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The convergence results between backward USSOR and Jacobi iterative matrices

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Abstract—In this paper, the backward USSOR iterative matrix is proposed. The relationship of convergence between the backward USSOR iterative matrix and Jacobi iterative matrix is obtained, which makes the results in the corresponding references be improved and refined. Moreover, numerical examples also illustrate the effectiveness of these conclusions.

Keywords—Backward USSOR iterative matrix, Jacobi iterative matrix, convergence, spectral radius

I. Introduction

To solve the equations

$$Ax = b, \tag{1}$$

where $A = [a_{ij}]$ is a given $n \times n$ complex matrix and nonsingular, iterative methods are always employed.

Let $A = D - C_L - C_U$ where $D = \operatorname{diag}(A)$ is a diagonal matrix obtained from A and nonsingular, $-C_L$ and $-C_U$ are strictly lower and upper triangular matrices obtained from A, respectively. We also let $L = D^{-1}C_L$, $U = D^{-1}C_U$. The equation (1) becomes the equivalent one

$$(I - L - U)x = D^{-1}Ax = D^{-1}b. \tag{2}$$

The Jacobi iterative matrix is

$$B = L + U = I - D^{-1}A.$$

The USSOR and some other iterative methods are studied in [3-5]. Here, we give the backward USSOR iterative matrix as follows:

$$\begin{aligned}\varphi_{\omega_1,\omega_2} &= (I - \omega_2 L)^{-1}[(1-\omega_2)I + \omega_2)U](I - \omega_1 U)^{-1} \\ &\quad [(1-\omega_1)I + \omega_1)L],\end{aligned}$$

or equivalently,

$$\begin{aligned}\varphi_{\omega_1,\omega_2} &= (I - \omega_2 L)^{-1}(I - \omega_1 U)^{-1}[(1-\omega_2)I + \omega_2)U] \\ &\quad [(1-\omega_1)I + \omega_1)L],\end{aligned} \tag{3}$$

with special values of ω_1, ω_2, we have

(1) When $\omega_1 = \omega_2 = \omega$, we obtain the backward SSOR iterative method;

(2) When $\omega_1 = \omega$, $\omega_2 = 0$, we obtain the backward SOR iterative method;

(3) When $\omega_1 = 1$, $\omega_2 = 0$, we obtain the backward G-S iterative method;

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The convergence relationship between the Gauss-Seidel iterative matrix and the Jacobi iterative matrix is studied in [1], and the generalized results are studied in [6]. Some eigenvalue relationships between other iterative matrices and Jacobi iterative matrix are studied with the p-cyclic case in [7-14]. In the following we consider the convergence results between the backward USSOR iterative matrix and the Jacobi iterative matrix, and obtain convergence relationships between some other backward iterative matrices and Jacobi matrix.

II. Preliminary

Definition 2.1([2]) *The splitting $A = M - N$ with A and M nonsingular is called a regular splitting if $M^{-1} \geq 0$ and $N \geq 0$. It is called a weak regular splitting if $M^{-1} \geq 0$ and $M^{-1}N \geq 0$.*

It is obvious that a regular splitting is a weak regular splitting.

Lemma 2.1([2]) *The nonnegative matrix $T \in R^{n\times n}$ is convergent, that is, $\rho(T) < 1$ if and only if $(I - T)^{-1}$ exists and $(I - T)^{-1} = \sum_{k=1}^{\infty} T^k \geq 0$.*

Lemma 2.2([2]) *Let $A = M - N$ be a weak regular splitting of A, $H = M^{-1}N$. Then the following statements are equivalent:*

(1) $A^{-1} \geq 0$; that is, A is inverse-positive.

(2)$A^{-1}N \geq 0$.

(3) $\rho(H) = \frac{\rho(A^{-1}N)}{1+\rho(A^{-1}N)}$ so that $\rho(H) < 1$.

Lemma 2.3([1]) *Let $A \geq 0$ be an irreducible $n \times n$ matrix. Then*

(1) A has a positive real eigenvalue equal to its spectral radius.

(2) To $\rho(A)$, there corresponds an eigenvector $x > 0$.

(3) $\rho(A)$ increases when any entry of A increases.

(4) $\rho(A)$ is a simple eigenvalue os A.

Lemma 2.4([1]) *Let $A = (a_{ij}) \geq 0$ be an irreducible $n \times n$ matrix. Then for any $x > 0$, either*

$$\min_{1\leq i\leq n} \frac{\sum_{j=1}^{n} a_{ij}x_j}{x_i} < \rho(A) < \max_{1\leq i\leq n} \frac{\sum_{j=1}^{n} a_{ij}x_j}{x_i},$$

or

$$\frac{\sum_{j=1}^{n} a_{ij}x_j}{x_i} = \rho(A), \qquad \forall i$$

III. Main results

Theorem 3.1 *Let the coefficient matrix A of (1) be irreducible with $a_{ii} \neq 0, \forall i$, $B = U + L \geq 0$ be the Jacobi matrix* *and $\varphi_{\omega_1,\omega_2}$ be the backward USSOR iterative matrix. Then, for $0<\omega_k<1, k=1,2,$ we have:*

(1) $\rho(B)>0$, $\rho(\varphi_{\omega_1,\omega_2})>(1-\omega_1)(1-\omega_2)$.

(2) *one and only one of the following mutually exclusive relations is valid:*

(i) $0<\rho(B)<1 \Longleftrightarrow (1-\omega_1)(1-\omega_2)<\rho(\varphi_{\omega_1,\omega_2})<1$.

(ii) $\rho(B)=1 \Longleftrightarrow \rho(\varphi_{\omega_1,\omega_2})=1$.

(iii) $\rho(B)>1 \Longleftrightarrow \rho(\varphi_{\omega_1,\omega_2})>1$.

Thus, the Jacobi iterative method and the backward USSOR iterative method are either both convergent, or both divergent.

Proof. Combining $\rho(\omega_2 L)=\rho(\omega_1 U)=0$ with lemma 2.1, we have $I-\omega_2 L)^{-1}\geq 0$,$(I-\omega_1 U)^{-1}\geq 0$, and

$$\begin{aligned}\varphi_{\omega_1,\omega_2} &= (I-\omega_2 L)^{-1}(I-\omega_1 U)^{-1}[(1-\omega_2)I+\omega_2)U] \\ &\quad [(1-\omega_1)I+\omega_1)L], \\ &= (I+\omega_2 L+\omega_2^2L^2+\cdots)(I+\omega_1 U+\omega_1^2U^2+\cdots) \\ &\quad [(1-\omega_1)(1-\omega_2)I+\omega_2(1-\omega_1)U+\omega_1(1-\omega_2)L \\ &\quad +\omega_1\omega_2 UL] \\ &\geq (1-\omega_1)(1-\omega_2)I+\omega_2(1-\omega_1)U+\omega_1(1-\omega_2)U.\end{aligned} \tag{4}$$

Since $a_{ii}\neq 0$ and A is irreducible, $I-L-U=D^{-1}A$ and $B=L+U$ are irreducible. By $0<\omega_k<1, k=1,2,$ we have $(1-\omega_1)(1-\omega_2)I+\omega_2(1-\omega_1)U+\omega_1(1-\omega_2)U\geq 0$ and irreducible. Thus, by (4), $\varphi_{\omega_1,\omega_2}\geq 0$ and irreducible. By lemma 2.3, there exists $\lambda=\rho(\varphi_{\omega_1,\omega_2})>0$ and corresponding vector $x=(x_1,x_2,\cdots,x_n)^T>0$, such that $\varphi_{\omega_1,\omega_2}x=\lambda x$, namely,

$$[(1-\omega_2)I+\omega_2 U][(1-\omega_1)I+\omega_1 L]x=\lambda(I-\omega_1 U)(I-\omega_2 L)x$$

Let $\eta=\omega_2(1-\omega_1)+\lambda\omega_1$, $\xi=\omega_1(1-\omega_2)+\lambda\omega_2$, and $\tau=(1-\omega_1)(1-\omega_2)$, by calculation,

$$\eta Ux+\xi Lx+(1-\lambda)\omega_1\omega_2 ULx=(\lambda-\tau)x, \tag{5}$$

that is,

$$\eta Ux+\xi Lx=(\lambda-1)\omega_1\omega_2 ULx+(\lambda-\tau)x. \tag{6}$$

(1) Since $B\geq 0$ is irreducible, by lemma 2.3, $\rho(B)>0$. If $\rho(\varphi_{\omega_1,\omega_2})\geq 1$, by $0<\omega_k<1, k=1,2,$ we know that $|\omega_k-1|<1, k=1,2,$ and $\tau=|(1-\omega_1)(1-\omega_2)|=|\omega_1-1||\omega_2-1|<1$, thus $\rho(\varphi_{\omega_1,\omega_2})\geq 1>1-\tau$. If $\rho(\varphi_{\omega_1,\omega_2})<1$, then $\lambda\geq\tau$ because the left side of (5) is nonnegative. By (5),

$$\eta Ux+\xi Lx\leq(\lambda-\tau)x. \tag{7}$$

If $\lambda=\tau$, by (7), we have $\eta Ux+\xi Lx\leq 0$, that is,

$$\eta\sum_{j=i+1}^{n}b_{ij}x_j+\xi\sum_{j=1}^{i-1}b_{ij}x_j\leq 0,\forall i.$$

Since $\eta>0,\xi>0$,$B=(b_{ij})\geq 0$ and $x>0$, we obtain that $B=0$. Thus, $\rho(B)=0$. This contradicts $\rho(B)>0$. So, $\rho(\varphi_{\omega_1,\omega_2})>1-\tau$.

(2) For mutually exclusive relations:

(i)If $0<\rho(B)<1$, let

$$M=(I-\omega_1 U)(I-\omega_2 L),$$

$$N=[\tau I+\omega_2(1-\omega_1)U+\omega_1(1-\omega_2)L+\omega_1\omega_2 UL],$$

then

$$\tilde{S}_{\tau,\omega_1,\omega_2}=M^{-1}N.$$

Since $M^{-1}=(I-\omega_2 L)^{-1}(I-\omega_1 U)^{-1}\geq 0$ and $N\geq 0$, $T=M-N$ is a regular splitting.

$$T=M-N=(1-\tau)(I-B)=(\omega_1+\omega_2-\omega_1\omega_2)(I-B).$$

By $B\geq 0$,$0<\rho(B)<1$, and $\omega_1+\omega_2-\omega_1\omega_2\leq 0$, we know that $T^{-1}=\frac{1}{1-\tau}(I-B)^{-1}=\frac{1}{\omega_1+\omega_2-\omega_1\omega_2}(I-B)^{-1}\geq 0$. By lemma 2.2, $\rho(\varphi_{\omega_1,\omega_2})=\rho(M^{-1}N)<1$. Combine this with the result in (1), we have $\tau<\rho(\varphi_{\omega_1,\omega_2})<1$.

If $\tau<\lambda=\rho(\varphi_{\omega_1,\omega_2})<1$, by (5), we have

$$\eta Ux+\xi Lx\leq(\lambda-\tau)x. \tag{8}$$

Since $\eta>0,\xi>0$, $b_{ii}=0,\forall i$,

$$\min\{\eta,\xi\}\sum_{j=1}^{n}b_{ij}x_j\leq\eta\sum_{j=i+1}^{n}b_{ij}x_j+\xi\sum_{j=1}^{i-1}b_{ij}x_j\leq(\lambda-\tau)x_i,\forall i,$$

that is,

$$\frac{\sum_{j=1}^{n}b_{ij}x_j}{x_i}\leq\frac{\lambda-\tau}{\min\{\eta,\xi\}},\qquad\forall i \tag{9}$$

By $\lambda<1$ and $1-\omega_k>0(k=1,2)$, there is

$$\lambda(1-\omega_k)<1-\omega_k,\ \ \lambda-1<-\omega_k+\lambda\omega_k,$$

$$0<\lambda-\tau=\lambda-1+\omega_1+\omega_2-\omega_1\omega_2<\omega_1+\omega_2-\omega_1\omega_2-\omega_k+\lambda\omega_k$$

Thus,

$$0<\frac{\lambda-\tau}{\eta}<1,\text{and }\ 0<\frac{\lambda-\tau}{\xi}<1. \tag{10}$$

Combining (9) with (10), we have

$$\frac{\sum_{j=1}^{n}b_{ij}x_j}{x_i}<1,\forall i.$$

By lemma 2.4, we obtain that $0<\rho(B)<1$.

(ii)If $\lambda=\rho(\varphi_{\omega_1,\omega_2})=1$, by (5), we have

$$(1-\tau)(U+L)x=(1-\tau)x,$$

namely, $Bx=x$. Since $x>0$, we have

$$\frac{\sum_{j=1}^{n}b_{ij}x_j}{x_i}=1,\forall i.$$

By lemma 2.4, we obtain that $\rho(B)=1$.

(iii) If $\lambda=\rho(\varphi_{\omega_1,\omega_2})>1$, by (6), we have

$$\eta Ux+\xi Lx\geq(\lambda-\tau)x. \tag{11}$$

Since $\eta>0,\xi>0$, $b_{ii}=0,\forall i$,

$$\max\{\eta,\xi\}\sum_{j=1}^{n}b_{ij}x_j\geq\eta\sum_{j=i+1}^{n}b_{ij}x_j+\xi\sum_{j=1}^{i-1}b_{ij}x_j\geq(\lambda-\tau)x_i,\forall i,$$

that is,

$$\frac{\sum_{j=1}^{n}b_{ij}x_j}{x_i}\geq\frac{\lambda-\tau}{\max\{\eta,\xi\}},\qquad\forall i \tag{12}$$

By $\lambda>1$ and $1-\omega_k>0(k=1,2)$, there is

$$\lambda(1-\omega_k)>1-\omega_k,\ \ \lambda-1>-\omega_k+\lambda\omega_k,$$

$$\lambda-\tau=\lambda-1+\omega_1+\omega_2-\omega_1\omega_2>\omega_1+\omega_2-\omega_1\omega_2-\omega_k+\lambda\omega_k,$$

Thus,

$$\frac{\lambda-\tau}{\eta} > 1, \text{and } \frac{\lambda-\tau}{\xi} > 1. \tag{13}$$

Combining (12) with (13), we have

$$\frac{\sum_{j=1}^{n} b_{ij}x_j}{x_i} > 1, \forall i.$$

By lemma 2.4, we obtain that $\rho(B) > 1$.

If $\rho(B) = 1$ and $\rho(\varphi_{\omega_1,\omega_2}) \neq 1$, by (1), we obtain that $\tau < \lambda = \rho(\varphi_{\omega_1,\omega_2}) < 1$ or $\rho(\varphi_{\omega_1,\omega_2}) > 1$. Thus, by (i) and (iii), we know that $0 < \rho(B) < 1$ or $\rho(B) > 1$. This contradicts $\rho(B) = 1$. So, $\rho(\varphi_{\omega_1,\omega_2}) = 1$.

If $\rho(B) > 1$ and $\rho(\varphi_{\omega_1,\omega_2}) \leq 1$, by (i) and (ii), we have $\rho(B) \leq 1$. This contradicts $\rho(B) > 1$. So, $\rho(\varphi_{\omega_1,\omega_2}) > 1$. Thus, the proof is completed. ■

With special values of ω_1, ω_2, we have the following corollaries:

Corollary 3.1 *Let the coefficient matrix A of (1) be irreducible, $B = U + L \geq 0$ be the Jacobi matrix and $\varphi_{\omega,\omega}$ be the backward SSOR iterative matrix. Then, for $0 < \omega < 1$, we have:*

(1) $\rho(B) > 0$, $\rho(\varphi_{\omega,\omega}) > (1-\omega)^2$.

(2)*one and only one of the following mutually exclusive relations is valid:*

(i) $0 < \rho(B) < 1 \Longleftrightarrow (1-\omega)^2 < \rho(\varphi_{\omega,\omega}) < 1$.

(ii) $\rho(B) = 1 \Longleftrightarrow \rho(\varphi_{\omega,\omega}) = 1$.

(iii) $\rho(B) > 1 \Longleftrightarrow \rho(\varphi_{\omega,\omega}) > 1$.

Thus, The Jacobi iterative method and the backward SSOR iterative method are either both convergent, or both divergent.

Corollary 3.2 *Let the coefficient matrix A of (1) be irreducible, $B = U + L \geq 0$ be the Jacobi matrix and $\varphi_{\omega,0}$ be the backward SOR iterative matrix. Then, for $0 < \omega < 1$, we have:*

(1) $\rho(B) > 0$, $\rho(\varphi_{\omega,0} > 1-\omega$.

(2)*one and only one of the following mutually exclusive relations is valid:*

(i)$0 < \rho(B) < 1 \Longleftrightarrow 1-\omega < \rho(\varphi_{\omega,0}) < 1$.

(ii) $\rho(B) = 1 \Longleftrightarrow \rho(\varphi_{\omega,0}) = 1$.

(iii) $\rho(B) > 1 \Longleftrightarrow \rho(\varphi_{\omega,0}) > 1$.

Thus, The Jacobi iterative method and the backward SSOR iterative method are either both convergent, or both divergent.

Corollary 3.3 *Let the coefficient matrix A of (1) be irreducible, $B = U + L \geq 0$ be the Jacobi matrix and $\varphi_{1,0}$ be the backward Gauss-Seidel iterative matrix. Then, we have:*

(1) $\rho(B) > 0$, $\rho(\varphi_{1,0}) > 0$.

(2) *one and only one of the following mutually exclusive relations is valid:*

(i)$0 < \rho(B) < 1 \Longleftrightarrow 0 < \rho(\varphi_{1,0}) < 1$.

(ii) $\rho(B) = 1 \Longleftrightarrow \rho(\varphi_{1,0}) = 1$.

(iii) $\rho(B) > 1 \Longleftrightarrow \rho(\varphi_{1,0}) > 1$.

Thus, The Jacobi iterative method and the backward Gauss-Seidel iterative method are either both convergent, or both divergent.

IV. Numerical example

Example 4.1 Let the coefficient matrix A of (1) be

$$A = \begin{bmatrix} 4 & -1 & -1 \\ 0 & 3 & -1 \\ 3 & 3 & -3 \end{bmatrix}.$$

The Jacobi iterative matrix is

$$B = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} \\ 1 & 1 & 0 \end{bmatrix}.$$

By caculation, we obtain $\rho(B) = \frac{1105}{1336} < 1$.

(1) Let $\omega_1 = 1$, $\omega_2 = 0$. We obtain the backward Gauss-seidel iterative matrix :

$$G_1 = \varphi_{1,0} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 1 & 1 & 0 \end{bmatrix},$$

and the Gauss-seidel iterative matrix

$$G_2 = \begin{bmatrix} 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{4} & \frac{7}{12} \end{bmatrix}.$$

$$\rho(G_1) = \frac{2}{3} < \frac{1138}{1621} = \rho(G_2) < \frac{1105}{1336} = \rho(B) < 1,$$

that is the backward Gauss-Seidel, Gauss-Seidel and Jacobi iterations converge and the backward Gauss-Seidel iteration is the best one.

(2) Let $\omega_1 = \omega = \frac{1}{2}$, $\omega_2 = 0$. We obtain the backward SOR iterative matrix :

$$G_1 = \varphi_{0.5,0} = \begin{bmatrix} \frac{55}{96} & \frac{13}{96} & \frac{7}{96} \\ \frac{1}{12} & \frac{7}{12} & \frac{1}{12} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix},$$

and the SOR iterative matrix

$$G_2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{8} & \frac{1}{8} \\ 0 & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{4} & \frac{5}{16} & \frac{31}{48} \end{bmatrix}.$$

$$(1-\frac{1}{2})^2 = \frac{1}{4} < \rho(G_1) = \frac{143}{161} < \frac{2408}{2699} = \rho(G_2) < 1,$$

that is, the backward SOR, SOR and Jacobi iterations converge and the backward SOR iteration is better than SOR iteration.

V. Conclusion

The convergence results between the backward USSOR and Jacobi iterative matrix is proposed, and The convergence results between some special cases of backward USSOR (such as backward SSOR, backward SOR, and backward Gauss-Seidel)and Jacobi iterative matrix are obtained. These results involve some special iterative methods which are proposed in the references. Numerical example also shows that the backward SOR and the backward Gauss-Seidel iterative methods are better than the corresponding methods under some circumstances.

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