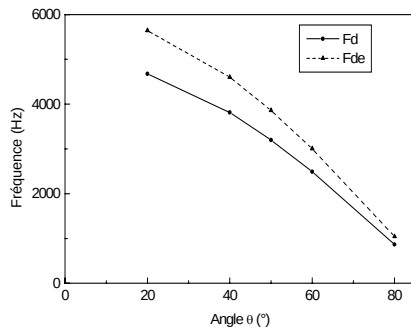


Fig.14 Curves of the laminar frequencies (F_d , F_{de}) with constant flow and variable angle, $Re=3537$, $D=6mm$

Figures 13 and 14 show that the deviance between the estimated frequencies (F_{de}) and the Doppler frequencies (F_d) is small, it converges towards zero for the important θ angles and proportionally to the pipe diameter (D).

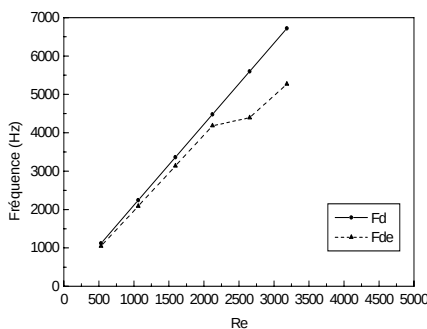


Fig.15 Curves of the frequencies (F_d , F_{de}) with constant angle and variable flow, $D=4mm$.

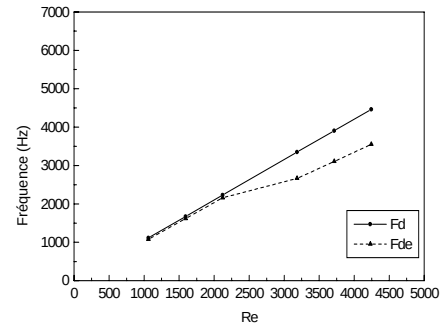


Fig.16 Curves of the frequencies (F_d , F_{de}) with constant angle and variable flow, $D=8mm$.

The value of the estimated Doppler frequency F_{de} enables us to evaluate the choice previously made ($F_2=F_d$). According to figures 15 and 16, the value of the estimated Doppler frequency is closer to the value of the Doppler frequency as the Reynolds number (Re) is small, beyond 2500, the difference is all the more important as the diameter is small.

IV. THE INFLUENCE OF FRICTION SPEEDS TO THE WALLS

According to the results previously obtained concerning the speed, we noticed that the error is important in turbulent flow. It is due, firstly, to the randomness of the movement of the liquid where the fluctuations tend to equalize speeds, which gives the impression that the movement is uniform and, secondly, to frictions of the particles to the walls along the pipe.

The relation which gives the speed of friction according to the mean velocity is [7]:

$$\text{-laminar flow: } V_f = 2(\bar{V})^{\frac{1}{2}} \left(\frac{\nu}{R} \right)^{\frac{1}{2}} \quad (16)$$

$$\text{-turbulent flow: } V_f = 0.182(\bar{V})^{\frac{7}{8}} \left(\frac{\nu}{R} \right)^{\frac{1}{8}} \quad (17)$$

where, $\bar{V}$ is the mean flow velocity, ν is the kinematic viscosity of the liquid and R the ray of the pipe.

V. COEFFICIENT OF FRICTION

According to the speed values V_f [7], frictions are present along the pipe, they decrease with the flow since the speed of the flow increases. These frictions are represented by a parameter without dimension C_f which connects the mean velocity to the speed of friction.

$$\text{laminar flow: } C_f = 2 \left(\frac{V_f}{V_{\max}} \right)^2 = 0.5 \left(\frac{V_f}{V} \right)^2 \quad (18)$$

$$\text{turbulent flow: } C_f = 2 \left(\frac{V_f}{V_{\max}} \right)^2 = 1.25 \left(\frac{V_f}{V} \right)^2 \quad (19)$$

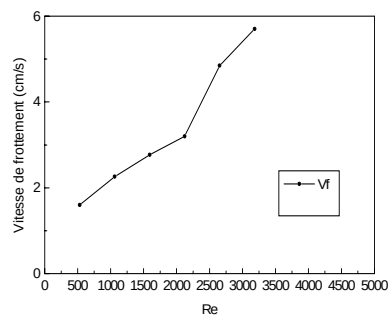


Fig.17 Curves of speeds of friction to constant angle and variable flow, $D=4\text{mm} = 60^\circ$

The calculation of the speed is made for three diameter values, and figure 17 shows the curve of the parietal speed according to the Reynolds number (Re).

We note that the speed V_f

- Increases according to the Reynolds number, thus proportionally to the volume throughput (Q_v).
- Proportional and definitely lower than the mean velocity.
- The increase of V_f induces errors in the calculation of the speed; it is the case of blood flow where the internal walls of the veins are rough and of irregular form. The results obtained partially justify the error rate made on turbulent speed.

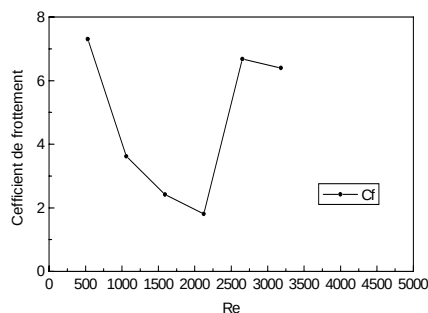


Fig.18 Curves of the coefficient of friction to constant angle and variable flow, $D=4\text{mm} = 60^\circ$

According to figure 18, the coefficient of friction decreases rapidly in laminar flow, and it decreases slightly in turbulent flow. The passage from the laminar flow to the turbulent flow is marked by an abrupt increase of C_f , which represents the particle fluctuation speeds, the beginning of turbulence and the randomness of the movement. The random movement of the particles causes serious errors in the calculation of the speed flow, whichever method of estimation we may use.

VI. CONCLUSION

The results which we obtained confirm the antagonism: Doppler - medical imagery; one cannot have a good precision of the frequency and of the speed at the same time, thus in

order to acquire a good Doppler signal, it is necessary to give up the clearness of the anatomical image. Regarding the constant flow and variable angle, we can point out the influence of some measurement parameters. For example for this application, the variation of θ , influences directly and simultaneously the frequency and the speed. The difficulty lies in the presence of two unknown parameters in one equation, which are the frequency and speed; these two parameters are interdependent and need to be calculated one according to the other. That proves the inherent problems of Doppler measurements. The speed estimation method which is suggested not only makes it possible to find the value of the speed, but also to extract useful information from the spectrum of the Doppler signal: the Doppler frequency of the (F_d) signal, the maximum Doppler frequency ($F_{d\max}$) which gives the highest speed that flow can reach. Knowing the latter [8] is very important since it is known that the degree of stenosis is directly related to the maximum speed of blood according to the northern - American trial of symptomatic carotid endarterectomy [9] and the European test [10] of arterial surgery of the carotid.

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NOMENCLATURE

F_d	Doppler frequency
F_{de}	Estimated Doppler frequency
$F_{d\max}$	Maximum Doppler frequency
F_m	Frequency of the maximum amplitude
K	Constant of manipulation
K_{L1}	Constant of manipulation in laminar flow
K_{T1}	Constant of manipulation in turbulent flow
F_e	Emission frequency
Q_v	Throughput volume
θ	Incidence angle
μ	Kinematic viscosity of the pipe
S	The pipe section
D	The pipe diameter
V_f	The speed of friction
V	Particle speed
ν	Kinematic viscosity of the liquid
R	The ray of the pipe
r	Ray of speed profile
Re	Renolds number
C	Speed in the medium
$V_{\max}$	Maximum speed of the flow
$\overline{V}$	The flow speed