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The Comparisons of Average Outgoing Quality Limit between the MCSP-2-C and MCSP-C

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***Abstract*—**This paper presents a comparison of average outgoing quality limit of the MCSP-2-C plan with MCSP-C when MCSP-2-C has been developed from MCSP-C. The parameters used in MCSP-2-C are: i (the clearance number), c (the acceptance number), m (the number of conforming units to be found before allowing c non-conforming units in the sampling inspection), f_1 and f_2 (the sampling frequency at level 1 and 2, respectively). The average outgoing quality limit (*AOQL*) values from two plans were compared and we found that for all sets of i, r, and c values, MCSP-2-C gives higher values than MCSP-C. For all sets of i, r, and c values, the average outgoing quality values of MCSP-C and MCSP-2-C are similar when p is low or high but is difference when p is moderate.

***Keywords*—**average outgoing quality, average outgoing quality limit, continuous sampling plan.

I. Introduction

A continuous sampling plan (CSP) is a sampling inspection plan for inspecting individual product units on a continuous basis. CSP involves alternating between two phases of inspection, i.e. 100% screening and sampling inspection. The original continuous sampling plan was the single-level continuous sampling plan that was presented by Dodge [1], namely CSP-1. This plan is the simplest and most famous and was used to develop other plans such as CSP-2 and CSP-3 by Dodge and Torrey [2], CSP-M by Lieberman and Solomon [3], TCSP-1 by Govindaraju and Balamurali [4], MLP-T-2 by Balamurali and Kalyanasundaram [5], CSP-C by Govindaraju and Kandnsamy [6] and MCSP-C by Balamurali and Subramani [7]. A review of various CSPs available in many statistical quality control textbooks for example Grant [8], Stephens [9], and Montgomery [10].

The MCSP-2-C plan is a two-level continuous sampling plan that has been developed as a single-level continuous sampling plan based on MCSP-C by Guayjarernpanishk and Mayureesawan [11]. MCSP-2-C has been proposed to reduce inspection or extended restart 100% inspection in the MCSP-C plan process. The operating procedure of the MCSP-2-C plan starts at 100% inspection, inspected one by one consecutively in the order of production.

When i successive units are found to conform then discontinue 100% inspection and start sampling inspection at level 1 which inspects only a fraction f_1 of the units selected at random. If a non-conforming unit is found within the first m sampled conforming units then starts sampling inspection at level 2, which inspects only a fraction f_2 until a total of c+1 non-conforming sampled units have been found then reverts to a 100% inspection.

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If c non-conforming units are found after the first m sampled units have been found to conform then inspection continues with a sampling rate f_1 until a total of c+1 non-conforming sampled units have been found then reverts immediately to a 100% inspection. The difference between MCSP-C and MCSP-2-C is if a non-conforming unit is found within the first m sampled conforming units then MCSP-C reverts to 100% inspection but MCSP-2-C starts sampling inspection at level 2 until a total of c+1 non-conforming sampled units have been found then reverts to a 100% inspection. The objectives of this paper are a comparison of average outgoing quality limit of the MCSP-2-C plan with MCSP-C and to give the values of p when average outgoing quality of the MCSP-C plan and MCSP-2-C are similar or different.

II. Design and Theory of the MCSP-2-C Plan

A. The Operating Procedure of the MCSP-2-C

The MCSP-2-C uses five parameters (i, c, m, f_1 and f_2) for inspection of the units being produced on the production line, which are defined by:

- i = the clearance number,
- c = the acceptance number,
- m = the number of conforming units to be found before allowing c non-conforming units in the sampling inspection,
- f_1 = the sampling frequency at level 1 or $f_1 = 1/r$,
- f_2 = the sampling frequency at level 2 or $f_2 = 2 f_1$.

The operating procedure of the MCSP-2-C plan is as follows:

- Step i. Start with 100% inspection of units in the order of production. When i successive units are found conforming, discontinue 100% inspection and start sampling inspection at level 1.
- Step ii. During the sampling inspection at level 1, inspect only a fraction f_1 of the units, selecting individual units one at a time in the order of production in such a way as to ensure an unbiased sample.
- Step iii. If c non-conforming units are found after the first m sampled units have been found conforming then continue sampling at level 1 until c+1 non-conforming sampled unit have been found, and then revert immediately to 100% inspection.
- Step iv. If a non-conforming unit is found within the first m sampled conforming units then start sampling inspection at level 2, inspect only a fraction f_2 until c+1 non-conforming sampled units have been found then return to Step i.
- Step v. Replace or correct all the non-conforming units found with conforming units.

B. The Performance Measures of the MCSP-2-C

A derivation of these performance measures assumed that the production process is under statistical control and based on the Markov Chain formulation.

Let p be the probability of non-conforming units and q =1-p, the following formulas for performance measures may be obtained:

The average number of units inspected in a 100% screening sequence following the finding of a non-conforming unit, u:

$$u = \frac{1-q^i}{pq^i} \tag{1}$$

The average number of units passed under the sampling inspection, v:

$$v = \frac{f_2(1+cq^m)+(c+1)f_1(1-q^m)}{pf_1 f_2} \tag{2}$$

The average cycle length, ACL:

$$ACL = \frac{f_1 f_2\left(1-q^i\right)+q^i f_2(1+cq^m)+(c+1)q^i f_1(1-q^m)}{pq^i f_1 f_2} \tag{3}$$

The average fraction inspected, AFI:

$$\begin{aligned} AFI = & \frac{f_1 f_2}{f_1 f_2\left(1-q^i\right)+q^i f_2\left(1+cq^m\right)+q^i f_1\left(c+1\right)\left(1-q^m\right)} \\ & + \frac{\left(c+1\right)q^i f_1 f_2}{f_1 f_2\left(1-q^i\right)+q^i f_2\left(1+cq^m\right)+q^i f_1\left(c+1\right)\left(1-q^m\right)} \\ & - \frac{q^{i+m} f_1 f_2}{f_1 f_2\left(1-q^i\right)+q^i f_2\left(1+cq^m\right)+q^i f_1\left(c+1\right)\left(1-q^m\right)} \end{aligned} \tag{4}$$

The average outgoing quality, AOQ:

$$\begin{aligned} AOQ = & \frac{pq^i\left(1-q^m\right)\left(1-f_1\right)f_2}{f_1 f_2\left(1-q^i\right)+q^i f_2\left(1+cq^m\right)+q^i f_1\left(c+1\right)\left(1-q^m\right)} \\ & + \frac{pq^{i+m}(c+1)(1-f_1)f_2}{f_1 f_2\left(1-q^i\right)+q^i f_2\left(1+cq^m\right)+q^i f_1\left(c+1\right)\left(1-q^m\right)} \\ & + \frac{pq^i(c+1)f_1\left(1-q^m\right)(1-f_2)}{f_1 f_2\left(1-q^i\right)+q^i f_2\left(1+cq^m\right)+q^i f_1\left(c+1\right)\left(1-q^m\right)} \end{aligned} \tag{5}$$

The average outgoing quality limit, $AOQL$:

$$AOQL = \underset{p}{Max}(AOQ). \tag{6}$$

Full details of the derivation of these performance measures can be found in Guayjarernpanishk and Mayureesawan [11].

C. Comparisons of Average Outgoing Quality Limit of MCSP-2-C with MCSP-C

In this section, the values of *AOQL* for MCSP-2-C were compared with the values of *AOQL* obtained for MCSP-C when the values of i were selected from 10, 15, 20, 30, 40 and 50, the values of $m = i$, the values of r were selected from 4 and 10 and the values of c were selected from 2 and 3.

The %Diff_*AOQL* values for comparing the *AOQL* values of MCSP-2-C plan with MCSP-C plan was defined by:

$$\%\text{Diff\_}AOQL = \left|\frac{AOQL(\text{MCSP-C})-AOQL(\text{MCSP-2-C})}{AOQL(\text{MCSP-C})}\right| \times 100\% \tag{7}$$

Where

AOQL(MCSP-2-C) = the *AOQL* values of MCSP-2-C plan,
AOQL(MCSP-C) = the *AOQL* values of MCSP-C plan.

The results for the comparisons are presented in the next section.

III. Results

A. The Comparisons of Average Outgoing Quality Limit

In Table I, the *AOQL* values of MCSP-2-C and MCSP-C and the percentage differences of the *AOQL* values between MCSP-2-C and MCSP-C for all sets of i, r, and c values are shown. We observed that the *AOQL* values of the two plans are different with the *AOQL* values of MCSP-2-C higher than the *AOQL* values of MCSP-C for all sets of i, r, and c values.

The comparisons of the percentage differences of the *AOQL* values between MCSP-2-C and MCSP-C for all sets of i, r, and c values are shown in Fig 1 to 3. We found that when i changes from 10 to 15, 20, 30, 40 and 50, respectively, the %Diff_*AOQL* values are slightly different at the same level of r and c. When r changes from 4 to 10, the %Diff_*AOQL* values are greater at the same level of i and c. When c changes from 2 to 3, the %Diff_*AOQL* values are similar at the same level of i at r = 4 but the %Diff_*AOQL* values are different at the same level of i at r = 10.

B. The Values of p

In this section, the *AOQ* values of MCSP-C and MCSP-2-C at c = 2 for all sets of p for each set of i and r are shown in Fig 4 to 7. We saw that for all sets of i and r at c = 2, for the low level of p, the *AOQ* values of MCSP-2-C are a little lower than MCSP-C. However at the high level of p, the *AOQ* values of MCSP-2-C are a little higher than MCSP-2-C and the *AOQ* values of MCSP-2-C are greater than the *AOQ* values of MCSP-C when p is at a moderate level. For all sets of r, the difference of the *AOQ* values between MCSP-C and MCSP-2-C are relatively small when the value of i increases. For all sets of i the difference of the *AOQ* values between MCSP-C and MCSP-2-C are relatively large when r increases.

In Table II, the values of p for the *AOQ* values of MCSP-C and MCSP-2-C are similar or different for all sets of i, r, and c values are shown. We found that the *AOQ* values of MCSP-C and MCSP-2-C are similar at the low or high level of p but the *AOQ* values of MCSP-C and MCSP-2-C are different at the moderate level of p.

TABLE I
THE *AOQL* VALUES OF MCSP-2-C AND MCSP-C AND THE PERCENTAGE DIFFERENCES OF THE *AOQL* VALUES BETWEEN MCSP-2-C AND MCSP-C (%Diff_*AOQL*)

i, r, c	*AOQL* MCSP-2-C	MCSP-C	% Diff_ *AOQL*

10, 4, 2	0.06981	0.06523	6.56
10, 4, 3	0.07350	0.06872	6.50
15, 4, 2	0.04801	0.04456	7.19
15, 4, 3	0.05062	0.04699	7.17
20, 4, 2	0.03659	0.03387	7.43
20, 4, 3	0.03860	0.03570	7.51
30, 4, 2	0.02479	0.02287	7.75
30, 4, 3	0.02617	0.02409	7.95
40, 4, 3	0.01980	0.01819	8.13
50, 4, 2	0.01507	0.01385	8.10
50, 4, 3	0.01592	0.01459	8.35
10, 10, 2	0.12991	0.10856	16.43
10, 10, 3	0.13682	0.11216	18.02
15, 10, 2	0.09021	0.07476	17.13
15, 10, 3	0.09517	0.07725	18.83
20, 10, 2	0.06909	0.05699	17.51
20, 10, 3	0.07295	0.05888	19.29
30, 10, 2	0.04702	0.03864	17.82
30, 10, 3	0.04970	0.03993	19.66
40, 10, 2	0.03564	0.02923	17.99
40, 10, 3	0.03770	0.03021	19.87
50, 10, 2	0.02871	0.02350	18.15
50, 10, 3	0.03033	0.02429	19.91

TABLE II
THE VALUES OF *P* FOR THE *AOQL* VALUES OF MCSP-C AND MCSP-2-C ARE SIMILAR OR DIFFERENT

i	*r*	*c*	The values of *p* similar	different
10	4	all	0 - 0.130 or 0.560 - 1	0.131 - 0.559
15	4	all	0 - 0.090 or 0.410 - 1	0.091 - 0.409
20	4	all	0 - 0.070 or 0.315 - 1	0.071 - 0.314
30	4	all	0 - 0.045 or 0.215 - 1	0.046 - 0.214
40	4	all	0 - 0.035 or 0.160 - 1	0.036 - 0.159
50	4	all	0 - 0.030 or 0.125 - 1	0.031 - 0.124
10	10	all	0 - 0.110 or 0.620 - 1	0.111 - 0.619
15	10	all	0 - 0.075 or 0.465 - 1	0.076 - 0.464
20	10	all	0 - 0.060 or 0.365 - 1	0.061 - 0.364
30	10	all	0 - 0.040 or 0.255 - 1	0.041 - 0.254
40	10	all	0 - 0.030 or 0.190 - 1	0.031 - 0.189
50	10	all	0 - 0.025 or 0.155 - 1	0.026 - 0.154

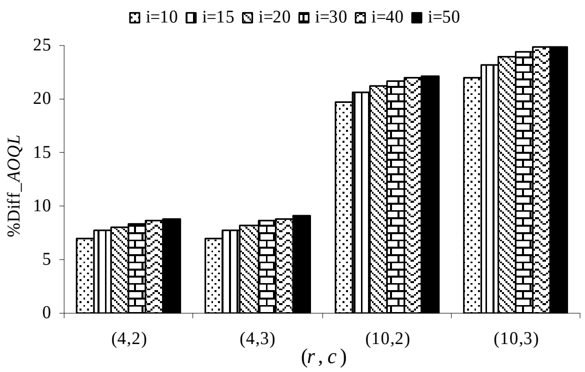

Fig. 1 The percentage differences of the *AOQL* values (%Diff_*AOQL*) between MCSP-2-C and MCSP-C for all sets of *i*.

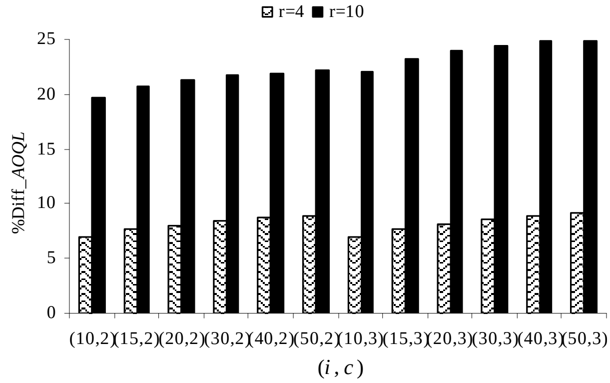

Fig. 2 The percentage differences of the *AOQL* values (%Diff_*AOQL*) between MCSP-2-C and MCSP-C for all sets of *r*.

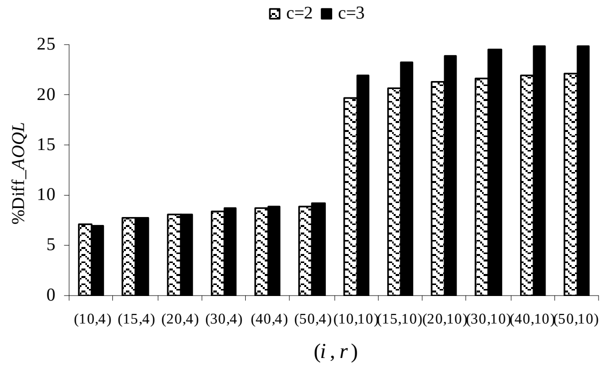

Fig. 3 The percentage differences of the *AOQL* values(%Diff_*AOQL*) between MCSP-2-C and MCSP-C for all sets of *c*.

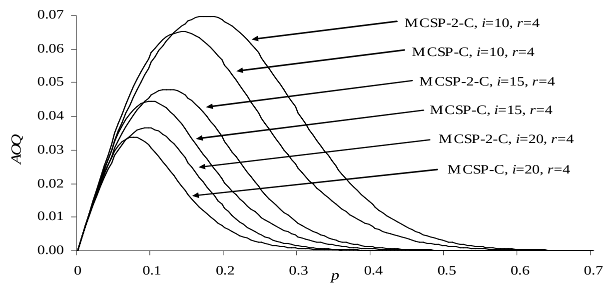

Fig. 4 The *AOQ* values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 4$ where $i = 10$, 15 and 20

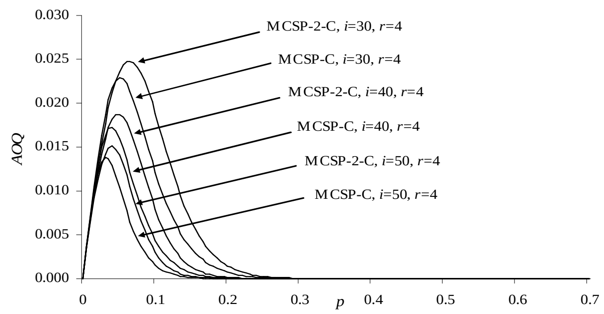

Fig. 5 The *AOQ* values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 4$ where $i = 30$, 40 and 50.

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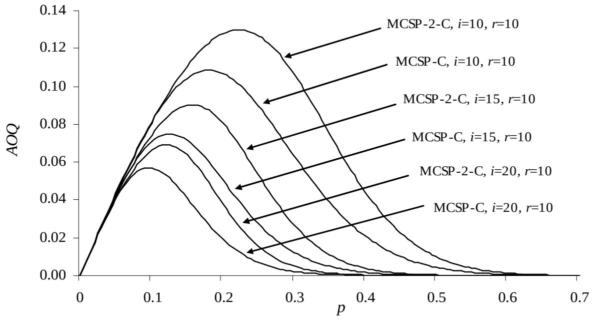

Fig. 6 The *AOQ* values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 10$ where $i = 10$, 15 and 20

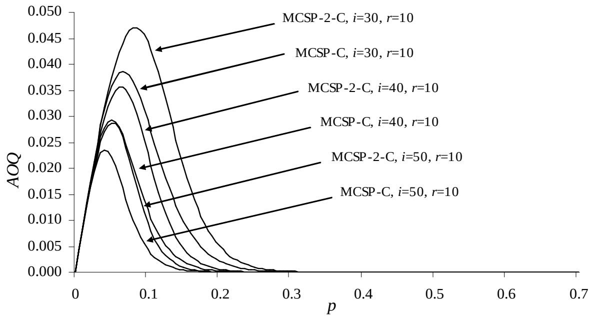

Fig. 7 The *AOQ* values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 10$ where $i = 30$, 40 and 50

IV. Discussions and Conclusions

The average outgoing quality limit (*AOQL*) is one of the performance measures which is the primary index for choosing the continuous sampling plans. So when considering the results of the *AOQL* comparisons, the operators may choose to use MCSP-C because this plan gives a lower number of non-conforming units that passed inspection and an easier operating process of inspection than MCSP-2-C. If sampling plans give high values of *AOQL* then they give low number of units inspected. In case the operators want to reduce the number of units inspected, they may choose the MCSP-2-C plan. We also observed that for values of i, there was a small effect on the differences of the *AOQL* values between MCSP-2-C and MCSP-C. However, for values of r, there was a great influence on the differences of the *AOQL* values. For values of c, there was no effect on the differences of the *AOQL* values when $r = 4$ but there was influence when $r = 10$.

When considering the low or high level of p, the two plans give similar *AOQ* values and the operators can choose MCSP-C or MCSP-2-C. At the moderate level of p, MCSP-C gives lower values of *AOQ* than MCSP-2-C, so they may choose MCSP-C. For values of i and r, there are also effects on the levels of p values for choosing the sampling plan.

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