

The Effect of Impact on the Knee Joint Due to the Shocks during Double Impact Phase of Gait Cycle

Jobin Varghese, V. M. Akhil, P. K. Rajendrakumar, K. S. Sivanandan

Abstract—The major contributor to the human locomotion is the knee flexion and extension. During heel strike, a huge amount of energy is transmitted through the leg towards knee joint, which in fact is damped at heel and leg muscles. During high shocks, although it is damped to a certain extent, the balance force transmits towards knee joint which could damage the knee. Due to the vital function of the knee joint, it should be protected against damage due to additional load acting on it. This work concentrates on the development of spring mass damper system which exactly replicates the stiffness at the heel and muscles and the objective function is optimized to minimize the force acting at the knee joint. Further, the data collected using force plate are put into the model to verify its integrity and are found to be in good agreement.

Keywords—Spring, mass, damper, impact, knee joint.

I. INTRODUCTION

KNEE joint is one of the most important and strongest joint among the below hip joints. It also has an important function in lowering and raising the upper body towards and away from the ground. During stiff-legged run, the time for colliding with the ground is short, and as a result, the force at the knee is higher [1]. Even though the force is higher at knee, a complaint bent knee helps to reduce the impact at the knee joint by increasing the time of impact. The ground reaction forces during heel strike transmits shocks through bones to the knee joint and results are the injuries like shin splints and fractures due to stress. [2], [3]. Stiffness of the body gives way for potentially dangerous impact shocks to transmit through the body [4], [5]. The knee joint makes 20-degree flexion while foot strikes on the ground and 30-degree while running [6] to absorb the shock. Large extended knee postures cause greater force at the knee joint [7].

The spring mass system gives good information to evaluate the running and walking process [8]. Landing period of running and hopping can be modeled as single linear springs [9]. Bouncing like a spring occurs during running, hopping and trotting [10]-[16]. This creates a ground reaction force and it can be calculated using force plates [1]. Hence, spring damper modeling best suits for studying human actions and characteristics.

We have studied the force acting at the knee joint by modeling a mass damper system with a forced excitation. The

prime objective is to minimize the maximum force acting at the knee as a result of shocks produced during double impact.

II. METHOD

Double impact phases of a gait cycle during climbing a hump can be modeled using a spring mass damper system as shown in Fig. 1 where, M_b is the weight of the thigh and half the mass of the upper body, and is acting on the knee joint. K_L and α_L are the stiffness and damping coefficients of the leg muscles. M_L is the mass of the leg, K_H is the stiffness of the heel, and F is the vertical reaction force acting on the heel from the ground. Half of the total body weight and thigh weight is acting on the knee joint. Reaction force is partially damped at heel, and the balance force is transmitted to the knee joint through leg muscles. Leg muscles can act as spring mass damper system, and hence, this force is again damped and further force is transmitted to the knee joint. This force can cause damage to the knee joint, if the magnitude is higher than that of the bearing capacity of the knee joint.

$$F_1 = K_H \times d_1 \quad (1)$$

$$F_2 = K_L \times d_2 \quad (2)$$

$$F_3 = \alpha_L \times \dot{d}_2 \quad (3)$$

Equations (1), (2), and (3) define the forces in spring with stiffness K_H and K_L , and in damper with damping coefficient α_L .

$$\ddot{q}_1 = (F_2 + F_3 - F_1 - F_R) / M_L \quad (4)$$

$$\ddot{q}_2 = -(F_2 + F_3 - F_B) / M_b \quad (5)$$

F_R is the ground reaction force, and F_B is body weight

$$d_1 = q_1 - f(t) \quad (6)$$

$$d_2 = q_2 - q_1 \quad (7)$$

$$\dot{d}_2 = \dot{q}_2 - \dot{q}_1 \quad (8)$$

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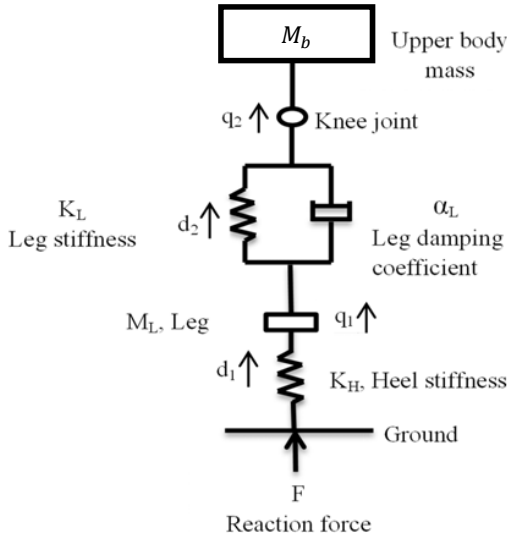


Fig. 1 A spring mass damper model of heel and leg muscles

where d_1 and d_2 are the displacements of springs K_H and K_L due to heel strike reaction force $F_R(t)$, $f(t)$ is the additional displacement of the spring K_H due to the hump on the path, q_1 and q_2 are the displacements of the leg mass M_L and body mass M_b , and \dot{q}_1 and \dot{q}_2 are the velocities of the M_L and the M_b .

Substituting (6)-(8) in (1)-(3)

$$F_1 = K_H \times (q_1 - f(t)) \quad (9)$$

$$F_2 = K_L \times (q_2 - q_1) \quad (10)$$

$$F_3 = \alpha_L \times (\dot{q}_2 - \dot{q}_1) \quad (11)$$

Substituting (9)-(11) in (4) and (5)

$$\ddot{q}_1 = \left(\begin{array}{c} K_L \times (q_2 - q_1) \\ + \alpha_L \times (\dot{q}_2 - \dot{q}_1) \\ - K_H \times (q_1 - f(t)) - F_R \end{array} \right) / M_L \quad (12)$$

$$\ddot{q}_2 = - \left(\begin{array}{c} K_L \times (q_2 - q_1) \\ + \alpha_L \times (\dot{q}_2 - \dot{q}_1) - F_B \end{array} \right) / M_b \quad (13)$$

where \ddot{q}_1 and \ddot{q}_2 are the accelerations of masses M_L and M_b . Rearranging (12) and (13)

$$\left(\begin{array}{c} -M_L \ddot{q}_1 + \alpha_L \times (\dot{q}_2 - \dot{q}_1) \\ -q_1(K_L + K_H) \\ + K_L q_2 + K_H f(t) \end{array} \right) = F_R \quad (14)$$

$$\left(\begin{array}{c} M_b \ddot{q}_2 + \alpha_L \times (\dot{q}_2 - \dot{q}_1) \\ + K_L (q_2 - q_1) \end{array} \right) = F_B \quad (15)$$

Differential equations (14) and (15) are formulated from (12) and (13) with variables q_1 and q_2 .

III. EXCITING FORCE

Force acting at the heel can also be defined as a sine function

$$F(t) = F_R \sin\left(\frac{\pi t}{T}\right) \quad \text{or} \quad F_R \cos(\omega t) \quad (16)$$

where F_R is the vertical reaction force at the heel. Hump can be defined as a sine wave.

$$f(t) = A \sin\left(\frac{\pi t}{T}\right) \quad \text{or} \quad A \sin\left(\frac{\pi t}{T}\right) \quad (17)$$

We have considered a flat surface $f(t) = 0$ by making amplitude $A=0$. Equations (14) and (15) resemble the spring mass model with mass M , damping coefficient C , spring constant K , acceleration \ddot{x} , velocity \dot{x} , displacement x , and excitation force F .

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F_R \cos(\omega t) \quad (18)$$

Converting (18) into general format, which is in terms of damping ratio ζ , natural circular frequency ω_n , and f is the vertical reaction force

$$\ddot{q}(t) + 2\zeta\omega_n\dot{q}(t) + \omega_n^2 q(t) = F_1(t) \quad (19)$$

where,

$$\omega_n = \sqrt{\frac{K}{M}}, \quad F_1(t) = \frac{F_R \cos(\omega t)}{M}, \quad \zeta = \frac{C}{2\sqrt{KM}}$$

By comparing (15) with (18), $M = M_b$, $C = \alpha_L$, $K = K_L$ are obtained. To solve the differential equation in (19), consider (20)

$$q = X \cos(\omega t - \theta) \quad (20)$$

$$q = A_s \cos(\omega t) + B_s \sin(\omega t) \quad (21)$$

where,

$$A_s = X \cos \theta, \quad B_s = X \sin \theta,$$

$$X = \sqrt{A_s^2 + B_s^2}, \quad \theta = \tan^{-1}\left(\frac{B_s}{A_s}\right) \quad (22)$$

Differentiating (21) with respect to time for calculating velocity and acceleration

$$\dot{q}(t) = -\omega A_s \sin(\omega t) + \omega B_s \cos(\omega t) \quad (23)$$

$$\ddot{q}(t) = -\omega^2 A_s \cos(\omega t) - \omega^2 B_s \sin(\omega t) \quad (24)$$

Substituting (23) and (24) in (19) and re-arranging the terms

$$0 = \left(-\omega^2 A_s + 2\zeta\omega_n\omega B_s + \omega_n^2 A_s - F_1(t) \right) \cos(\omega t) \\ + \left(-\omega^2 B_s - 2\zeta\omega_n\omega A_s + \omega_n^2 B_s \right) \sin(\omega t) \quad (25)$$

Equation (25) must be valid for all values of t . When $t=0$ and $t = \frac{\pi}{2\omega}$, (25) simplifies to

$$(\omega_n^2 - \omega^2)A_s + 2\zeta\omega_n\omega B_s = F_1(t) \quad (26)$$

$$(-2\zeta\omega_n\omega)A_s + (\omega_n^2 - \omega^2)B_s = 0 \quad (27)$$

Solving (26) and (27)

$$A_s = \frac{(\omega_n^2 - \omega^2)F_1(t)}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} \quad (28)$$

$$B_s = \frac{2\zeta\omega_n\omega F_1(t)}{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2} \quad (29)$$

Substituting (28) and (29) in (20) and also solving $X = \sqrt{A_s^2 + B_s^2}$ for q in (20), we get

$$q = \frac{F_1(t)}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \cdot \cos \left[\omega t - \tan^{-1} \left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} \right) \right] \quad (30)$$

General solution of the differential equation (15) is (30) with $M, K,$ and α .

Table I has the parameters of the leg. Since there is no damper, $C=0$ and hence $\zeta = 0$. Stiffness of the heel is 140 kN/m.

Body part	Heel	Leg	Upper body
Stiffness	140 kN/m	200 kN/m	-
Damping coefficient	-	920 Ns/m	-
Leg mass	-	0.0465 W	0.422W

W is the total weight of the human body. Heel is modeled as spring alone and leg as spring mass damper system. The values are extracted from [17], [18].

Maximum heel displacement during heel strike is 4.07 mm, and the average value is 2.07 mm [19]. The vibration at heel after strike follows a sine wave path with a frequency of 0.1 to 70 HZ and thus $2.07 = d_1 \sin(\omega_1 t)$

$$\omega_1 = \sin^{-1} \left(\frac{(4.07)}{d_1} \right) / \left(\frac{1}{70} \right) \quad (31)$$

Time period of excitation of mass damper system can be found out by dividing distance (q_1) with velocity \dot{q}_1 .

$$q_1 = \frac{F_R}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}} \cdot \cos \left[\omega t - \tan^{-1} \left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} \right) \right] \quad (32)$$

Substituting $M_L, K_L, K_H, \alpha_L, f(t),$ and F_R from (14) into (30) to obtain q_1 .

$$\dot{q}_1(t) = -\omega A_s \sin(\omega t) + \omega B_s \cos(\omega t) \quad (33)$$

$$t = \frac{q_1}{\dot{q}_1} \quad (34)$$

Angular velocity for the excitation is

$$\omega_2 = \frac{2\pi}{t} \quad (35)$$

q_2 can be found out from (30)

$$q_2 = \frac{F_b}{(\omega_n^2 - \omega_2^2)^2 + (2\zeta\omega_n\omega_2)^2} \times \cos \left[\omega_2 t - \tan^{-1} \left(\frac{2\zeta\omega_n\omega_2}{\omega_n^2 - \omega_2^2} \right) \right] \quad (36)$$

Substituting the values of $M_b, K_L, \alpha_L,$ and F_B from (15) into (30) to obtain the solution for q_2

The force acting at the knee joint is

$$F = \frac{-(F_2 + F_3)}{M_b} \quad (37)$$

This force acting at the knee joint, which may damage the knee joint, has to be minimized.

The constraint which is to limit the force acting at the knee joint should be less than 20 N.

$$\text{Max } F(t) \leq 20 \quad (38)$$

IV. TRANSMISSIBILITY FACTOR

The primary objective is to minimize the transmissibility factors. The minimum transmissibility factor suggests the minimum transmission of force vibration to the knee joint. It is defined as the ratio of force acting at the knee joint, F , to the reaction force

$$\text{Minimize } \frac{\max \text{ abs } F(t)}{F_{\text{Reaction}}} \quad (39)$$

subject to

- $20 - \text{Max } F(t) \geq 0$
- $K_h \geq 140 \frac{kN}{m}, K_L \geq 200 \frac{kN}{m}$
- $\alpha_L \geq 920 \text{ Ns/m}$

V. RESULT AND DISCUSSION

Fig. 2 shows the amount of force transmitted to the knee joint after damping in heel and leg muscles. This is really a dangerous force, since high magnitudes or cyclic stress can even lead to damage or failure of knee joint. It is well clear from the figure that a force of magnitude about 12 N is transferred to the knee joint after having a stiffness of $140 \frac{kN}{m}$ at the heel joint and $200 \frac{kN}{m}$ at the leg muscles and damping factor 920 Ns/m. As this is a vertical reaction force, it transfers through the bones, and a stress will be developed in the bones.

So, it is a high necessity that the shin and shank bones should be strong enough to withstand this extra force. From Fig. 3, we can see that a slight displacement is made at the knee joint during the phase of double impacts. But, as the knee joint has the capability to withstand the bending moment due to this displacement, it arrests this movement. However, this effect adversely affects the natural functioning of the knee joint and may have sprain after a long period. As disturbance force is the reaction force that was measured from a gait lab, the profile of force curve and displacement curve follow the same curvature, but vary in magnitude.

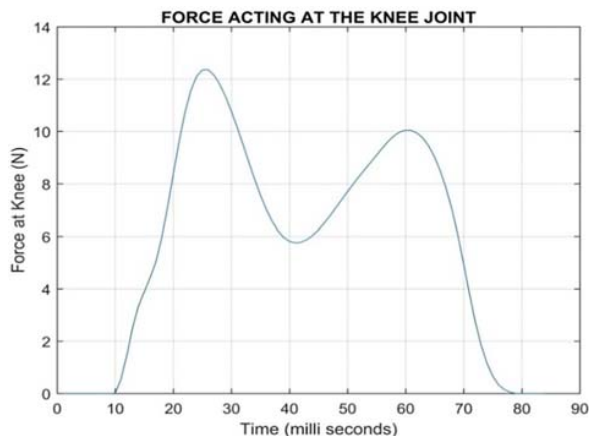


Fig. 2 Force transmitted to the knee joint after damping at heel and leg muscles during double support phase

VI. CONCLUSION

A spring mass damper model is used to study the force that is transmitted to the knee joint after damping at heel and leg muscles. The maximum force that is to be transmitted during a heel strike is being minimized. The force plate data from the gait lab are used to test the muscle model that is built using spring mass damper system, and the model shows a good agreement with the real data. The stiffness, frequency, and damping coefficients are adjusted in order to replicate the human muscle characteristics.

In future, additional stiffness and damping can be considered for ankle joints and can be modeled in order to study the dynamic effects at the knee. This study can be extended to study forces acting at the knee joint of the exoskeleton so that it can be designed to withstand the additional force transmitted to the knee joint during double impact phase.

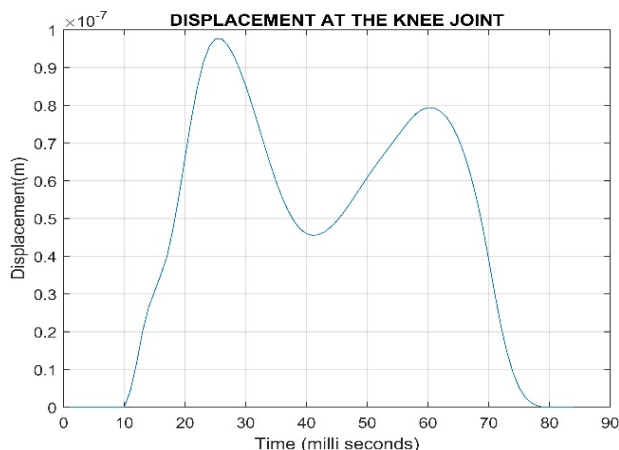


Fig. 3 Displacement at knee joint due to the force acting at heel during heel strike

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