International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:4, No:4, 2010

The Decentralized Nonlinear Controller of Robot Manipulator with External Load Compensation

Sun Lim, Il-Kyun Jung

***Abstract*—**This paper describes a newly designed decentralized nonlinear control strategy to control a robot manipulator. Based on the concept of the nonlinear state feedback theory and decentralized concept is developed to improve the drawbacks in previous works concerned with complicate intelligent control and low cost effective sensor. The control methodology is derived in the sense of Lyapunov theorem so that the stability of the control system is guaranteed. The decentralized algorithm does not require other joint angle and velocity information. Individual Joint controller is implemented using a digital processor with nearly actuator to make it possible to achieve good dynamics and modular. Computer simulation result has been conducted to validate the effectiveness of the proposed control scheme under the occurrence of possible uncertainties and different reference trajectories. The merit of the proposed control system is indicated in comparison with a classical control system.

***Keywords*—**Robot manipulator control, nonlinear controller, Lyapunov based stability, Interconnection compensation.

I. Introduction

ROBOT manipulator is a typical nonlinear complex system . In its conventional controller design such as PI or PD controller, the control algorithm is based on nonlinear interconnection term compensation to eliminate the interaction. This approach requires a detailed model of the robot manipulators and rich priced sensor, Joint position and Joint torque sensor. Typically, a set of second-order differential equations is obtained to characterize the dynamic behavior of rigid robot arms. The torque or forces acting on arm joints are the input to the system equation. The interaction term is compensated by exactly known value which is measured or transmitted by sensor and industrial communication module. If system has a useful space, all joint sensor information is transmitted by physical signal line. But under compact modular size, this data line is eliminated. Thus in this study we proposed the decentralized control mechanism.

In the design of convention control systems, approximately linearized robot manipulator models are employed. Normally, the system is simplified as one single-machine to end-effector laod. Then conventional controllers are designed based on the simplified centralized feedback-linearization model. It is obvious that when a major fault in other joint occurs, the behavior of the robot manipulator may change significantly. Conventional centralized linear controllers do not guarantee the system stability under such condition [1].

The main objective of this paper is to develop a systematic approach for designing decentralized adaptive nonlinear control.

The proposed controller is applied to a two-degree freedom robot joint manipulator example system. In order to illustrate closed-loop performance, we consider a symmetrical other joint fault or unstable state. The simulation results show that the proposed decentralized adaptive nonlinear controller can achieve highly perfect tracking and system stability enhancement.

Authors are with the Korea Electronics Technology Institute (*KETI*)
e-mail: sunishot@ keti.re.kr).

II. Mathematical Model

The dynamic model of the Robot manipulator is modified from the traditional model of n-joint rigid robot manipulator system, its dynamics are described by[4],[5]

$$M(q)\ddot{q} + H(q)\dot{q} + G(q) + \tau_d(t) = u(t) \tag{1}$$

where q , $\dot{q}$ and $\ddot{q}$ are all the $n\times 1$ vectors of joint angular positions, corresponding velocity and acceleration. $M(q)$ is the $n\times n$ symmetric positive definite inertial matrix, $H(q)$ is the $n\times l$ vector containing Coriolis, centrifugal forces, $\tau_d(t)$ is the external load torque and $G(q)$ is the $n\times 1$ vector gravity torques, $u(t)$ is the $n\times 1$ vector of applied joint torques.

For simplicity equation, 2-degree joint rigid robot manipulator system are described by in state variable form as [2],[5]

$$x_1 = q_1,\ x_2 = \dot{q}_1,\ x_3 = q_2,\ x_4 = \dot{q}_2$$

$$\dot{x}_1 = x_2 \tag{2}$$

$$\dot{x}_2 = \frac{1}{a_1}[bx_2(x_2 + x_4)(1 + \frac{a_2^2}{a_1 a_2 - a_2^2}) + \gamma_1 g + u_1$$

$$- \frac{a_2}{(a_1 a_2 - a_2^2)^2} a_1(\gamma_2 g - bx_4 + u_2) - a_2(\gamma_1 g + u_2))]$$

(3)

$$\dot{x}_3 = x_4 \quad (4)$$

$$\dot{x}_4 = \frac{1}{a_1 a_2 - a_2^2}[a_1(\gamma_2 g - b x_4^2 + u_2) - a_2(b x_2(x_2 + x_4) + \gamma_1 g + u_2)] \quad (5)$$

where

$$a_1 = (m_1 + m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos(x_3) + J_1$$

$$a_2 = m_2 r_2^2 + 2m_2 r_1 r_2 \cos(x_3) + J_2$$

$$b = m_2 r_1 r_2 \sin(x_3)$$

$$\gamma_1 = -((m_1 + m_2) r_1 \cos(x_3) + m_2 r_2 \cos(x_1 + x_3))$$

$$\gamma_2 = -m_2 r_2 \cos(x_1 + x_3)$$

From above equation (1.2)~(1.5), the entire system have the highly nonlinear term and interaction term. So previous control approaches don't guarantee the stability and performance.

Given the desired motion trajectory $q_{di}(t)$, the objective is to synthesize a decentralized control input $u_i(t)$ such that the output individual joint angle $y = q_i(t)$ tracks $q_{di}(t)$ as closely as possible in spite of unknown inter connection term and model uncertainty, i.e.,

$$\lim_{t\to\infty}\begin{bmatrix} q_1(t) \\ q_2(t)\end{bmatrix} = \begin{bmatrix} q_{1d}(t) \\ q_{2d}(t)\end{bmatrix}, \ \forall x_i \le \alpha < \infty$$

To begin the controller design, practical and reasonable assumptions on the system have to be made. In general, the system is subjected to parametric uncertainties due to the variations of m_i, J_i. For simplicity, in this paper, we only consider the parametric uncertainties of important parameters like m_i and nominal value of the disturbance d, d_0. Other parametric uncertainties can be dealt with in the same way if necessary. In order to use parameter adaptation to reduce parametric uncertainties to improve performance, it is necessary to linearly parameterize the state space equation (1.2)~(1.5) in terms of a set of unknown parameters. To this end define the unknown parameters set $\theta_1 = [\theta_1 \quad \theta_2 \quad \theta_3]^T$ as

$$\theta_1 = \frac{1}{a_1}\left\{\left(1 + \frac{a_2^2}{a_1 a_2 - a_2^2}\right) b\right\},$$

$$\theta_2 = \gamma_1,$$

$$\theta_3 = \frac{a_2}{(a_1 a_2 - a_2^2)^2} a_1(\gamma_2 g - b x_4 + u_2) - a_2(\gamma_1 g + u_2))$$

The state space equation (1.2)~(1.5) can thus be linearly parameterized in terms of θ as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \theta_1 x_2^2 + \theta_1 \phi_1(x) x_2 + \theta_2 g + u_1 + \theta_3 \quad (6)$$

Since the extents of parametric uncertainties and uncertain nonlinear interaction term are known, we make the following practical assumption

A1: parametric uncertainties and uncertain interaction term satisfy $\theta \in \Omega_\theta = \{\theta : \theta_{\min} < \theta < \theta_{\max}\}$, where

$$\theta_{\min} = [\theta_{1\min} \quad \theta_{2\min} \quad \theta_{3\min}]^T,$$

$$\theta_{\max} = [\theta_{1\max} \quad \theta_{2\max} \quad \theta_{3\max}]^T$$

A2: Interconnection term is bounded.

In the robot manipulator dynamic described by (1.6), u_1 is control input, x_1 is output. Thus, this equation is highly nonlinear decoupled system. Since there is no direct relation between the output and input. It is difficult to design the control input so that the system output can track the desired trajectory accurately without other subsystem information(x_j, $j = 3 \sim n$)

III. Decentralized Adaptive Nonlinear Control

In this study, the nonlinear state feedback theory is used to eliminate this coupling relationship between the control input and the system output to simplify the design of torque controller. According (1.6), the dynamic model of the robot manipulator subsystem can be represented as follows:

Subsystem1:

$$\dot{x} = f(x) + u_1 + \alpha \quad (7)$$

where $x = [x_1 \quad x_2]^T$

$$f(x) = [x_2 \quad \theta_{10} x_2^2 + \theta_2 \phi_1 x_{20}]^T$$

$$\alpha = [0 \quad \Delta\theta_1 x_2^2 + \Delta\theta_2 \phi_1 x_2 + \theta_3 g + \theta_4]^T$$

Subscript i0 means the nominal value.

Define a new error state variable e as follows:

$$e = [e_0 \quad e_1 \quad e_2]^T$$

where $e_0 \equiv \int_0^t z_1(\tau) - q_d(\tau) d\tau$, $e_1 \equiv z_1 - q_d(t)$, $e_2 \equiv z_2 - \dot{q}_d(t)$. After the transformation of coordinates and inserting the new control input (1.8), the dynamic model shown in (1.7) can be rewritten as follows:

$$u_1 = -\theta_{10} z_2^2 + \theta_{10} \phi_{10} z_2 + v_{f1} \quad (8)$$

We set the new control input v_{f1} as

$$v_{f1} = v_{f1a} + v_{f1b} \quad (9)$$

We will design an adaptive nonlinear controller without consideration of the interconnection terms.

$$\dot{e} = Ae + B\left(v_{f1} + \alpha\right) \quad (10)$$

where

$$A=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix},\ B=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Assume that the system parameter variation and external interconnection term are absent, i.e., $\alpha = 0$, (1.10) can be rewritten as

$$\dot{e} = Ae + Bv_{f1} \tag{11}$$

According to the optimal control technique [3], a performance index $J\left(e(t),t\right)$ is defined as follows:

$$J\left(e(t),t\right)=\theta\left(e(t_f),t_f\right)+\int_t^{t_f}\Phi\left(e(\tau),v_{f1a}(\tau)\right)d\tau$$

with the Lagrangian

$$\Phi\left(e(\tau),v_{f1a}(\tau)\right)=\frac{1}{2}e^TQe+\frac{1}{2}v_{f1a}^TRv_{f1b}$$

where $Q\in R^{3\times3}$ is positive-real definite matrix and R is a positive value.

The first control objective is to design a control input v_{f1a} that minimizes the performance index $J\left(e(t),t\right)$ according to the error dynamic shown in (). This is equivalent to solve the Riccati equation can be derived [3]

$$\begin{aligned}&Q+2PA-PBR^{-1}B^TP\\&=Q+PA+A^TP-PBR^{-1}B^TP=0\end{aligned} \tag{12}$$

Consequently, the stability of the error dynamics show in (11) can be guaranteed by a judicious choice of the control gain. Namely, it result in $q_1(t)\to q_{d1}(t)$ as $t\to\infty$. However, if uncertainties occur, i.e., the parameters of the system deviate from the nominal value or an interconnection term is added into the system, the optimal PID feedback linearization control cannot guarantee the performance. Moreover, the stability of the controlled system may be destroyed. To ensure the entire system stability, an adaptive control system, which is composed of the decentralized optimal PID feedback linearization control shown in () and an adaptive uncertainty and interconnection term is summarized in the following theorem.

Theorem: Consider the error dynamics shown in (10), an adaptive control system is designed as (13) with an adaptive uncertainty and interconnection term observer shown in (14), then the entire stability of the decentralized adaptive control system can be guaranteed

$$v_{f1a}=-R^{-1}B^TPe-\hat{\alpha} \tag{13}$$

$$\dot{\hat{\alpha}}=\Gamma e^TPB \tag{14}$$

where Γ is a positive constant.

Proof:

Define a Lyapunove function candidate as

$$V\left(e(t),\tilde{\alpha}(t)\right)V\left(e(t),\tilde{\alpha}(t)\right)=\frac{1}{2}e^TPe+\frac{1}{2\Gamma}\tilde{\alpha}^T\tilde{\alpha} \tag{15}$$

where $\tilde{\alpha}=\hat{\alpha}-\alpha$ is defined as the estimated error. Differentiating (15) with respect to time and using the Riccati equation , it can be obtained that

$$\begin{aligned}\dot{V}\left(e(t),\tilde{\alpha}(t)\right)&=\frac{1}{2}\dot{e}^TPe+\frac{1}{2}e^TP\dot{e}+\frac{1}{\Gamma}\dot{\hat{\alpha}}^T\tilde{\alpha}\\&=\frac{1}{2}\left(e^TA^T+v_{f1}^TB^T+\alpha^TB^T\right)Pe\\&+\frac{1}{2}e^TP\left(Ae+Bv_{f1}+B\alpha\right)+\frac{1}{\Gamma}\dot{\hat{\alpha}}\tilde{\alpha}\\&=\frac{1}{2}e^T\left(A^TP+PA\right)e+e^TPBv_{f1}+e^TPB\alpha+e^TPB\alpha\frac{1}{\Gamma}\dot{\hat{\alpha}}\tilde{\alpha}\end{aligned}$$

Substituting (13) and (14) into upper equation, the following result can be concluded:

$$\begin{aligned}&\dot{V}\left(e(t),\tilde{\alpha}(t)\right)\\&=\frac{1}{2}e^Te^T\left(A^TP+PA\right)e+e^TPB\left(-R^{-1}B^TPe-\hat{\alpha}\right)\\&+e^TPB\alpha+\frac{1}{\Gamma}\left(\Gamma e^TPB\right)\tilde{\alpha}\\&=\frac{1}{2}e^Te^T\left(A^TP+PA-2PBR^{-1}B^TP\right)e-e^TPB\hat{\alpha}\\&+e^TPB\alpha+e^TPB\tilde{\alpha}\\&=\frac{1}{2}e^T\left(A^TP+PA-2PBR^{-1}B^TP\right)e\end{aligned} \tag{16}$$

According to the Riccati equation shown in (12) , the derivative of Lypunove function can be represented as

$$\dot{V}\left(e(t),\tilde{\alpha}(t)\right)=-\frac{1}{2}e^T\left(Q+PBR^{-1}B^TP\right)e\le 0 \tag{17}$$

Since $\dot{V}\left(e(t),\tilde{\alpha}(t)\right)\le 0$, $\dot{V}\left(e(t),\tilde{\alpha}(t)\right)$ is a negative semi-definite function, that is,

$V\left(e(t),\tilde{\alpha}(t)\right)\le V\left(e(0),\tilde{\alpha}(0)\right)$ which implies $e(t)$ and $\tilde{\alpha}(t)$ are bounded functions. Define a function

$$L(t)\equiv e^T\left(Q+PBR^{-1}B^TP\right)e\big/2\le-\dot{V}\left(e(t),\tilde{\alpha}(t)\right)$$

and integrate function $L(t)$ w.r.t time

$$\int_0^t L(\tau)d\tau\le V\left(e(0),\tilde{\alpha}(0)\right)-V\left(e(t),\tilde{\alpha}(t)\right)$$

Because $V\left(e(0),\tilde{\alpha}(0)\right)$ is a bounded function, and $V\left(e(t),\tilde{\alpha}(t)\right)$ is a nonincreasing and bounded function, the following result can be concluded:

$$\lim_{t\to\infty}\int_0^t L(\tau)d\tau<\infty \tag{18}$$

Also $\dot{L}(t)$ is bounded, so Barbalat's Lemma [4],[5], it can be shown that

$$\lim_{t\to\infty} L(t) = 0$$

I.e, $e(t) \to 0$ as $t \to \infty$. As result, the stability of the decentralized adaptive nonlinear control system can be guaranteed.

IV. Simulation

The system under study is shown in Fig. 2 and the data of the system are induced in Table 1. In order to investigate the effectiveness and performance, simulation has been carried out in Matlab to evaluate the proposed decentralized adaptive nonlinear control.

Symbol	Quantity	Value
m_1	Joint1 mass	0.5 kg
m_2	Joint2 mass	1.5 kg
J_1	Joint1 moment inertia	5 kgm
J_2	Joint2moment inertia	5 kgm
g	gravity	9.8 m/s^2
r_1		1.0 m
r_2		0.8 m

In addition, following the design procedure in Section 3, the nonlinear control gain design is given by the following steps,

Step 1) specify nominal value of robot manipulator parameter

Step 2) Select a weight matrix $Q = 100 \times I^{3\times3}$ and $R = 1$, and solve the Riccati equation.

Step 3) For simplicity, the tuning gain of the adaptation law.

The joint position and desired tracking response are depicted in Fig1.

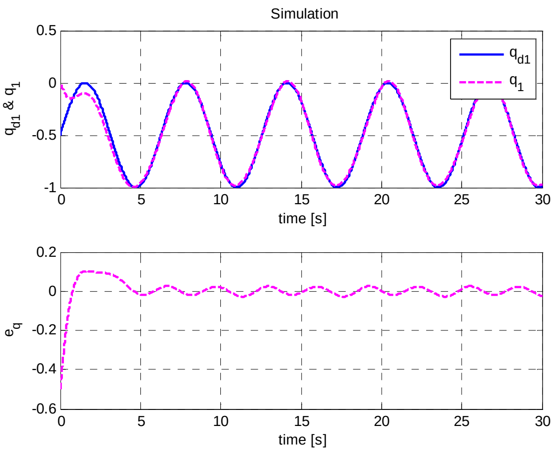

Fig. 1.The Joint position and desired trajectory (upper), the error response w.r.t. time (lower)

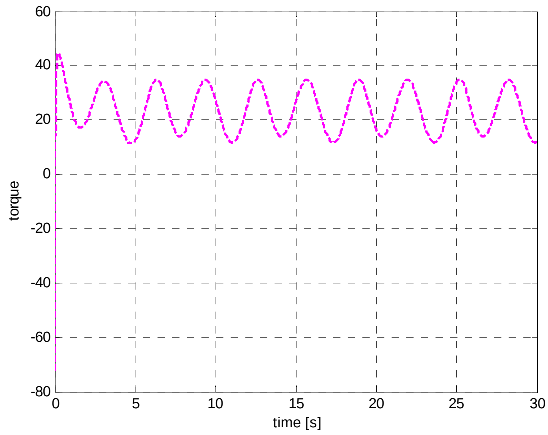

Fig. 2. The control input, joint torque , response w.r.t. time

The control input i.e., Joint torque is depicted in Fig2. Observing the simulated results shown in the time interval 0~5 second, good decoupled property can be obtained.

Now, the proposed decentralized adaptive nonlinear controller requires only the physical limit value. So this assumption is reasonable for engineering condition.

V. Conclusion

This paper has successfully demonstrated the application of a newly decentralized adaptive nonlinear control scheme to the control a robot manipulator. We use an adaptive controller in order to deal with interconnection term between individual joint mechanism subsystems without use of the other subsystem's information.

The proposed nonlinear controller is practically realizable sine it uses only local measurement which are easy to measure.

References

[1] Tzyh-Jong, "Effect Motor Dynamic on Nonlinear Feedback Robot Arm Control," *IEEE Trans. On Robotics and automatio, Vol 7, no. 1.*

[2] Zhimei Chen, "Sliding Mode Control of Robot Manipulators Based on Neural Network Reaching Law," *IEEE International Conferenc on Control an Automation,. Guangzhou, China –May 30 to June 1, 2007*

[3] F.L. Lewis and V.L. Syrmos, "Optimal Control", New York, Wiley, 1995

[4] K.J. Astrom and B. Wittenmark, "Adaptive Control", Addison-Wesley, 1995

[5] J.J.E. Slotine and W.Li," Appied Nonlinear Control", Englewood Cliffs, NJ: Prentical-Hall, 1991

[6] Richard M. Murray and Zexiang Li, " A Mathematical introduction to robotic manipulation", CRC press, 1994

[7] Jhon J. Craig, "Introduction to Robotics 3rd", Prentice-Hall, 1995