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The Cracks Propagation Monitoring of a Cantilever Beam Using Modal Analysis

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***Abstract*—**Cantilever beam is a simplified sample of a lot of mechanical components used in a wide range of applications, including many industries such as gas turbine blade. Due to the nature of the operating conditions, beams are subject to variety of damages especially crack propagates. Crack propagation may lead to catastrophic failure during operation. Therefore, online detection of crack presence and its propagation is very important and may reduce possible significant cost of the whole system failure. This paper aims to investigate the effect of cracks presence and crack propagation on one end fixed beam`s vibration. A finite element model will be developed for the blade in which the modal response of the structure with and without crack will be studied.

***Keywords*—**Blade, crack propagation, health monitoring, modal analysis.

I. INTRODUCTION

CRACKS and crack propagation have an effect on system behavior, due to its dynamic characteristic changes. As a result, the mechanical system components that are used in many industries such as oil and petrochemical, aerospace, nuclear power plant and so on, do not work normally and the system works with low efficiency. It leads to waste of energy, time and financial resources. To prevent these, researchers have been studying materials properties, blade cooling systems, thermodynamics and aerodynamics design [1]-[7]. Also, if the crack propagates to another edge of a component, regardless of its reasons, it can break the component and it can break down the system. One important aspect of a component is the vibration can cause a crack on it. If the vibration frequency amplitude be more than standard amplitude, the system would be able to work under bad conditions. If the resonance occurs, then the system fails and many components will be severely damaged, and it costs the system irreparable damages.

So far, many researchers have worked on several cracks in different structures [9]-[14]. High vibration of the turbine blades can result in a more than allowance mechanical stresses which may lead to high cycle fatigue.

Cracks and crack propagation and their significant effects on the blades vibration, which can make condition more severe and increase the possibility of being in a resonance range, are the subjects of this paper. Therefore, the vibratory modes in which blade are in resonance condition, should be studied and this is done by modal analysis method. Since the turbine is working, there are numbers of complex vibratory systems that can appear in the form of forced vibration in the presence of exciter factors. Mazoor has divided blades vibration to three categories: First of all, those are affected by free vibrations. Second, blades which are under force vibration, and the third category consists of blades, which are involved in the self-exciting phenomenon [8]. Orhan analyzed free and forced vibration of cracked beam [9]. J. Wauer investigated vibrational characteristic of cracked rotating blade [10]. In another work, Shukla and Harsha studied comparative modal analysis of steam turbine blade with analytical method and experimental method, so cracked and un-cracked mode shapes are compared with each other. They examined Modal-Test on the third stage blades of the compressor [11]. Beams also have been the subject of many researches. For example, W.M. Ostachowicz and M. Krawczuk analyzed the effect of cracks on the natural frequencies of a cantilever beam [12]. Francois Leonard et al. investigated cracks that occurred in metal beams obtained under controlled fatigue-crack propagation [13]. M. Behzad et al. studied a continuous model for flexural vibration of beams with an edge crack perpendicular to the neutral plane. They assumed that the displacement field is a superposition of the classical Euler-Bernoulli beam's displacement and of a displacement due to the crack [14]. In this paper, using the Finite Element method as well as modal analysis of a blade of a gas turbine, which is installed in the National Iranian South Oil Company (NISOC), the natural frequency of the blade is determined. At the next step, the Finite Element method is applied to calculate the frequency of a cracked blade. Then we observe the crack growth. So, we can investigate some ways to detect or prevent cracks and their growth in blades. In this regard we prefer to assume the blade as a cantilever beam.

II. THEORY

In the proposed model, the beam is assumed to have a transverse crack edge clamped at left end and it is free at right end and has a uniform structure with a constant rectangular cross-section. Moreover, the system is based on the Euler Bernoulli beam theory and the crack is considered to be an open crack.

A. Governing Equation of Free Vibration

The free bending vibrations of a Euler–Bernoulli beam of a constant rectangular cross-section are given by the following differential equation:

$$EI \frac{d^4 y}{dx^4} - m \omega_i^2 y = 0, \tag{1}$$

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where m (kg/m) is the mass of the beam per unit length, ω_i is the natural frequency of the *i*th mode (rad/s), E is the modulus of elasticity (N/m^2) and, I is the area moment of inertia (m^4) [15], [16]. By defining $\lambda_i^4 = \omega_i^2 m/EI$, (1) is rearranged as a fourth-order differential equation as:

$$\frac{d^4y}{dx^4} - \lambda_i^4 y = 0, \tag{2}$$

Then the general solution for (2) is:

$$y = A\cos\lambda_i x + B\sin\lambda_i x + C\cosh\lambda_i x + D\sinh\lambda_i x \tag{3}$$

where A, B, C, D are constants and λ_i is a frequency parameter [15]. Since the bending vibration is applied, edge crack is modeled as a rotational spring with a lumped stiffness and the crack is assumed open. Based on this modeling, the beam is divided into two segments that are on the left and the right-hand sides of the crack, respectively. Once this equation is solved by applying beam boundary conditions and compatibility relations, the natural frequency of the i^t mode for free ends (4) and cantilever beam (5) are calculated as:

$$\omega_{i_o} = \sqrt{\frac{EI}{ml^4}}\ , \tag{4}$$

$$\omega_i = r_i c_i \sqrt{\frac{EI}{ml^4}}\ , \tag{5}$$

where ω_{i_o} is the ith mode frequency of the un-cracked beam and c_i is a known constant depending on the mode number and the status of the end of the beam (for clamped-free beam, c_i is 3.516 and 22.034 for the first and second mode, respectively). ω_i is the ith mode frequency of the cracked beam. r_i is the ratio between the natural frequencies of the cracked and un-cracked beam. L is length of the beam [16], [17]. Now we can consider frequency response function of a damped single degree of freedom system (SDOF) and frequency response function of a damped multi degree of freedom system (MDOF).

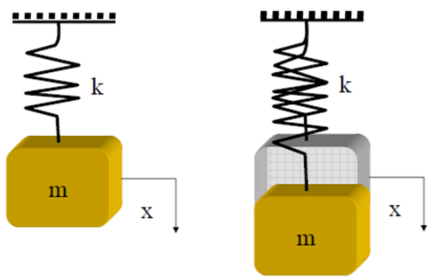

Fig. 1 Mass and spring as a SDOF

B. Frequency Response of a Damped SDOF System

The degrees of freedom of a system are defined by the minimum number of independent coordinates necessary to describe the positions of all parts of the system at any instant of time. For example, the spring–mass system shown in Fig. 1 is a SDOF since a single coordinate $x_{(t)}$ is sufficient to describe the position of the mass from its equilibrium position at any instant of time [15].

First of all, we need to determine the equation of motion for the system [17]. Free body diagram of the mass-spring system in dynamic balance is determined as Fig. 2:

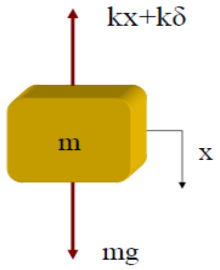

Fig. 2 Free body diagram of a mass-spring system

Using Newton's 2nd law, the equation of motion is determined as:

$$\sum F = m\ddot{x}\ , \tag{6}$$

where F (N) is force balance, m (kg) is representing the mass, and $\ddot{x}$ (m/s^2) is the second derivation of distance (m). So, by using free body diagram we have:

$$Mg - (kx + k\delta) = m\ddot{x} \tag{7}$$

Then the equation of motion will appear as:

$$m\ddot{x} + kx = 0\ , \tag{8}$$

In order to determine the displacement of mass, x, we need to solve (8) that is 2nd order differential equation. So considering the $x_{(t)}$ equation as:

$$x(t) = Xe^{i\omega t}\ , \tag{9}$$

and substituting (9) into (8), we reach to:

$$-m\omega^2 X + kX = 0\ , \tag{10}$$

So we have:

$$(k - m\omega^2)X = 0\ , \tag{11}$$

As X ≠ 0, therefore:

$$\omega = \sqrt{\frac{k}{m}}, \tag{12}$$

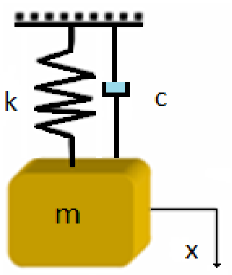

Fig. 3 Mass, spring and damper as a system

In the presence of viscous damping as shown in Fig. 3, the equation of motion for free vibration changes as:

$$m\ddot{x} + c\dot{x} + kx = 0, \tag{13}$$

where the $c\dot{x}$ is the damping force. Now considering:

$$x(t) = Xe^{st}, \tag{14}$$

and substituting (14) into (13) leads to:

$$ms^2 + cs + k = 0, \tag{15}$$

Solution of (15) results in:

$$s_{1,2} = -\omega_0\zeta \pm i\omega_0\sqrt{1-\zeta^2}, \tag{16}$$

where

$$\zeta = c/(2\sqrt{km}), \tag{17}$$

in which, ζ is the damping ratio and the diagonal element c represents the generalized damping of the various modes of the system. If we choose:

$$\alpha = \zeta\omega_0, \tag{18}$$

and:

$$\omega_d = \omega_0\sqrt{1-\zeta^2}, \tag{19}$$

where ω_d is defined as the damped frequency. Therefore, displacement is determined as:

$$x(t) = Xe^{-\alpha t}e^{i\omega_d t}, \tag{20}$$

The equation of motion for forced vibration with viscous damping is given as:

$$m\ddot{x} + c\dot{x} + kx = f(t), \tag{21}$$

Assume:

$$x(t) = Xe^{i\omega t}, \tag{22}$$

and:

$$f(t) = Fe^{i\omega t}, \tag{23}$$

By substituting (22) and (23) in (21), we have:

$$(-\omega^2 m + i\omega c + k)Xe^{i\omega t} = Fe^{i\omega t}, \tag{24}$$

So, the frequency response function (FRF) is determined as:

$$H(\omega) = \frac{X}{F} = \frac{1}{(k-\omega^2 m) + i\omega c}, \tag{25}$$

The part $(k-\omega^2 m)$ is the real part and the part $i\omega c$ is the imaginary part of (25). The magnitude of a SDOF system is given as (26) and shown in Fig. 4:

$$|H(\omega)| = \frac{1}{\sqrt{(k-\omega^2 m)^2 + (\omega c)^2}}, \tag{26}$$

Then the phase angle is (27) and Fig. 5:

$$\prec H(\omega) = Arc\tan g\left(\frac{-\omega c}{k-\omega^2 m}\right), \tag{27}$$

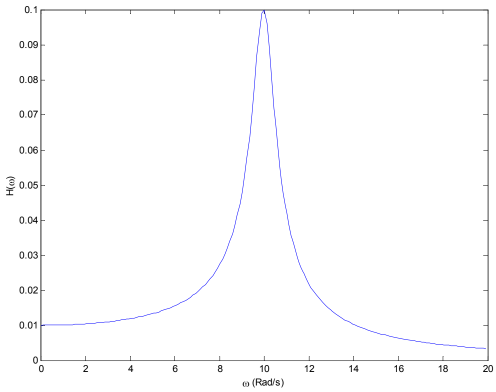

Fig. 4 FRF of the single degree of freedom

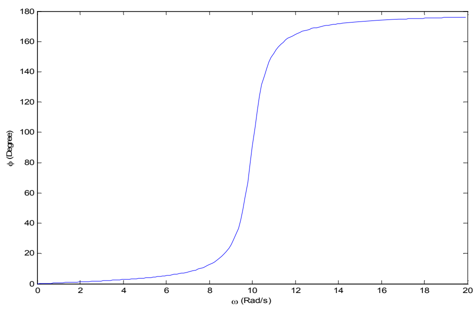

Fig. 5 The phase angle of the system

The relation between receptance and mobility is:

$$x(t) = Xe^{i\omega t}, \tag{28}$$

$$v(t) = \dot{x}(t) = i\omega Xe^{i\omega t} = Ve^{i\omega t}, \tag{29}$$

$$Y(\omega) = \frac{V}{F} = \frac{i\omega X}{F} = i\omega H(\omega) \ , \tag{30}$$

$$|Y(\omega)| = \omega |H(\omega)|, \tag{31}$$

$$\theta_Y = \theta_H + 90, \tag{32}$$

The relation between receptance and accelerance is:

$$x(t) = Xe^{i\omega t}, \tag{33}$$

$$a(t) = \ddot{x}(t) = -\omega^2 Xe^{i\omega t} = Ae^{i\omega t}, \tag{34}$$

$$A(\omega) = \frac{A}{F} = \frac{-\omega^2 X}{F} = -\omega^2 H(\omega) \ , \tag{35}$$

$$|A(\omega)| = \omega^2 |H(\omega)|, \tag{36}$$

$$\theta_A = \theta_H + 180, \tag{37}$$

C. Frequency Response of a Damped MDOF System

Consider the general equation of motion for a MDOF with a viscous damping [10]:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\}, \tag{38}$$

$$\alpha = \zeta\omega_0, \quad \omega_d = \omega_0\sqrt{1-\zeta^2} \tag{39}$$

$$\omega_0^2 = k/m\,, \ \zeta = c/(2\sqrt{km}) \tag{40}$$

Stiffness and mass matrix are:

$$[C] = \beta[K] + \gamma[M] \tag{41}$$

and then:

$$[\Psi]^T[C][\Psi] = \beta[k_r] + \gamma[m_r], \tag{42}$$

Let's define:

$$\{x\} = [\Psi]\{p\}, \tag{43}$$

Subsisting in (43) in (38) and considering (f =0), we have:

$$[m_r]\{\ddot{p}\} + [c_r]\{\dot{p}\} + [k_r]\{p\} = \{0\}, \tag{44}$$

So for the damped system:

$$\omega_d = \omega_r\sqrt{1-\zeta_r^2}\ , \ \zeta_r = 0.5\beta\omega_r + 0.5\gamma/\omega_r\,, \tag{45}$$

The reacceptance matrix can be defined as (46):

$$H_{jk}(\omega) = \sum_{r=1}^{N} \frac{\psi_{jr}\psi_{kr}}{(k_r - \omega^2 m_r) + i\omega c_r}, \tag{46}$$

For MDOF, the relation between reacceptance and mobility is (30) and the relation between receptance and interance is (35) too.

III. Finite Element Modeling

The ANSYS 15.0 finite element software [18] is used to develop the model and extract free vibration of the un-cracked and cracked cantilever beam. For this purpose, in this study, free vibration of a one end fixed (cantilever) beam with a V-shaped edge crack is studied. First, the key points are created and then line segments are formed and combined to create an area. Finally, this area is extruded one crack configurations as shown in Figs. 6-8 are prepared to find out how the crack affects dynamic behavior of the beam keeping. Crack location from cantilever end constant, crack depth is increased gradually from 20% to 80% of the width to observe the effect of crack depth on natural frequencies and forced response of the beam. Additionally, depth of the crack on the top surface is chosen constant as "4.5 mm" and crack positions were varied as 300 mm from cantilever beam end (middle position of the beam). Then the authors choose the depth of the cracks to be "18 mm" as 80%, while the length and cross-sectional area of the beam are ''600 mm,'' and ''22.5×2.57 mm^2,'' respectively. As for the material properties the modulus of elasticity "E" is "70×10^9 N/m^2", the density is ''2700 kg/m^3'' and the Poisson's ratio is ''Z (0.3).'' A 20-nodes three-dimensional structural solid element under "SOLID186" is selected to model the beam. Figs. 9-11 show the finite element model of the beam.

IV. Results

Free vibration analysis of a cracked beam is done for various crack conditions to obtain natural frequencies and dynamic responses of the beam. These results are used to identify the depth and location of the cracks. In this regard, after codes are written in Matlab2015a [19], we calculate the receptance of all modes and then plot them. H_{11} plot is shown in Figs. 12, 13. However, it is suggested that these theoretical data to be compared to experimental Results (see Table I).

V. Conclusion

We conclude that as the crack depth increases:

(1) Natural frequency of the beam will experience a sharper decrease in comparison to its normal condition.
(2) The numbers that are determined from codes in Matlab, the results from the exact solution and what are determined from the finite element model, are almost equal, with negligible errors.

TABLE I
COMPARISON OF EXACT SOLUTION, FEM, ERROR

Mode num.	Exact solution	Frequency (Hertz) obtained in FEM		Frequency (Hertz) obtained in FEM		% Error (20% cracked)	% Error (80% cracked)
	Normal cantilever beam	Normal beam	20% Cracked beam	Normal beam	80% Cracked beam		
1	36.8901	38.0585	37.3806	38.0585	37.5803	1.78	1.26
2	231.2029	248.6633	236.7190	248.6633	244.3782	4.80	1.72

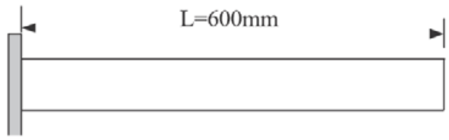

Fig. 6 A Cantilever beam without crack

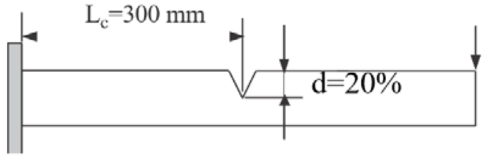

Fig. 7 Single cracked in 20% of width

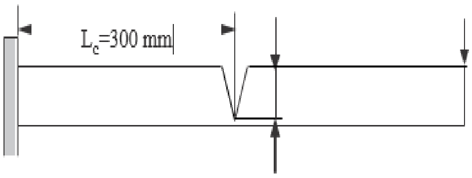

Fig. 8 Single cracked in 80% of width

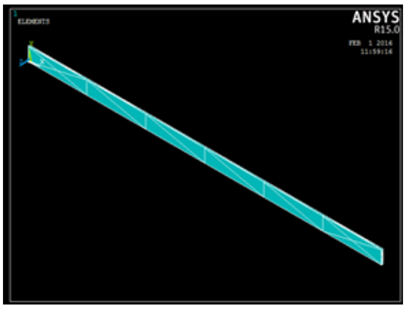

Fig. 9 Finite element modeling of the normal beam

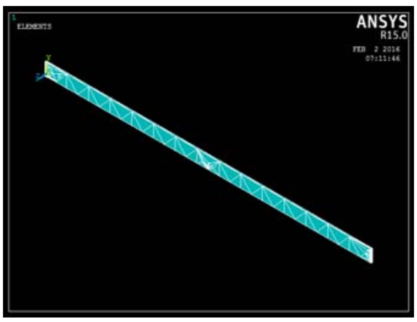

Fig. 10 Finite element modeling of the 20% cracked beam

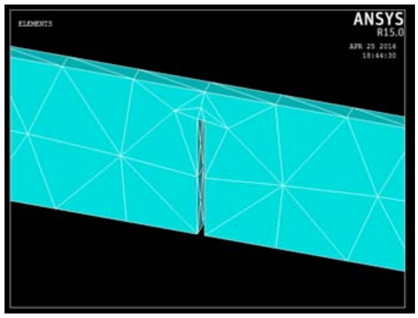

Fig. 11 A part of finite element modeling of the 80% cracked beam

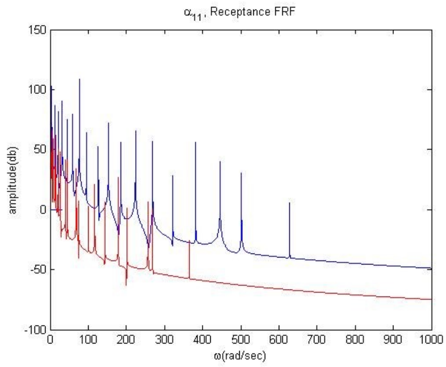

Fig. 12 The measured receptance for 20% cracked beam

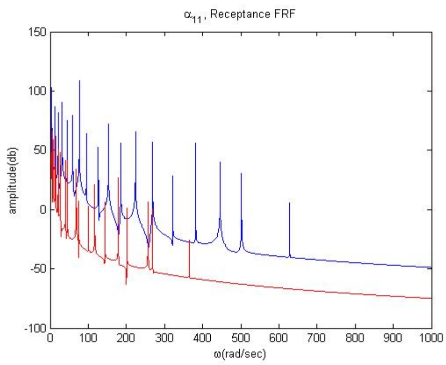

Fig. 13 The measured reacceptance for 80%cracked beam

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