

The Customization of 3D Last Form Design Based On Weighted Blending

Shih-Wen Hsiao, Chu-Hsuan Lee, Rong-Qi Chen

Abstract—When it comes to last, it is regarded as the critical foundation of shoe design and development. Not only the last relates to the comfort of shoes wearing but also it aids the production of shoe styling and manufacturing. In order to enhance the efficiency and application of last development, a computer aided methodology for customized last form designs is proposed in this study. The reverse engineering is mainly applied to the process of scanning for the last form. Then the minimum energy is used for the revision of surface continuity, the surface of the last is reconstructed with the feature curves of the scanned last. When the surface of a last is reconstructed, based on the foundation of the proposed last form reconstruction module, the weighted arithmetic mean method is applied to the calculation on the shape morphing which differs from the grading for the control mesh of last, and the algorithm of subdivision is used to create the surface of last mesh, thus the feet-fitting 3D last form of different sizes is generated from its original form feature with functions remained. Finally, the practicability of the proposed methodology is verified through later case studies.

Keywords—3D last design, Customization, Reverse engineering, Weighted morphing, Shape blending.

I. INTRODUCTION

LAST has played an important role on the shoe design and development, shoe designers can directly design shoe styles on the surface of last. 2D patterns which aids shoe manufacturing is generated through the design process, then applying last to these 2D patterns that will be sewed can complete the production of a pair of shoes. Therefore, during the design stage, the appropriate selection of last types and sizes can realize the design concept effectively and aid shoes manufacturing simultaneously. This will achieve the state of art for selecting appropriate, customer-oriented and function-based last for shoe design and development.

Nevertheless, the development and the manufacture of last are regarded as the kind of free form; this leads form design and size adjustment to be relatively related problem. Although the size of the last can be measured by some feature positions, the last form design of surface has still heavily depended on artificers' experience and rule of thumb, thus a last for wearable and comfort shoe design can be obtained. This causes the last forms different due to the different users' requirements, especially for the design concerns of different functions. To

effectively preserve and utilize the last form, the application of reverse engineering for scanning last is a common mean to reconstruct the 3D model of last, and further more generates lasts with different sizes through the grading method even in the different shoe size systems [1]. With this module, amounts of lasts can be manufactured; this benefits customers to select their own shoes designed by the lasts, which suited to themselves. However, not every person has perfect feet form, more, developing a same type of last form in different sizes for people with different countries and areas can be a great work, and some specific requirements of sizes for particular users shall also be concerned.

In order to enhance the efficiency and application of last development, a methodology of applying 3D reconstruction to 3D last morphing is proposed in the study, the aim of flexibly adjusting last size to fit the form of feet can be reached by the brick piling data structure. In the first stage of research processes, different functional last forms are scanned by reverse engineering methodology, and the data of scan point is obtained. Then the data of these scan points are defined on the segments of surface reconstruction, which partitioned by feature curves. Furthermore, the grid points for fitting process are generated which allows B-spline surface be applied to the surface reconstruction in every section. After the revising calculation of surface reconstruction based on the minimum energy, a 3D last model that consisted of the control mesh of B-spline surface can be constructed. Based on the proposed form reconstruction module and through systematical methodology which is applied to the scaling for the sizes of 3D last model, the generation and shape morphing of 3D last model in different sizes can be mainly completed by the application of shape blending algorithm based on weighted arithmetic mean method which processes scaling calculation differs from grading toward the last mesh. Thus not only the required features of different functions can be remained in the 3D last model, but also a new 3D last model with the requirements of different sizes can be generated by shape morphing. In addition, various 3D last forms can be designed and developed concurrently through the module, especially for lasts with different heel heights.

In the end, the result of this study is applied to the constructed application of 3D last model system. Plus, the practicality of methodology is verified by several cases study. With the frame of the proposed methodology, a 3D last surface with high quality can be concurrently designed and various 3D last forms in different sizes can also be constructed by shape morphing based on weighted blending method. This can not

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only be an effective module but also enhance the applying level of last development.

II. RELATED STUDIES

From the previous research of last, constructing 3D last form with 2D profiles is a relative common method [2]. Since the practical 3D last models, which make produced shoes wearable, with different features can be constructed by blending and adjusting the 2D profiles, the last in different feet sizes for various customers can be generated through sizing and grading [3]. A standardized 3D last model which also accurately reconstructs different shoe size systems of last can be developed through the methodology and fulfill the requirements of public-oriented shoe design. On the other hand, in order to enhance the fitness of the shoes, it is necessary that a shoe last designed by using the data of numerous foot measurement [1]. The proposed method scans the users' feet form through reverse engineering method to attain the accurate feet measurement [4]; at the meanwhile the data offers reliable criteria for grading of 3D last model to obtain the more accurate blending result that benefits the shoe last design. Nevertheless, the feature profile for last design also benefits the analysis and evaluation of fitness for shoe lasts and human feet [5]; this may be a critical foundation for a shoe selecting system of customers. As far as the users' wearing circumstance as concerned, the animation of walking feet can be simulated by virtual system [6], [7] and examine the comfort representation while wearing and functional analysis in each part of the designed shoes. In this way, the feature profile concerned by shoe last design is the key reference material for the position of grid points. Through the simulation of users' wearing circumstance, the examination for fruitful results of the shoe design can be conducted and the quality of the product design can be enhanced as well. Except for presenting the feet measurement to aid shoe design, the last is also applied to flat 3D surfaces of model into 2D patterns [8], [9], and each 2D pattern for shoe design can be sampling. Therefore, once the related problem of 3D last form design as mentioned above can be considered and the form information of surface flatten can be widely offered, then the development of last aided design and manufacture can be much more benefited.

In the process of surface reconstruction from scan data, using parametric surfaces to represent the entire 3D shape is a highly efficient procedure, which can also provide a variety of geometric information to help solving problems. Before surface reconstruction can begin, figuring out how to divide the scan data into appropriate blocks using shape features [10], [11] and re-simplifying the entire scan data using slicing methods [12], [13] are the most important preparations. When parametric surfaces are needed for the shape reconstruction of already partitioned blocks, using interpolation to obtain the minimum energy is a relatively common method [14]-[18]; it can also be widely used in the reconstruction of various parametric surfaces. However, ensuring both real-time and accuracy are challenges for algorithm design. In addition, gradually obtaining optimal results by adjusting the control points of the parametric surfaces is a solution from a different viewpoint

[19], [20], while using a genetic algorithm for the reconstruction of parametric surfaces is an attempt to use a new method [21]. When all parametric surfaces of the entire shape have been obtained, the continuity of surface convergence will be an important challenge as it directly affects the quality of the surfaces, thereby raising different requirements related to different levels of continuity [22]-[27]. To alter the surface control grid by using parameters, and thus obtain different results under conditions in line with the same continuity is a more flexible method [28], but ensuring that such correction will not lead to transition errors is another obstacle to overcome. Finally, to make it easier to alter the form of parametric surfaces, reducing the number of surface control points is a feasible option [29]. However, if shape-blending methods can be applied to systematically change the shapes using parameters [30], [31], the varied requirements in shoe design concerning 3D lasts suitable for different individual sizes can thus be met.

Through the previous research, it is necessary to propose a form reconstruction module that corresponds to the features of 3D last form, not only the last form can be standardized but also the last form with various sizes can be developed concurrently. During the process, the continuity of the connections among the surfaces of the last model is necessary to be considered, while the last form shall be flexibly adjusted with the original features remained to concurrently generate the last forms which differ from the result of grading. Through the methodology proposed in this study, the problems mentioned above can be effectively solved, and various 3D last forms can be systematically developed and the efficiency and application of last development can also be enhanced.

III. MATH

A. The Reconstruction Module of 3D Last Form

Last has an enormous influence on shoe design and development, to enhance the developing efficiency and application, a reconstruction module of last form based on feature curves is proposed in this study. In the reconstruction module, various 3D last model can be systematically generated by weighted blending with the original features remained. In general, last form belongs to a kind of free form that has no certain representation, but feature curves of model are commonly applied to accurately represent the measurement or each portion of feet which makes the produced last practical and cause shoes wearable, thus a reconstruction module of last form will be developed based on feature curves, as shown in Fig. 1.

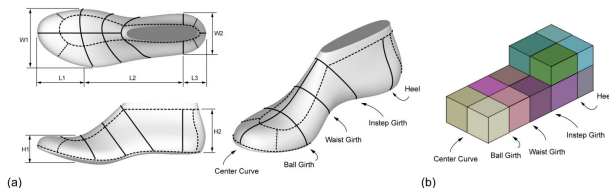


Fig. 1 The form reconstruction module of 3D last (a) the application of feature curves and aided curves (b) the 3D last model composed of brick piling data structure

In the proposed module, the three girths measurement of feet: Ball girth, waist girth, instep girth, and the other feature curves such as the center curve and heel girth, etc. are used (Fig. 1 (a)). Thus some important size parameters can be obtained by measuring the relative position of these feature curves on the last, including the last length which composed of L1, L2 and L3; the last width which respectively represented by W1 and W2. When the three girths need to be adjusted, the required girth can be generated by adjusting the height of H1 and H2 with the respective width remained. Among these feature curves, waist girth is often represented for functional feature which will not always fit the feet form, thus H1 and H2 are indeed the critical parameters for the adjustment of girth size. In order to systematically process the size scaling of 3D last model with these parameters, several dotted curves will be adopted to the form reconstruction as the aided curves for data partitioning, this cause the primary form model partitioned by feature curves and then transform to the 3D last form model composed of brick piling data structure (Fig. 1 (b)). While conducting the shape blending calculation, those key size parameters mentioned before can correspond to length, width and height for each brick, thus each profile can be used to generate a new 3D last form model by adjusting the scaling ratio of the corresponding length, width and height based on the positions of each profile.

Shape morphing is the process of continual transformation between two shapes or forms that also includes the conversion of size. In order to make the process continuity contained, in this study, the weighted arithmetic mean method in (1) is applied to calculate the degree of scaling in every axial directions and then generates the output of weighted blending with the specific form feature remained.

$$\begin{aligned} S_i &= w_a \times S_a + w_b \times S_b \\ w_a &= \frac{i}{n} \\ w_b &= 1 - w_a \end{aligned} \quad (1)$$

where n is the step number for the degree of shape morphing process, S_a and S_b are the dimensions of form a and form b respectively, w_a and w_b are the weights of form a and form b respectively, and S_i is the dimension of i -th form of shape morphing.

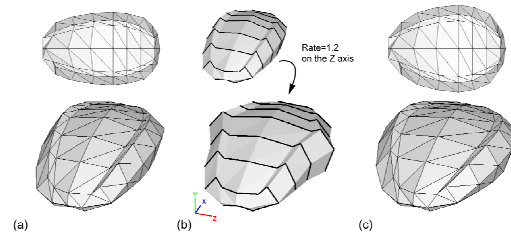


Fig. 2 The scaling transformation based on weighted blending (a) the primary form (b) the scaling of feature profile (c) the form after blending

When weighted blending is applied to the 3D shape morphing based on the brick piling data structure, the result as Fig. 2 can be obtained. Fig. 2 is a 3D model, which composed of brick piling data structure, while adjusting the scaling ratio of the continuous profile, a new blending form can be generated by calculating the scaling ratios of another profiles based on weighted blending method (Figs. 2 (b) and (c)). Through this module, the result differs from grading can be flexibly generated with the primarily geometric features remained.

B. Principle of Catmull-Clark Subdivision

Catmull-Clark Subdivision was proposed by Catmull and Clark [32], [33], the basic principle of Subdivision is to repeatedly and regularly input new vertices on an original mesh, and a smooth surface can then be obtained. At meanwhile, in the process of subdivision, a hierarchical relation between old and new vertices will generated, and number of level depends on design requirement. Catmull-Clark Subdivision is a quadrilateral subdivision induced by uniform bi-cubic B-spline surfaces, which means the surface acquired through this subdivision and the calculated B-spline surface generated through basis function composed by knot vector will be the same. This makes generation of forms much freer, and the restriction of application for B-spline surface calculated by basis function based on control points can be avoided.

In the process of Catmull-Clark subdivision, due to this algorithm is a kind of down to top calculating module, new position of new vertices are merely calculated through a quadrilateral interactive relation after implement of subdivision. Thus vertices coordinate of new level is obtained through vertices coordinate of original one that has calculated the weight value of related points, as shown as Fig. 3. Different calculation will be adopted due to different position where new vertices sited; the content is illustrated as follows:

1. The New Vertices Replacement for Original Vertices Not on Boundary

While new vertices correspond to the condition, there are three common situations (Fig. 3 (a)): three quadrilateral combinations, four quadrilateral combinations and six quadrilateral combinations. Even though the number of combination is different, through (2), new positioning coordinate can still be acquired by weight calculation of related point of different position on the upper level.

$$v'_{2i,2j} = (1-\beta-\gamma) \cdot v_{i,j} + \frac{\beta}{n} \cdot \sum_{d=0}^{n-1} p_d + \frac{\gamma}{n} \cdot \sum_{d=0}^{n-1} q_d$$

$$\beta = \frac{3}{2n}, \gamma = \frac{1}{4n}$$
(2)

In the equation, n represents the number of quadrilateral combination, v represents vertices on original mesh, v' represents new vertices, then p and q are related points of different and relative positions, β and γ represents weight coefficient.

2. New Vertices on the Original Mesh Line Not on Boundary

The principle is based on subdivision of quadrilateral mesh. In this condition, there are i and j two possible direction for the position of new vertices generation (Fig. 3 (b)), the new vertices on mesh line of I direction can be calculated by (3), and the other side is (4).

$$v'_{2i+1,2j} = \frac{1}{16}(v_{i,j-1} + v_{i+1,j-1} + 6v_{i,j} + 6v_{i+1,j} + v_{i,j+1} + v_{i+1,j+1})$$
(3)

$$v'_{2i,2j+1} = \frac{1}{16}(v_{i-1,j} + v_{i-1,j+1} + 6v_{i,j} + 6v_{i,j+1} + v_{i+1,j} + v_{i+1,j+1})$$
(4)

3. New Vertices Inside the Original Quadrilateral Mesh

If new vertices are in the situation (Fig. 3 (c)), then the four vertices of quadrilateral mesh are regarded as related points, new position coordinates are calculated by (5).

$$v'_{2i+1,2j+1} = \frac{1}{4}(v_{i,j} + v_{i,j+1} + v_{i+1,j} + v_{i+1,j+1})$$
(5)

4. The New Vertices Replacement for Original Vertices on Boundary

In this situation, according to mesh direction of new vertices, the adjacent and related points on the same boundary can be imported to (6) or (7), as shown in Fig. 3 (d).

$$v'_{2i,2j} = \frac{1}{8}(v_{i-1,j} + 6 \cdot v_{i,j} + v_{i+1,j})$$
(6)

$$v'_{2i,2j} = \frac{1}{8}(v_{i,j-1} + 6 \cdot v_{i,j} + v_{i,j+1})$$
(7)

5. The New Vertices on the Original Mesh Line on Boundary

In condition of the vertices of this kind, only the two vertices of mesh line are imported to (8) or (9) with the consideration of direction (Fig. 3 (e)), the new coordinate can then be acquired.

$$v'_{2i+1,2j} = \frac{1}{2}(v_{i,j} + v_{i+1,j})$$
(8)

$$v'_{2i,2j+1} = \frac{1}{2}(v_{i,j} + v_{i,j+1})$$
(9)

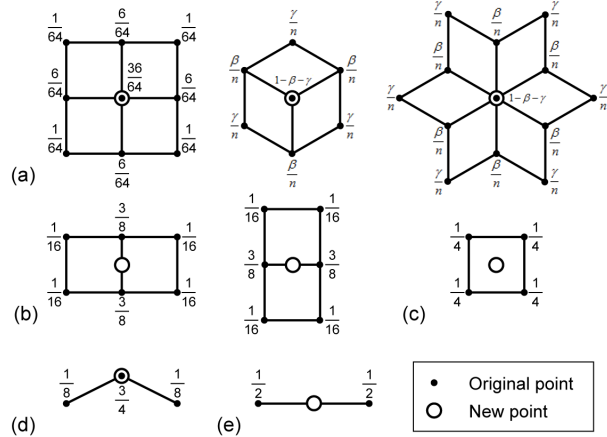


Fig. 3 The possible position of new vertices and weight value of related points in the process of Catmull-Clark subdivision

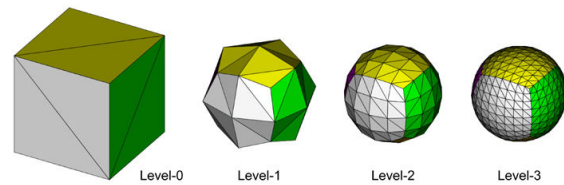


Fig. 4 The 3D meshes of form obtained with different levels of subdivision

According to the calculation principle mentioned above, after implementing Catmull-Clark subdivision to control mesh of a cube, the process and result for a cube gradually transforming to a ball are acquired (Fig. 4).

IV. CASE STUDIES

An application for 3D last design system based on the frame of proposed methodology was constructed in this study, as shown in Fig. 8. Based on the application, the form reconstruction module of 3D last could be reconstructed and various 3D lasts could be generated by shape morphing, thus the efficiency of last development could be enhanced, the context as illustrated as follows:

A. The Reconstruction of 3D Last Model

When last manufacturing artificers completed the primary sample of last, the 3D last model based on feature curves could be reconstructed by the proposed methodology in this study. First, the reverse engineering method was applied to scan the form of real last, and the functionally significant features could be digitalized (Fig. 5 (a)). Although the scanned points represented the whole 3D last model completely, these points were useless and impractical in other fields. In the attempt to increase applying options and offer more valuable service of these scanned points, applying the scanned points to frame of methodology, the 3D last model could be reconstructed by the form reconstruction module based on feature curves. The very first thing was to partition the feature curves and aided curves which adopted in the module on the surface of scanned model

(Fig. 5 (b)), due to the intersecting of feature curves so that these curves could be separated into segments of several sizes, then the same amount of points could be scattered on segments of the same direction based on the relative positions for circumferential frames of data bricks (Fig. 5 (c)). Next, in every axial viewpoint, projecting the points on the frames onto the surface of the assigned viewpoint and applying the blending method to calculate the grid points inside the area on the framed surface, and then re-projected the point data onto the 3D model (Fig. 5 (d)); the grid points for surface blending calculation could be acquired.

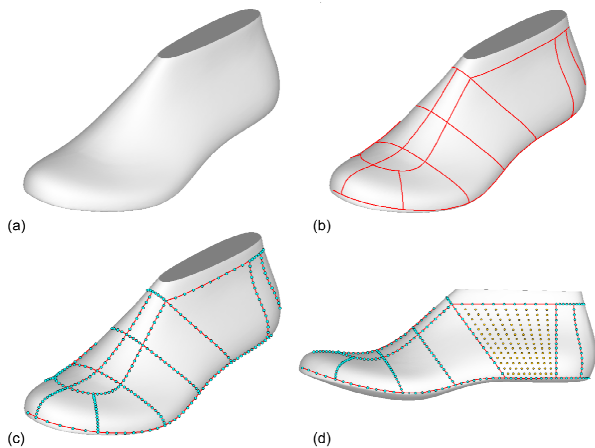


Fig. 5 The acquisition of the grid points which were applied to surface fitting calculation in the form reconstruction module based on feature curves (a) the scanned model based on reverse engineering method (b) partitioning the required curves on the model (c) scattering the grid points on the segments (d) the projection on the model after the generation of grid points

The data of grid points acquired from the model were partitioned into several sections of different sizes based on feature curves (Fig. 6 (a)), and based on the surface fitting algorithm, the control mesh of the B-spline surface for the grid points in every section could be calculated gradually (Fig. 6 (b)). After the control mesh of every section were calculated, the interrelated grid position of the adjacent sections could be slightly adjusted by the continuity reversion method based on the minimum energy proposed in this study, this caused the surface sections which partitioned by the feature curves as an unity, and the required 3D last surfaces could be generated by inputting different parameter t of the B-spline surface (Fig. 6 (c)). At the meanwhile, the zebra texture could be utilized to examine the continuity of the surfaces (Fig. 6 (d)), through the representation of the zebra texture, the practicality of the continuity revision method could be verified. The form surface with high quality benefited the application of 3D last model in several fields.

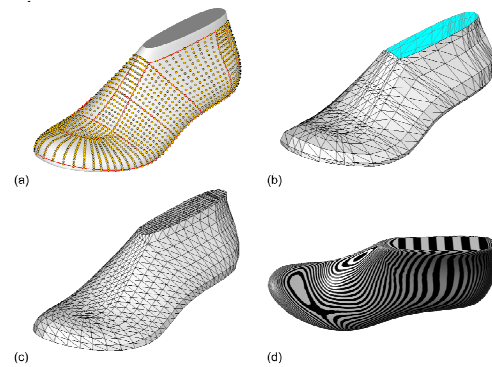


Fig. 6 The surface reconstruction of 3D last model (a) the scanned grid points on the model (b) the control mesh of the B-spline surface (c) B-spline surface (d) the examination of the surface continuity by application of zebra texture

B. The Shape Blending of 3D Last Based On Weighted Blending

When the appearance of scanned last model was reconstructed by the proposed reconstruction module of the last form, the whole 3D last was partitioned into several bricks of different sizes based on the feature curves and aided curves. Each brick had grid points for representative feature in each dimension or direction, due to the brick piling data structure, each grid point could be calculated with the weighted blending by the relative index of grid points in the assigned data structure.

Take weighted blending of the 3D last for example, the main spotlight was focused on the size scaling transformation among feature curves, and thus the continuous profiles for calculation could be extracted from the brick piling data structure (Fig. 7 (a)). After measuring the size parameters for blending, the required degree of scaling in each axis for each profile could be calculated by algorithm, as shown in Fig. 7 (b). During the process, the scaling of X axis was more complicated, because the length of last was separated into three segments in the study, this led the scaling of X-axis should be conducted by composing each sub-brick first, and then re-pile the bricks into the unity of brick set. And the degree of scaling for the Z axis could be directly converted by the input of parameters; the Scale Z of each profile could also be calculated through weighted arithmetic mean method. One issue should be more concerned that the scaling of Z axis could have the impact on the three girths of the last. Therefore, when completed the scaling degree of Z axis, the target girth could be obtained by calculating the required scaling degree of Y axis based on the iteration method and the primary girth. While the system completed the series of calculations, the new 3D last model could then be shape blended with the primary features remained (Fig. 7 (c)). Further, the size parameters for blending could be measured from the scanned data of 3D foot (Fig. 8 (a)). And then the customized 3D last model is shape blended (Fig. 8 (b)).

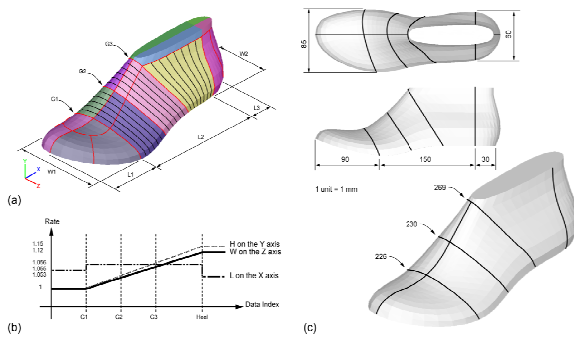


Fig. 7 The shape blending of the 3D last model based on weighted blending (a) the continuous profiles among each feature curve (b) the scaling degree of each axial direction (c) the new 3D last model of shape blending

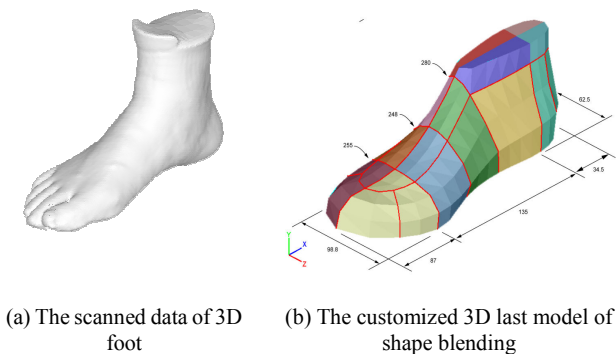


Fig. 8 The customized 3D last model of shape blending from the scanned data of 3D foot

V. CONCLUSIONS

In order to enhance the efficiency and application of last development, a methodology of computer aided design for customized last forms was proposed in this study. Through the verification with case studies, the scaling result differed from the grading could be generated by the 3D last form reconstruction module, which could make the last much more fitted to the feet form with the primary functional features that always uncovered by the experiential artificers could be remained, and the 3D last model of various sizes could then be generated concurrently. On other hands, the form reconstruction module was developed which based on the feature curves of the last, thus the suitable description and the size measurement of common last could be applied to other related studies.

The shoe design is covered by the consideration of ergonomics, aesthetics and functions, etc. And the last plays an important role in the field of shoe design. From then until now, the design and development of the last have much more depended on the experience of the last artificer, even though in the computer aided design system. The proposed methodology in this study was capable to generate various 3D last models for the functional significant forms of shoe last by the systematic method, and enhance the efficiency of the last development. At the meanwhile, the form reconstruction module based on the

feature curves can be applied to the evaluation of the related digital system, this not only causes the ability of last aid the design and manufacture of shoes, but also offers more valuable service for the human oriented life, and these are the important contributions of this study.

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