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The Competitive Newsvendor Game with Overestimated Demand

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***Abstract*—**The tradition competitive newsvendor game assumes decision makers are rational. However, there are behavioral biases when people make decisions, such as loss aversion, mental accounting and overconfidence. Overestimation of a subject's own performance is one type of overconfidence. The objective of this research is to analyze the impact of the overestimated demand in the newsvendor competitive game with two players. This study builds a competitive newsvendor game model where newsvendors have private information of their demands, which is overestimated. At the same time, demands of each newsvendor forecasted by a third party institution are available. This research shows that the overestimation leads to demand steal effect, which reduces the competitor's order quantity. However, the overall supply of the product increases due to overestimation. This study illustrates the boundary condition for the overestimated newsvendor to have the equilibrium order drop due to the demand steal effect from the other newsvendor. A newsvendor who has higher critical fractile will see its equilibrium order decrease with the drop of estimation level from the other newsvendor.

***Keywords*—**Bias, competitive newsvendor, Nash equilibrium, overestimation.

I. Introduction

SINCE Arrow and Harris [1] first introduced the newsvendor problem in 1951, it has become an important and fundamental problem in operations research. In original setting, a single newsvendor has to decide the optimal order under a known demand probability distribution with the cost of order and the penalty of stock-out. The newsvendor is assumed to maximize his expected profit by trading off the losses of stock out and leftover. The newsvendor model contributes to the inventory management in the fashion and sport goods industries, as well as the retailing of the manufacturing products [2]. With the development of game theory, it is possible to analyze the equilibrium decision making between multiple players. In 1988, Polar first introduced the competition between two players in the newsvendor game with independent demand of each newsvendor [3]. In Polar's setting, each player's product is a substitute for the other with a known substitute rate when stocking out. Polar proved the existence and uniqueness of the Nash equilibrium in the competitive newsvendor game. Lippman and McCardle [4] extended newsvendor game with the competition between two newsvendors while demands are splitting from total demand.

Recently, studies of newsvendor game have been extended with irrational behaviors of newsvendor as it has been found that an individual's behavior is impacted by psychological biases, such as loss aversion and overconfidence. Loss aversion indicates that people prefer to avoid the loss rather than gain the profit [5]. Wang and Webster [6], [7] first studied the loss aversion newsvendor problem and game, and showed that the loss aversion newsvendor trends to order more compared with the unbiased newsvendor. When overconfidence is involved, it is more complex due to the existence of 3 kinds of overconfidence: 1) overestimate that people overrate their performance in prediction; 2) overprecise that people overrate the stability of their performance in prediction; and 3) overplacement that people overrate their performance compared to competitors in prediction.

It has been found that demand forecast of the new product trends to be overestimated. Compared the results from information acceleration method to forecast the sales of Toyota Celica with the actual sales in 1991, it turns out the information acceleration model have overestimated the sales by 10% [8]. There are a few explanations of the overestimation of demand. It has been shown that the boom and bust effect of the new product can lead to the overestimation by the newsvendors, wholesalers and manufacturers in the growth stage of demand [9]. For example, on January 2016, Apple, the largest the smart phone company in the world, cut iPhone's production by 30% in the next quarter compared to previous overestimated plan. Huang and Vir Singh [10] stated that company tends to overestimate the potential market of a new product. While the overconfidence bias has been discussed in a few competitive newsvendor games, this study makes the first step to analysis the impact of the overestimation on the competitive newsvendor game.

This research contributes to the newsvendor game by building a complex newsvendor game model with private overestimated demand information held by each player and demand information forecasted by a third party without the conflict of interest. Each newsvendor determine his order quantities based on the overestimated demand of his own firm and the other newsvendor's demand by the forecast from the third party institution. It also has been assumed that when stock out occurs, consumers will switch to the other newsvendor by customer search. This study indicates that if only one newsvendor overestimates his demand, the equilibrium order of the overestimated newsvendor increases while the unbiased newsvendor has equilibrium order decreased. In this process,

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the total supply of the product will increase. It also has been found when both newsvendors are overestimated, one of the newsvendors may have order dropped. The condition for order drop is related to overestimated levels of both players.

The layout of this paper is as follows: In Section II, after a brief discussion of unbiased model, overestimated competitive newsvendor game is analyzed with one overestimated player and two overestimated players in sequences. Section III draws the conclusion of the research and discusses the future study direction.

II. Mathematical Model

A. The Basic Competitive Newsvendor Problem

Research on mathematical model of the newsvendor problem can be dating back to 1988 when Edgeworth [11] studied the optimal cash reserves needed in the banks for random withdraws from depositors. In the competitive newsvendor game setting, there are two independent newsvendors selling the homogeneous product to consumers. Newsvendor i sets the product price as r_i with an average cost as c_i. In a single period, the random demand of the product for newsvendor i, D_i, follows a known distribution with a cumulative distribution function (CDF) $F_i(\cdot)$ and a probability distribution function (PDF) $f_i(\cdot)$. The CDF of demand $F_i(\cdot)$ is assumed to be continuous differentiable, invertible, and increasing. The product has no salvage value left at the end of period. At the same time, there are no penalty cost for the unsatisfied demand due to limited inventory. However, unsatisfied demand of newsvendor i will be transferred to the other newsvendor. The newsvendors have to determine an order quantity to maximize their expected profit at the beginning of the period. The expected profit function of newsvendor i is shown in (1)

$$\pi_i(Q_i, Q_j) = E_D\left(r_i \min\left(D_i + (D_j - Q_j)^+, Q_i\right) - c_i Q_i\right) \quad (1)$$

Based on the first order condition, the best response of the newsvendor i to maximize profits respecting to the newsvendor j's action is $B_i(Q_j) = F^{-1}_{D_i+(D_j-Q_j)^+}\left(\frac{r_i-c_i}{r_i}\right)$ where $\frac{r_i-c_i}{r_i}$ is the critical fractile. There exists a unique Nash equilibrium for competitive newsvendors (Q_i^*, Q_j^*), which meets (2):

$$\int_0^{Q_i^*} f_i(x) F_j(Q_i^* + Q_j^* - x)dx = \frac{r_i-c_i}{r_i} \quad (2)$$

Fig. 1 gives the Nash equilibrium point (Q_j^*, Q_i^*) and the best response curve of the newsvendors, while demands of the product for both newsvendors are exponential distributed. The best response of newsvendor i decreases with order quantity of newsvendor j. Therefore, the existence and uniqueness of the Nash equilibrium in the competitive newsvendor game can be guaranteed.

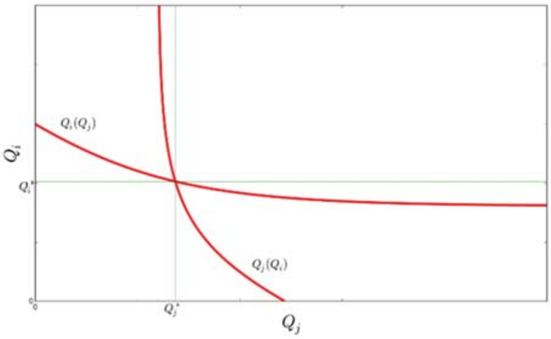

Fig. 1 The Nash equilibrium of competitive newsvendor game

B. The Competitive Newsvendor with Overestimation Bias

Compared with traditional competitive newsvendor game， newsvendors are assumed to be overestimated on their demand. Given that the demand forecasted by a third party institution D_i is available for all newsvendors, newsvendor i holds the belief that it should have a higher demand D_i' which follows a CDF $F_i'(\cdot)$ and CDF $f_i'(\cdot)$. Let D_i' be the overestimated demand, which follows the same distribution with unbiased demand with a higher mean value. Suppose the overestimated demand information is private. In this study, we let the forecasted demand for each newsvendor from the third party institution follows exponential distribution with a mean of $1/\lambda_i$. The overestimated demand of newsvendor i follows exponential distribution with a mean of $1/\lambda_i'$, where $\lambda_i > \lambda_i'$. Let $\Delta\lambda_i = \lambda_i - \lambda_i'$. Overestimation level of the newsvendor i is defined as $OL_i = \Delta\lambda_i/\lambda_i$

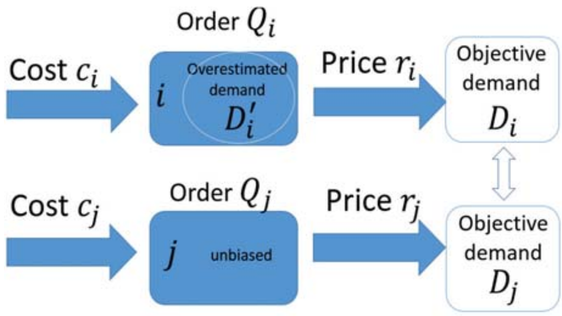

Fig. 2 Model structure with only one overestimated newsvendor

In the first place, we discuss the case that only one newsvendor is overestimating its demand, while the other newsvendor remains unbiased. Fig. 2 shows the model structure while only one newsvendor is overestimated. Therefore, the overestimated newsvendor will determine its equilibrium order by its own overestimated demand information and unbiased demand information provided by a third party. While the other newsvendor only responses to the public objective demand information.

Let newsvendor i's order quantity be biased by overestimation as Q_{i0}. The payoff function of overestimated newsvendor i and unbiased newsvendor j are shown in (3) and (4):

$$\pi_i(Q_{i0}, Q_{j0}) = E_D\left(r_i \min\left(D_i' + (D_j - Q_{j0})^+, Q_{i0}\right) - c_i Q_{i0}\right) \quad (3)$$

$$\pi_j(Q_{i0}, Q_{j0}) = E_D\left(r_j \min\left(D_j + (D_i - Q_{i0})^+, Q_{j0}\right) - c_j Q_{j0}\right) \quad (4)$$

According to the first order condition, the Nash equilibrium of competitive newsvendor's order (Q_{io}^*, Q_{jo}^*) satisfies (5) and (6):

$$\int_0^{Q_{jo}^*} f_i'(x) F_j'(Q_{io}^* + Q_{jo}^* - x)dx = \frac{r_i - c_i}{r_i} \quad (5)$$

$$\int_0^{Q_{io}^*} f_j(x) F_j(Q_{jo}^* + Q_{io}^* - x)dx = \frac{r_j - c_j}{r_j} \quad (6)$$

There still exists a unique Nash equilibrium of overestimated competitive newsvendor game with only one overestimated newsvendor. The proving follows the original newsvendor game.

Proposition 1. Overestimation effect: the overestimated newsvendor can steal demand from the unbiased newsvendor, but the overall supply of the product increases.

$$Q_{io}^* > Q_i^*, \quad Q_{jo}^* < Q_j^* \quad (7)$$

$Q_{io}'(\lambda_i) < 0$ is proved on Appendix A. Therefore, the change of newsvendor i 's equilibrium order should be positive $(Q_{io}^* - Q_i^*) = Q_i'(\lambda_i)|_{Q_i(Q_j)}(-\Delta\lambda_i) > 0$. Meanwhile, the difference of newsvendor j's equilibrium order is negative, $(Q_{jo}^* - Q_j^*) = Q_j'(Q_i)|_{Q_j(Q_i)}(Q_{io}^* - Q_i^*) < 0$. Considering that newsvendor j's best response order is negative correlated with order of newsvendor i, $-1 \le Q_j'(Q_i)|_{Q_j(Q_i)} \le 0$, the overall supply of the product by newsvendor increases, $Q_{io}^* + Q_{jo}^* - (Q_i^* - Q_j^*) = (Q_j'(Q_i)|_{Q_j(Q_i)} + 1) * (Q_{io}^* - Q_i^*) > 0$. Fig. 3 shows the Nash equilibrium (Q_{jo}^*, Q_{io}^*) in the overestimated newsvendor game with one overestimated newsvendor compared to the original Nash equilibrium (Q_i^*, Q_j^*).

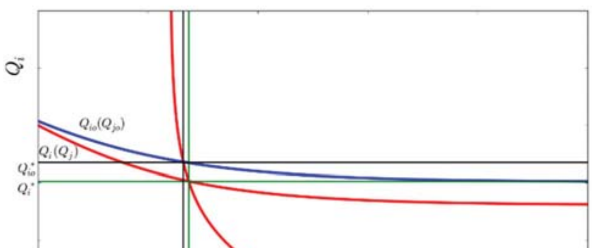

Fig. 3 Nash equilibrium with over-estimated newsvendor i

Proposition 1 indicates the effect of overestimation on the newsvendor equilibrium in the competitive newsvendor game. The overestimated newsvendor can take advantage of the overestimation by reducing competitor's order quantity. Then, this study discusses the situation while both two newsvendors are overestimated. Fig. 4 shows the model structure when both newsvendors are overestimated.

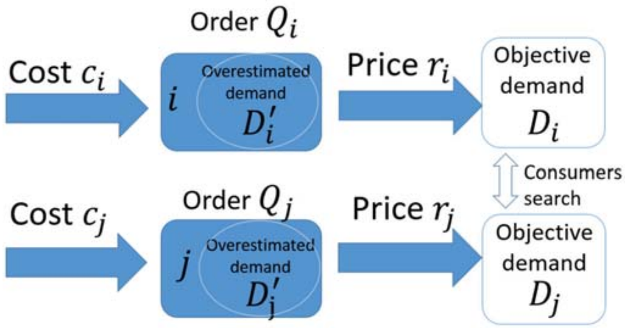

Fig. 4 Model structure with two overestimated newsvendors

Under this setting, the payoff function of newsvendors can be modeled in (8).

$$\pi_i(Q_{i0}, Q_{j0}) = E_D\left(r_i \min\left(D_i' + (D_j - Q_{j0})^+, Q_{i0}\right) - c_i Q_{i0}\right) \quad (8)$$

Therefore, the Nash equilibrium of competitive overestimated newsvendor with overestimated newsvendors (Q_{io}^*, Q_{jo}^*) should be the set satisfies (9):

$$\int_0^{Q_{jo}^*} f_i'(x) F_j'(Q_{io}^* + Q_{jo}^* - x)dx = \frac{r_i - c_i}{r_i} \Delta Q_{j2} = \frac{\partial Q_j}{\partial \lambda_j}|_{Q_j(Q_i)} * (-\Delta\lambda_j) = \frac{Q_j(1-e^{-\alpha Q_i})}{\alpha(1-e^{-\alpha Q_i} + \alpha Q_j e^{-\alpha Q_i})} \cdot (-\Delta\lambda_j) \quad (9)$$

Fig. 5 shows the movement of the Nash equilibrium. Since, the two players are stealing demand from each other according to proposition 1, at most one newsvendor has equilibrium order reduced. The equilibrium $((Q_{io}^*, Q_{jo}^*)$ with overestimation may located in region I where newsvendor j has order dropped, in region II where both newsvendors have order increased, or in region III where newsvendor i has its equilibrium order reduced.

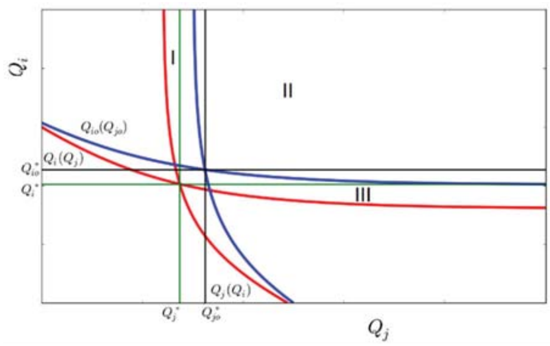

Fig. 5 Nash equilibrium with both estimated newsvendors

Proposition 2. When both newsvendors have a same unbiased demand ($\lambda_i = \lambda_j = \alpha$), overestimated newsvendor j's equilibrium order decreases when their ratio of overestimation level is larger than a threshold value (7) which is decreasing with newsvendor i's critical fractile (c_i/p_i) and increasing with newsvendor j 's critical fractile (c_j/p_j).

$$\frac{OL_i}{OL_j} \geq \frac{e^{\alpha Q_i^*}-1}{\alpha Q_i^*} + \frac{e^{\alpha Q_i^*}-1}{e^{\alpha Q_j^*}-1} = l \tag{10}$$

Proof. Denote l as the right hand side of (10). Let ΔQ_{j1} be the change of equilibrium order of newsvendor j causing by newsvendor i's overestimation. Let ΔQ_{j2} be newsvendor j's equilibrium order change by newsvendor j's overestimation.

$$\Delta Q_{j1} = \frac{\partial Q_i}{\partial \lambda_i}|_{Q_i(Q_j)} * (-\Delta\lambda_i) * \frac{\partial Q_j}{\partial Q_i}|_{Q_i(Q_j)} = \frac{Q_i\left(1-e^{-\alpha Q_j}\right)}{\alpha\left(1-e^{-\alpha Q_j}+\alpha Q_i e^{-\alpha Q_j}\right)} \cdot \frac{\alpha Q_j e^{-\alpha Q_i}}{\left(1-e^{-\alpha Q_i}+\alpha Q_j e^{-\alpha Q_i}\right)} \cdot (-\Delta\lambda_i) \tag{11}$$

$$\Delta Q_{j2} = \frac{\partial Q_j}{\partial \lambda_j}|_{Q_j(Q_i)} * \left(-\Delta\lambda_j\right) = \frac{Q_j(1-e^{-\alpha Q_i})}{\alpha(1-e^{-\alpha Q_i}+\alpha Q_j e^{-\alpha Q_i})} \cdot \left(-\Delta\lambda_j\right) \tag{12}$$

The total change of newsvendor j 's equilibrium order is $(\Delta Q_{j1} + \Delta Q_{j2})$. Take (11) and (12) into $\Delta Q_{j1} + \Delta Q_{j2} < 0$: Take this boundary condition with respect to Q_i^*

$$\frac{\partial l}{\partial Q_i^*} = \frac{(\alpha^2 Q_i^* e^{\alpha Q_i^*} - \alpha e^{\alpha Q_i^*} + \alpha)}{(\alpha Q_i^*)^2} + \frac{e^{\alpha Q_i^*}\left(e^{\alpha Q_j^*}-1\right) - \alpha Q_j'(Q_j^*) * e^{\alpha Q_j^*} * e^{\alpha Q_i^*} - 1}{\left(e^{\alpha Q_j^*}-1\right)^2} \tag{13}$$

with $Q_j'(Q_j^*) < 0$, the second term of (10) is always larger than zero, we can only discuss the second term of the right hand side of (13).

$$\lim_{Q_i^* \to 0} (\alpha^2 Q_i^* e^{\alpha Q_i^*} - \alpha e^{\alpha Q_i^*} + \alpha) = 0 \tag{14}$$

$$\frac{\partial(\alpha^2 Q_i^* e^{\alpha Q_i^*} - \alpha e^{\alpha Q_i^*} + \alpha)}{\partial Q_i^*} = \alpha^3 Q_i^* e^{\alpha Q_i^*} > 0 \tag{15}$$

Therefore, $\partial \frac{OL_i}{OL_j} / \partial Q_i^* > 0$ always holds. With the relationship $\partial Q_i^* / \partial(\frac{r_i - c_i}{r_i}) > 0$ and $\partial Q_i^* / \partial(\frac{r_j - c_j}{r_j}) > 0$, (16) can be determined:

$$\frac{\partial l}{\partial\left(\frac{r_i - c_i}{r_i}\right)} > 0 \qquad \frac{\partial l}{\partial\left(\frac{r_j - c_j}{r_j}\right)} < 0 \tag{16}$$

when both newsvendors have the same demand forecast by the third party, Proposition 2 shows whether the equilibrium order reduce is dependent on the ratio of overestimation levels. The newsvendor who has a higher critical fractile has a lower obstacle for equilibrium order decreasing. With a lower critical fractile, lower the overestimation level ratio needed to reduced equilibrium order of the other newsvendor.

As shown in Fig. 3, the model of two overestimated competitive newsvendors could leads both newsvendors to have order increase. When both newsvendors have the same demand forecast by the third party ($\lambda_i = \lambda_j = \alpha$), the condition of newsvendor j has a higher order increase than newsvendor j $\Delta Q_j - \Delta Q_i > 0$ is (16).

$$\frac{OL_i}{OL_j} < \frac{\alpha Q_j^* Q_i^* e^{-\alpha Q_j^*}\left(1-e^{-\alpha Q_i^*}\right) + Q_j^*(1-e^{-\alpha Q_i^*})(1-e^{-\alpha Q_j^*} + \alpha Q_j^* e^{-\alpha Q_i^*})}{\alpha Q_j^* Q_i^* e^{-\alpha Q_i^*}\left(1-e^{-\alpha Q_j^*}\right) + Q_i^*(1-e^{-\alpha Q_j^*})(1-e^{-\alpha Q_i} + \alpha Q_i^* e^{-\alpha Q_j^*})} \tag{16}$$

Denote the right hand side of (16) as k. Since the express of (16) is complex but asymmetry, a case study is conducted to illustrate how the value of k correlated with original equilibrium orders. The results are shown in the Fig. 6.

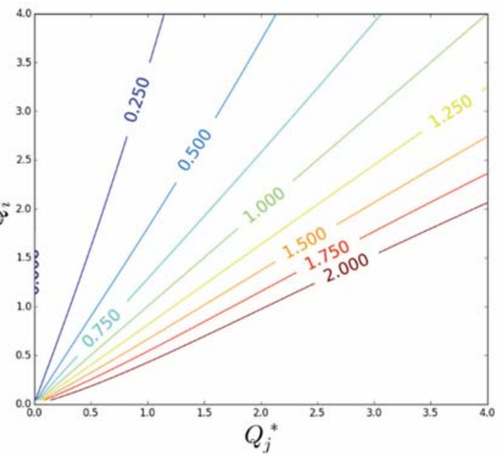

Fig. 6 The value of k respect to Q_i^* and Q_j^*

Fig. 6 shows that the value of k is increasing with the ratio of original equilibrium order (Q_j^*/Q_i^*). The newsvendor who has larger original equilibrium order still has advantage in overestimation effect. With the same overestimation level, the newsvendor who had larger original equilibrium order would have larger equilibrium order increased than the newsvendor who has smaller equilibrium order. Additionally, if both newsvendors had same original equilibrium order value, the newsvendor with larger overestimation level will have larger equilibrium order increase.

III. Conclusions

Competitive newsvendor game under behavior biased is an important research topic in operation research. This research examines the impact of overestimation of demand on the competitive newsvendor game. Firstly, this paper discusses the case that one player is overestimating his demand. The Nash equilibrium solution shows that the biased newsvendor has a larger equilibrium order compared with the unbiased competitive newsvendor. At the same time, the other unbiased newsvendor has a lower equilibrium order quantity but the overall supply of product by two newsvendor increases. Then, let that both two newsvendors are overestimated. In this case, it leads to two outcomes 1) both newsvendors have the equilibrium order increase 2) one newsvendor has order fall while the other newsvendor's order increases. It has been found that the boundary condition when the newsvendor's equilibrium order remain unchanged is increasing with its own critical fractile and decreasing with the other newsvendor's critical fractile. It indicates that the newsvendor with a higher critical fractile is more likely to have equilibrium order drop. In both situations, the overall product supply still rises which may also lead to the losses for overestimated newsvendors.

This paper contributes to behavioral operations research by demonstrating the overestimated newsvendor's behaviors in the competitive newsvendor game. The overestimation can be an aggressive strategy to force the opponent to reduce their inventory level. The newsvendor will receive more orders through the customer search due to the stock out of the opponent. It should be pointed out that under overestimation strategy, the high level of inventory leads to the risk of unsold products. For the future study, other behavior biases should be investigated in competitive newsvendor game, for example, overprecise.

APPENDIX

A. Proof of $Q'_{io}(\lambda_i) < 0$

If $\lambda_i = \lambda_j$:

$$\frac{\partial B_i(Q_j)}{\partial \lambda_i} = Q_i e^{-\lambda_i Q_i} - Q_i e^{-\lambda_j (Q_i + Q_j)} = Q_i e^{-\lambda_i Q_i}\left(1 - e^{-\lambda_i Q_j}\right) \geq 0$$

It also can be observed that $\frac{\partial B_i(Q_j)}{\partial \lambda_i} = 0$ when $Q_j = 0$.

If $\lambda_i \neq \lambda_j$:

$$\frac{\partial B_i(Q_j)}{\partial \lambda_i} = Q_i e^{-\lambda_i Q_i} + \frac{\lambda_j}{(\lambda_i - \lambda_j)^2} e^{-\lambda_j (Q_i + Q_j)} - \frac{\lambda_j}{(\lambda_i - \lambda_j)^2} e^{-\lambda_i Q_i - \lambda_j Q_j} - \frac{\lambda_i Q_i}{\lambda_i - \lambda_j} e^{-\lambda_i Q_i - \lambda_j Q_j}$$

Let $z = \frac{\partial B_i(Q_j)}{\partial \lambda_i}$.

When newsvendor j does order the product with

$$z|_{Q_j=0} = -\frac{\lambda_j Q_i}{\lambda_i - \lambda_j} e^{-\lambda_i Q_i} + \frac{\lambda_j}{(\lambda_i - \lambda_j)^2} e^{-\lambda_j Q_i} - \frac{\lambda_j}{(\lambda_i - \lambda_j)^2} e^{-\lambda_i Q_i}$$
$$= \frac{\lambda_j}{\lambda_i - \lambda_j}\left(\frac{e^{-\lambda_j Q_i} - e^{-\lambda_i Q_i}}{\lambda_i - \lambda_j} - Q_i e^{-\lambda_i Q_i}\right) > 0$$

assume newsvendor j order infinite number of product:

$$z|_{Q_j=\infty} = Q_i e^{-\lambda_i Q_i} > 0$$

The change of value of z when newsvendor's order increase on the best response function if newsvendor:

$$\Delta z = \left(\frac{\partial z}{\partial Q_i}, \frac{\partial z}{\partial Q_i}\right) \cdot (\Delta Q_i, \Delta Q_j)$$

with $-1 < \frac{\Delta Q_i}{\Delta Q_j} < 0$, and $\Delta Q_i < 0$, consider the case that Q_j is increasing, the value of Δz satisfies following equation:

$$\Delta z \geq \left(\lambda_j Q_i - \frac{\lambda_i}{\lambda_i - \lambda_j}\right)\left(e^{-\lambda_i Q_i} - e^{-\lambda_i Q_i - \lambda_j Q_j}\right)(-\Delta Q_i)$$

If $\lambda_i - \lambda_j > 0$, $\Delta z > 0$.

If $\lambda_i - \lambda_j < 0$ and $Q_i > \frac{1}{\lambda_i - \lambda_j}$, then$\Delta z > 0$.

If $\lambda_i - \lambda_j < 0$, $Q_i < \frac{1}{\lambda_i - \lambda_j}$ $\Delta z < 0$. However, since $\lim_{Q_i \to \infty} z > 0$, it can be concluded that $\Delta z > 0$. Therefore $Q'_{io}(\lambda_i) < 0$ always hold.

REFERENCES

[1] K. J. Arrow, T. Harris, and J. Marschak, "Optimal inventory policy," *Econometrica: Journal of the Econometric Society,* pp. 250-272, 1951.
[2] G. Gallego and I. Moon, "The Distribution Free Newsboy Problem: Review and Extensions," *The Journal of the Operational Research Society,* vol. 44, pp. 825-834, 1993.
[3] M. Parlar, "Game theoretic analysis of the substitutable product inventory problem with random demands," *Naval Research Logistics (NRL),* vol. 35, pp. 397-409, 1988.
[4] S. A. Lippman and K. F. McCardle, "The competitive newsboy," *Operations research,* vol. 45, pp. 54-65, 1997.
[5] D. Kahneman and A. Tversky, "Prospect theory: An analysis of decision under risk," *Econometrica: Journal of the Econometric Society,* pp. 263-291, 1979.
[6] C. X. Wang and S. Webster, "The loss-averse newsvendor problem," *Omega,* vol. 37, pp. 93-105, 2009.
[7] C. X. Wang, "The loss-averse newsvendor game," *International Journal of Production Economics,* vol. 124, pp. 448-452, 2010.
[8] G. L. Urban, B. D. Weinberg, and J. R. Hauser, "Premarket forecasting of really-new products," *The Journal of Marketing,* pp. 47-60, 1996.
[9] M. Paich and J. D. Sterman, "Boom, bust, and failures to learn in experimental markets," *Management Science,* vol. 39, pp. 1439-1458, 1993.
[10] Y. Huang, P. Vir Singh, and K. Srinivasan, "Crowdsourcing new product ideas under consumer learning," *Management Science,* vol. 60, pp. 2138-2159, 2014.
[11] F. Y. Edgeworth, "The mathematical theory of banking," *Journal of the Royal Statistical Society,* vol. 51, pp. 113-127, 1888.