The Control of a Highly Nonlinear Two-wheels Balancing Robot: A Comparative Assessment between LQR and PID-PID Control Schemes

A. N. K. Nasir, M. A. Ahmad, R. M. T. Raja Ismail

Abstract**—**The research on two-wheels balancing robot has gained momentum due to their functionality and reliability when completing certain tasks. This paper presents investigations into the performance comparison of Linear Quadratic Regulator (LQR) and PID-PID controllers for a highly nonlinear 2–wheels balancing robot. The mathematical model of 2-wheels balancing robot that is highly nonlinear is derived. The final model is then represented in state-space form and the system suffers from mismatched condition. Two system responses namely the robot position and robot angular position are obtained. The performances of the LQR and PID-PID controllers are examined in terms of input tracking and disturbances rejection capability. Simulation results of the responses of the nonlinear 2–wheels balancing robot are presented in time domain. A comparative assessment of both control schemes to the system performance is presented and discussed.

Keywords**—**PID, LQR, Two-wheels balancing robot.

I. Introduction

THE research on two-wheeled balancing robot has gained momentum over the last decade due to the nonlinear and unstable dynamics system. Various control strategies had been proposed by numerous researchers to control the two-wheeled balancing robot such that the robot able to balance itself. Two wheels balancing robot is a good platform for researchers to investigate the efficiency of various controllers in control system. The research on two wheels balancing robot is based on inverted pendulum model. Thus, a two wheels balancing robot needs a good controller to control itself in upright position without the needs from outside.

Motion of two wheels balancing robot is governed by under-actuated configuration, i.e., the number of control inputs is less than the number of degrees of freedom to be stabilized [1], which makes it difficult to apply the conventional robotics approach for controlling the systems. Due to these reasons, increasing effort has been made towards control designs that guarantee stability and robustness for mobile wheeled inverted pendulums. Although two wheels balancing robot are intrinsically nonlinear and their dynamics will be described by nonlinear differential equations, it is often possible to obtain a linearized model of the system. If the system operates around an operating point, and the signals involved are small signals, a linear model that approximates the nonlinear system in the region of operation can be obtained. Several techniques for the design of controllers and analysis techniques for linear systems were applied. In [2], motion control was proposed using linear state-space model. In [3], dynamics was derived using a Newtonian approach and the control was design by the equations linearized around an operating point. In [4], the dynamic equations were studied, with the balancing robot pitch and the rotation angles of the two wheels as the variables of interest, and a linear controller was designed for stabilization under the consider of its robustness in [5]. In [6], a linear stabilizing controller was derived by a planar model without considering vehicle yaw. The above control laws are designed on the linearized dynamics which only exhibits desirable behavior around the operating point, and do not have global applicability. In [7], the exact dynamics of two wheels inverted pendulum was investigated, and linear feedback control was developed on the dynamic model. In [8], a two-level velocity controller via partial feedback linearized and a stabilizing position controller were derived; however, the controller design is not robust with respect to parameter uncertainties. In [9], a controller using sliding mode approach was proposed to ensure robustness versus parameter uncertainties for controlling both the position and the orientation of the balancing robot.

The mathematical model is established through a modeling process where the system is identified based on the conservation laws and property laws. This process is crucial since a controller is design solely based on this mathematical model. Thus, an accurate equation must be derived in order for the controller to response accordingly.

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This paper presents investigations of performance comparison between conventional (PID-PID) and modern control (Linear Quadratic Regulator) schemes for a two wheels balancing robot. The mathematical model of the two wheels balancing robot system is presented in differential equation form. The dynamic model of the system with the permanent magnet DC motors dynamic included is derived based on [3] and [10]. Performances of both control strategy with respect to balancing robot outputs angular position θ and linear position x are examined. Comparative assessment of both control schemes to the two balancing robot system performance is presented and discussed.

II. Dynamic Model

Modeling is the process of identifying the principal physical dynamic effects to be considered in analyzing a system, writing the differential and algebraic equations from the conservative laws and property laws of the relevant discipline, and reducing the equations to a convenient differential equation model. This section provides a description on the modeling of the two wheels balancing robot, as a basis of a simulation environment for development and assessment of both control schemes. The robot with its three degrees of freedom is able to linearly move which is characterized by position x, able to rotate around the y-axis (yaw) with associated angle δ and able to rotate around z-axis (pitch) where the movement is described by angle θ. List of parameters for the two wheels balancing robot are shown in Table I. These parameters are based on the project conducted by Ooi (2003) as stated by [10]. The inputs of the system are the voltages V_{aR} and V_{aL} which both are applied to the two motors which located on right side and left side of the robot as shown in Fig. 1. In order to obtain the dynamic model of the balancing robot some assumptions and limitations are introduced:

1) Motor inductance and friction on the motor armature is neglected.
2) The wheels of the robot will always stay in contact with the ground.
3) There is no slip at the wheels.
4) Cornering forces are also negligible.

Fig. 2 shows a free body diagram of the balancing robot which contributed to the nonlinear dynamic equations of the system. Equation (1) represents linear acceleration in x direction, equation (2) represents angular acceleration about y-axis and equation (3) represents angular acceleration about z-axis.

TABLE I
List of Parameters of two-wheels balancing robot based on [11]

Symbol	Parameter	Value
D	distance between contact patches of the wheels	0.2 m
g	gravitational force	9.81 $m.s^{-2}$
Jp	chassis's inertia	0.0041 $kg.m^2$
$Jp\delta$	chassis's inertia during rotation	0.00018 $kg.m^2$
Jw	wheel's inertia	0.000039 $kg.m^2$
ke	back emf constant	0.006087 Vs/rad
km	motor torque constant	0.006123 Nm/A
l	distance between center of the wheels and the robot's CG	0.07 m
Mp	body's mass	1.13 kg
Mw	wheel's mass	0.03 kg
R	nominal terminal resistance	3 Ω
r	wheel's radius	0.051 m

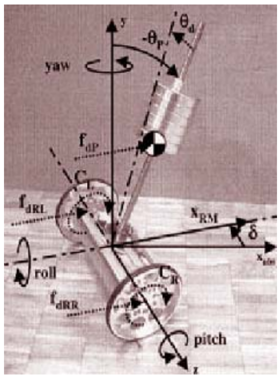

Fig. 1 A mobile balancing robot (Grasser et al., 2002)

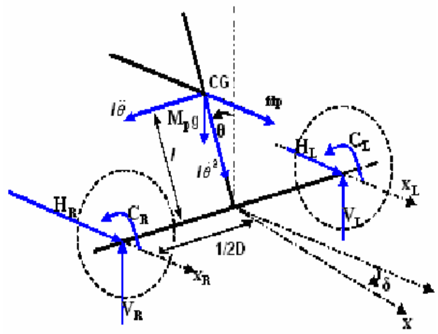

Fig. 2 Free body diagram of balancing robot

As can be seen from equations (1), (2), and (3), all nonlinear terms are remain in the equations. All these equations are used to design the proposed controllers which will be described in the section III.

$$\ddot{x} = -\frac{2k_e k_m}{\alpha R \beta}\left[\frac{1}{r} + \frac{M_p l \cos\theta}{\gamma}\right]\dot{x} + \frac{M_p^2 g l^2 \sin\theta\cos\theta}{\alpha\gamma\beta\theta}\theta + \frac{k_m}{\alpha R \beta}\left[\frac{1}{r} + \frac{M_p l \cos\theta}{\gamma}\right]V_{aR} + \frac{k_m}{\alpha R \beta}\left[\frac{1}{r} + \frac{M_p l \cos\theta}{\gamma}\right]V_{aL} + \frac{1}{\alpha\beta} f_{drR} + \frac{1}{\alpha\beta} f_{drL} + \frac{1}{\alpha\beta}\left[1 + \frac{M_p l^2 \cos^2\theta}{\gamma}\right] f_{dp} + \frac{M_p l \dot{\theta}^2 \sin\theta}{\alpha\beta}. \quad (1)$$

$$\ddot{\theta} = \frac{2k_e k_m}{\gamma R \beta}\left[1 + \frac{M_p l \cos\theta}{\alpha r}\right]\dot{x} + \frac{M_p g l \sin\theta}{\gamma\beta\theta}\theta - \frac{k_m}{\gamma R \beta}\left[1 + \frac{M_p l \cos\theta}{\alpha r}\right]V_{aR} - \frac{k_m}{\gamma R \beta}\left[1 + \frac{M_p l \cos\theta}{\alpha r}\right]V_{aL} - \frac{M_p l \cos\theta}{\alpha\gamma\beta} f_{drR} + \frac{M_p l \cos\theta}{\alpha\gamma\beta} f_{drL} + \frac{l\cos\theta}{\gamma\beta}\left[1 - \frac{M_p}{\alpha}\right] f_{dp} - \frac{M_p^2 l^2 \dot{\theta}^2 \sin\theta\cos\theta}{\alpha\gamma\beta}. \quad (2)$$

$$\ddot{\delta} = -\frac{k_m D}{2J_{p\delta} r R} V_{aR} + \frac{k_m D}{2J_{p\delta} r R} V_{aL} - \frac{D}{2J_{p\delta}} f_{drR} + \frac{D}{2J_{p\delta}} f_{drL}. \quad (3)$$

The symbols of α, β, and γ in equations (1), (2), and (3) are defined as in equation (4), (5), and (6):

$$\alpha = 2M_w + \frac{2J_w}{r^2} + M_p \quad (4)$$

$$\beta = \frac{\alpha\gamma - M_p^2 l^2 \cos^2\theta}{\alpha\gamma} \quad (5)$$

$$\gamma = J_p + M_p l^2 \quad (6)$$

III. CONTROLLER DESIGN & SIMULATION

In this section, two control schemes (LQR and PID) are proposed and described in detail. Furthermore, the following design requirements have been made to examine the performance of both control strategies.

1) The system overshoot (*%OS*) of robot position, x is to be at most 25%.
2) The Rise time (T_r) of robot position, x less than 5 s.
3) The settling time (T_s) of robot position, x and robot angle θ is to be less than 10 seconds.
4) Steady-state error is within 2% of the initial value.

A. PID Controller

PID stands for Proportional-Integral-Derivative. This is a type of feedback controller whose output, a control variable (CV), is generally based on the error (e) between defined set point (SP) and some measured process variable (PV). Each element of the PID controller refers to a particular action taken on the error. In order to demonstrate the performance of the PID controller in locating the balancing robot to its desired position and angle, the collocated sensor signal of the position of the robot about roll axis, $x(s)$ and angular position of the robot about yaw axis $\theta(s)$ are fed back and compared to the reference position, $x_f(s)$ and angle $\theta_f(s)$ respectively. Initially, the angular position of the robot which is position about pitch axis is set 50 degrees or 0.8727 radians. In this study, two PID controllers are required to control the position on the roll axis and the angular position about the yaw axis. The position and angular position errors are regulated through the proportional, integral and derivative gain for each PID. Block diagram of the PID controller is shown in Fig. 3, where $u_1(s)$ and $u_2(s)$ represent the applied voltage at the right motor and left motor respectively. Both of the inputs of the balancing robot are limited to 20 volts to –20 volts. The control signal $u_1(s)$ and $u_2(s)$ in Fig. 3 can be represented as in equations (7) and (8) respectively:

$$u_{\text{PID}}(s)_{\text{position}} = -\left(K_{P1} + \frac{K_{I1}}{s} + K_{D1}s\right)[r(s) - r_f(s)] \quad (7)$$

$$u_{\text{PID}}(s)_{\text{angle}} = -\left(K_{P2} + \frac{K_{I2}}{s} + K_{D2}s\right)[\theta(s) - \theta_f(s)] \quad (8)$$

where s is the Laplace variable. Hence the closed-loop transfer function is obtained as in equation (9) and (10).

$$\frac{r(s)}{r_f(s)} = \frac{\left(K_{P1} + K_{D1}s + \frac{K_{I1}}{s}\right)G(s)}{1 + \left(K_{P1} + K_{D1}s + \frac{K_{I1}}{s}\right)G(s)} \quad (9)$$

$$\frac{\theta(s)}{\theta_f(s)} = \frac{\left(K_{P2} + K_{D2}s + \frac{K_{I2}}{s}\right)G(s)}{1 + \left(K_{P2} + K_{D2}s + \frac{K_{I2}}{s}\right)G(s)} \quad (10)$$

In this paper, the Ziegler-Nichols approach is utilized to design both PID controllers. Analyses the tuning process of the proportional, integral and derivative gains using Ziegler-Nichols technique shows that the optimum response of PID controller for controlling linear position is achieved by setting $K_{P1} = -8$, $K_{I1} = -0.921$ and $K_{D1} = -6$, while for controlling angular position, $K_{P2} = -63$, $K_{I2} = -60$ and $K_{D2} = -11$. All the PID1 and PID2 controller parameters must be tuned simultaneously to achieve the best responses as desired.

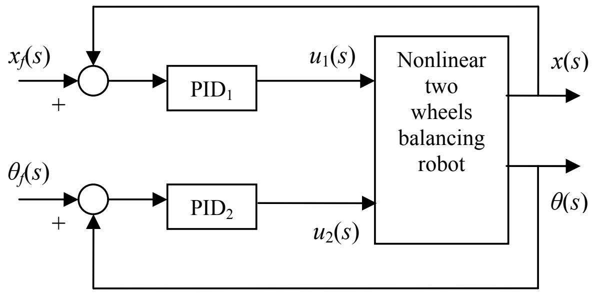

Fig. 3 Block diagram of PID controller

B. LQR Controller

LQR is a method in modern control theory that uses state-space approach to analyze such a system. Using state-space methods it is relatively simple to work with a multi-output system. The system can be stabilized using full state feedback. The schematic of this type of control system is shown in Fig. 4.

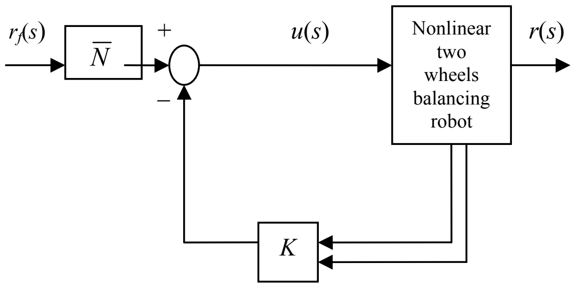

Fig. 4 The LQR control structure

For a LTI system, the technique involves choosing a control law $u = \psi(x)$ which stabilizes the origin while minimizing the quadratic cost function which can be presented as in equation (11).

$$J = \int_0^\infty x(t)^T Qx(t) + u(t)^T Ru(t)dt \tag{11}$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$. The term "linear-quadratic" refers to the linear system dynamics and the quadratic cost function. A famous and somewhat surprising result due to Kalman is that the control law which minimizes J always takes the form $u = \psi(x) = Kx$. The optimal regulator for a LTI system with respect to the quadratic cost function above is always a linear control law. With this observation in mind, the closed-loop system takes the states space form as represented in equation (12).

$$\dot{x} = (A - BK)x \tag{12}$$

Substituting the control law into equation (11), the cost function J is represented as in equation (13).

$$J = \int_0^\infty x(t)^T Qx(t) + (-Kx(t))^T R(-Kx(t))dt \tag{13}$$

Equation (13) can be further simplified as represented equation (14).

$$J = \int_0^\infty x(t)^T (Q + K^T RK)x(t)dt \tag{14}$$

In designing LQR controller, LQR function in matlab m-file can be used to determine the value of the vector K which determines the feedback control law. This is done by choosing two parameter values, input R and $Q=C'xC$ where C is from state equation of the linearized model. The controller can be tuned by changing the nonzero elements in Q matrix which is done in m-file code. Consequently, by tuning the values of nonzero elements in matrix Q as shown in (15) and matrix R as shown in (16), the values of matrix K are obtained as represented in equation (17).

$$Q = [1\ 0\ 0\ 0\ 0\ 0;\ 0\ 1\ 0\ 0\ 0\ 0;\ 0\ 0\ 100\ 0\ 0\ 0;$$

$$0\ 0\ 0\ 100\ 0\ 0;\ \ 0\ 0\ 0\ 0\ 0.1\ 0;\ \ 0\ 0\ 0\ 0\ 0\ 0.01] \tag{15}$$

$$R = [1\ 0;\ 0\ 1] \tag{16}$$

The matrix R and matrix Q must be properly tuned to minimized the quadratic cost function as represented in (14).

$$K=[-5.9000\ \ -6.8040\ \ -34.1774\ \ -7.9286\ \ -1.7071\ \ -8.0903;$$

$$-5.9000\ \ -6.8040\ \ -34.1774\ \ -7.9286\ \ -0.7071\ \ -7.0903] \tag{17}$$

The matrix K is the LQR controller parameter which determines feedback control law $u = \psi(x) = Kx$.

IV. Result and Analysis

In this section, the simulation results of the proposed controller, which is performed on the model of a two wheels balancing robot are presented. Comparative assessment of both control strategies to the system performance are also discussed in details in this section.

Two wheels balancing robot systems with LQR and PID controller block diagram produced two responses, angular position θ and linear position x. As stated earlier, the initial value of the angular position θ of the balancing robot was set to 0.5 radians. It means that the initial condition of the robot is very unstable. Fig. 5 shows the comparison of the balancing robot linear position response between LQR and PID controller graphically. In this figure, the response for the linear position of the robot with PID controller is represented by straight line or blue color line and the response for the linear position of the robot with LQR controller is represented by dotted line or red color line. Fig. 5 shows that both of the controllers are capable to control the linear position of the nonlinear two wheels balancing robot.

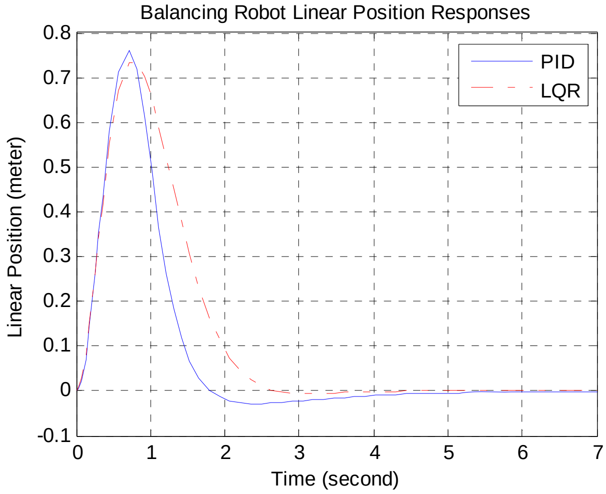

Fig. 5 Two-Wheel Balancing Robot Linear Position Response

Table II shows the summary of the performance characteristics of the balancing robot linear position between LQR and PID controller quantitatively. Based on the data tabulated in Table II, LQR has the fastest settling time of 2.38 seconds while PID has the slowest settling time of 2.68 seconds. An extra of 0.3 seconds is required for the PID controller balancing robot to balance itself. Similarly, for the maximum overshoot, LQR controller has the best overshoot which is the lowest overshoot between two controllers. The maximum displacement of the balancing robot when LQR control signal applied to the system is 0.73 meters while maximum displacement of the balancing robot when PID control signal applied to the system is 0.77 meters. A distance of minimum 0.04 meters is required for the PID controller balancing robot to balance itself. Despite the large initial values for the displacement, the proposed LQR controller is able to bring itself to the vertical position. In term of the rise time, balancing robot with PID controller has the fastest rise time 0.37 seconds while balancing robot with LQR controller needs an extra time of 0.02 seconds to rise from 10% to the 90% of the initial value. In term of steady state error, both of the controllers had shown very outstanding performance by giving zero error at time 6 seconds and more. The responses of the balancing robot linear position have acceptable overshoot and undershoot.

Fig. 6 shows the balancing robot with LQR and PID controller angular position responses. It shows that the result has got similar pattern and not much different.

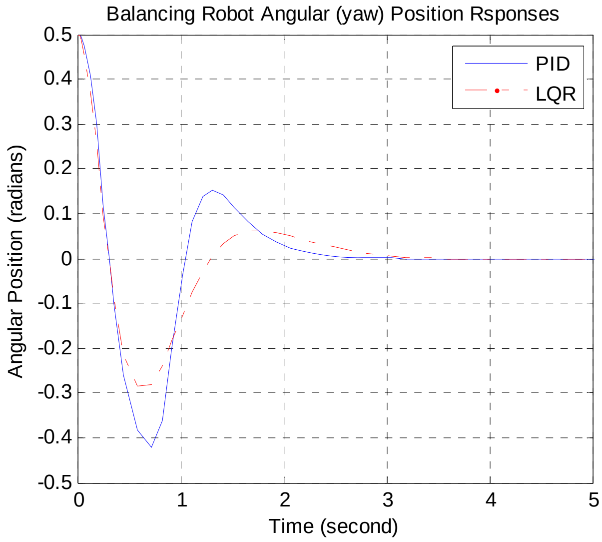

Fig. 6 Two-Wheels Balancing Robot Angular Position Response

The initial value of the balancing robot angular position is 0.5 radians. The robot needs to balance itself by eliminating the angular position so that the body of the robot remains vertically straight in upright position. Fig. 6 shows that both of the LQR and PID controllers are capable of controlling the nonlinear unstable balancing robot.

Table III shows the summary of the performance characteristics of the balancing robot angular position between LQR and PID controller quantitatively. Based on the data tabulated in Table III, PID has the fastest settling time of 2.45 seconds while LQR has the slowest settling time of 2.59 seconds. An extra time of 0.14 seconds is required for the PID controller balancing robot to balance itself. In contrast, for the maximum undershoot, LQR controller has the best undershoot which is the lowest undershoot between two controllers. The maximum angular displacement of the balancing robot when LQR control signal applied to the system is –0.29 radians while maximum angular displacement of the balancing robot when PID control signal applied to the system is –0.38 radians. An extra angle of minimum 0.01 meters is required

TABLE II
SUMMARY OF PERFORMANCE CHARACTERISTICS OF THE BALANCING ROBOT LINEAR POSITION BETWEEN LQR AND PID

Time Response Spesification	LQR	PID
Rise Time	0.39 sec	0.37sec
Settling Time	2.38 sec	2.68 sec
Steady state error	0.00	0.00
Maximum overshoot	0.73meter	0.77 meter

TABLE III
SUMMARY OF PERFORMANCE CHARACTERISTICS OF THE BALANCING ROBOT ANGULAR POSITION BETWEEN LQR AND PID

Time Response Spesification	LQR	PID
Rise Time	0.22 sec	0.26sec
Settling Time	2.59 sec	2.45 sec
Steady state error	0.00	0.00
Maximum undershoot	0.29radians	0.38 radians

for the PID controller balancing robot to balance itself. Despite the large initial values for the displacement, the proposed LQR controller is able to bring itself to the vertical position. In term of the rise time, balancing robot with LQR controller has the fastest rise time 0.22 seconds while balancing robot with PID controller needs an extra time of 0.04 seconds to rise from 10% to the 90% of the initial value. In term of steady state error, both of the controllers had shown very outstanding performance by giving zero error at time 4 seconds and more.

The responses of the balancing robot angular position have acceptable overshoot and undershoot. Performance characteristics for linear and angular position represented in bar chart form are shown in Fig. 7 and Fig. 8 respectively.

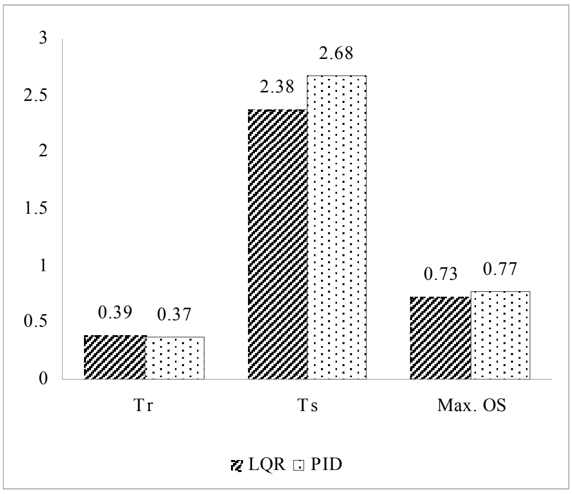

Fig. 7 Performance characteristics for linear position

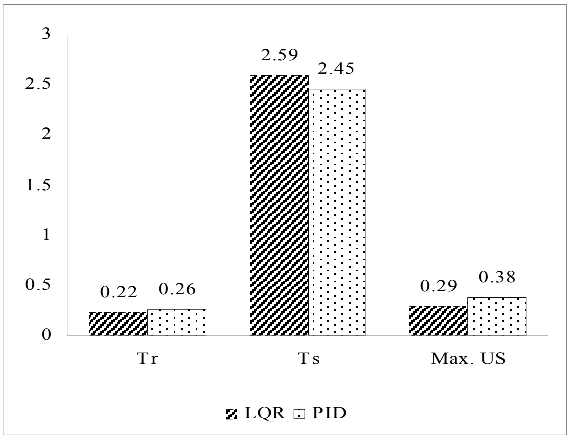

Fig. 8 Performance characteristics for angular position

V. Conclusion

In this paper, two controllers such as LQR and PID are successfully designed. Based on the results and the analysis, a conclusion has been made that both of the control method, modern controller (LQR) and conventional controller (PID) are capable of controlling the nonlinear two wheels balancing robot angular and linear position. All the successfully designed controllers were compared. The responses of each controller were plotted in one window and are summarized in Table II and Table III. Simulation results and bar charts in Fig. 7 and Fig. 8 show that LQR controller has better performance compared to PID controller in controlling the nonlinear balancing robot system. Further improvement need to be done for both of the controllers. PID controller should be improved so that the maximum overshoot and maximum undershoot for the linear and angular positions do not have very high range as required by the design criteria. On the other side, LQR controller can be improved so that it's settling time for angular position might be reduced as faster as PID controller.

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