

The Comparisons of Average Outgoing Quality Limit between the MCSP-2-C and MCSP-C

P. Guayjarernpanishkand, T. Mayureesawan

Abstract—This paper presents a comparison of average outgoing quality limit of the MCSP-2-C plan with MCSP-C when MCSP-2-C has been developed from MCSP-C. The parameters used in MCSP-2-C are: i (the clearance number), c (the acceptance number), m (the number of conforming units to be found before allowing c non-conforming units in the sampling inspection), f_1 and f_2 (the sampling frequency at level 1 and 2, respectively). The average outgoing quality limit (AOQL) values from two plans were compared and we found that for all sets of i , r , and c values, MCSP-2-C gives higher values than MCSP-C. For all sets of i , r , and c values, the average outgoing quality values of MCSP-C and MCSP-2-C are similar when p is low or high but is difference when p is moderate.

Keywords—average outgoing quality, average outgoing quality limit, continuous sampling plan.

I. INTRODUCTION

A continuous sampling plan (CSP) is a sampling inspection plan for inspecting individual product units on a continuous basis. CSP involves alternating between two phases of inspection, i.e. 100% screening and sampling inspection. The original continuous sampling plan was the single-level continuous sampling plan that was presented by Dodge [1], namely CSP-1. This plan is the simplest and most famous and was used to develop other plans such as CSP-2 and CSP-3 by Dodge and Torrey [2], CSP-M by Lieberman and Solomon [3], TCSP-1 by Govindaraju and Balamurali [4], MLP-T-2 by Balamurali and Kalyanasundaram [5], CSP-C by Govindaraju and Kandansamy [6] and MCSP-C by Balamurali and Subramani [7]. A review of various CSPs available in many statistical quality control textbooks for example Grant [8], Stephens [9], and Montgomery [10].

The MCSP-2-C plan is a two-level continuous sampling plan that has been developed as a single-level continuous sampling plan based on MCSP-C by Guayjarernpanishk and Mayureesawan [11]. MCSP-2-C has been proposed to reduce inspection or extended restart 100% inspection in the MCSP-C plan process. The operating procedure of the MCSP-2-C plan starts at 100% inspection, inspected one by one consecutively in the order of production.

When i successive units are found to conform then discontinue 100% inspection and start sampling inspection at level 1 which inspects only a fraction f_1 of the units selected at random. If a non-conforming unit is found within the first m sampled conforming units then starts sampling inspection at level 2, which inspects only a fraction f_2 until a total of $c+1$ non-conforming sampled units have been found then reverts to a 100% inspection.

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If c non-conforming units are found after the first m sampled units have been found to conform then inspection continues with a sampling rate f_1 until a total of $c+1$ non-conforming sampled units have been found then reverts immediately to a 100% inspection. The difference between MCSP-C and MCSP-2-C is if a non-conforming unit is found within the first m sampled conforming units then MCSP-C reverts to 100% inspection but MCSP-2-C starts sampling inspection at level 2 until a total of $c+1$ non-conforming sampled units have been found then reverts to a 100% inspection. The objectives of this paper are a comparison of average outgoing quality limit of the MCSP-2-C plan with MCSP-C and to give the values of p when average outgoing quality of the MCSP-C plan and MCSP-2-C are similar or different.

II. DESIGN AND THEORY OF THE MCSP-2-C PLAN

A. The Operating Procedure of the MCSP-2-C

The MCSP-2-C uses five parameters (i , c , m , f_1 and f_2) for inspection of the units being produced on the production line, which are defined by:

i = the clearance number,

c = the acceptance number,

m = the number of conforming units to be found before allowing c non-conforming units in the sampling inspection,

f_1 = the sampling frequency at level 1 or $f_1 = 1/r$,

f_2 = the sampling frequency at level 2 or $f_2 = 2f_1$.

The operating procedure of the MCSP-2-C plan is as follows:

- Step i. Start with 100% inspection of units in the order of production. When i successive units are found conforming, discontinue 100% inspection and start sampling inspection at level 1.
- Step ii. During the sampling inspection at level 1, inspect only a fraction f_1 of the units, selecting individual units one at a time in the order of production in such a way as to ensure an unbiased sample.
- Step iii. If c non-conforming units are found after the first m sampled units have been found conforming then continue sampling at level 1 until $c+1$ non-conforming sampled unit have been found, and then revert immediately to 100% inspection.
- Step iv. If a non-conforming unit is found within the first m sampled conforming units then start sampling inspection at level 2, inspect only a fraction f_2 until $c+1$ non-conforming sampled units have been found then return to Step i.
- Step v. Replace or correct all the non-conforming units found with conforming units.

B. The Performance Measures of the MCSP-2-C

A derivation of these performance measures assumed that the production process is under statistical control and based on the Markov Chain formulation.

Let p be the probability of non-conforming units and $q=1-p$, the following formulas for performance measures may be obtained:

The average number of units inspected in a 100% screening sequence following the finding of a non-conforming unit, u :

$$u = \frac{1-q^i}{pq^i} \quad (1)$$

The average number of units passed under the sampling inspection, v :

$$v = \frac{f_2(1+cq^m) + (c+1)f_1(1-q^m)}{pf_1f_2} \quad (2)$$

The average cycle length, ACL :

$$ACL = \frac{f_1f_2(1-q^i) + q^if_2(1+cq^m) + (c+1)q^if_1(1-q^m)}{pq^if_1f_2} \quad (3)$$

The average fraction inspected, AFI :

$$AFI = \frac{f_1f_2}{f_1f_2(1-q^i) + q^if_2(1+cq^m) + q^if_1(c+1)(1-q^m)} + \frac{(c+1)q^if_1f_2}{f_1f_2(1-q^i) + q^if_2(1+cq^m) + q^if_1(c+1)(1-q^m)} - \frac{q^{i+m}f_1f_2}{f_1f_2(1-q^i) + q^if_2(1+cq^m) + q^if_1(c+1)(1-q^m)} \quad (4)$$

The average outgoing quality, AOQ :

$$AOQ = \frac{pq^i(1-q^m)(1-f_1)f_2}{f_1f_2(1-q^i) + q^if_2(1+cq^m) + q^if_1(c+1)(1-q^m)} + \frac{pq^{i+m}(c+1)(1-f_1)f_2}{f_1f_2(1-q^i) + q^if_2(1+cq^m) + q^if_1(c+1)(1-q^m)} + \frac{pq^i(c+1)f_1(1-q^m)(1-f_2)}{f_1f_2(1-q^i) + q^if_2(1+cq^m) + q^if_1(c+1)(1-q^m)} \quad (5)$$

The average outgoing quality limit, $AOQL$:

$$AOQL = \max_p(AOQ) \quad (6)$$

Full details of the derivation of these performance measures can be found in Guayjarernpanishk and Mayureesawan [11].

C. Comparisons of Average Outgoing Quality Limit of MCSP-2-C with MCSP-C

In this section, the values of $AOQL$ for MCSP-2-C were compared with the values of $AOQL$ obtained for MCSP-C

when the values of i were selected from 10, 15, 20, 30, 40 and 50, the values of $m=i$, the values of r were selected from 4 and 10 and the values of c were selected from 2 and 3.

The %Diff_ $AOQL$ values for comparing the $AOQL$ values of MCSP-2-C plan with MCSP-C plan was defined by:

$$\% \text{Diff_} AOQL = \left| \frac{AOQL(\text{MCSP-C}) - AOQL(\text{MCSP-2-C})}{AOQL(\text{MCSP-C})} \right| \times 100\% \quad (7)$$

Where

$AOQL(\text{MCSP-2-C})$ = the $AOQL$ values of MCSP-2-C plan,
 $AOQL(\text{MCSP-C})$ = the $AOQL$ values of MCSP-C plan.

The results for the comparisons are presented in the next section.

III. RESULTS

A. The Comparisons of Average Outgoing Quality Limit

In Table I, the $AOQL$ values of MCSP-2-C and MCSP-C and the percentage differences of the $AOQL$ values between MCSP-2-C and MCSP-C for all sets of i , r , and c values are shown. We observed that the $AOQL$ values of the two plans are different with the $AOQL$ values of MCSP-2-C higher than the $AOQL$ values of MCSP-C for all sets of i , r , and c values.

The comparisons of the percentage differences of the $AOQL$ values between MCSP-2-C and MCSP-C for all sets of i , r , and c values are shown in Fig 1 to 3. We found that when i changes from 10 to 15, 20, 30, 40 and 50, respectively, the %Diff_ $AOQL$ values are slightly different at the same level of r and c . When r changes from 4 to 10, the %Diff_ $AOQL$ values are greater at the same level of i and c . When c changes from 2 to 3, the %Diff_ $AOQL$ values are similar at the same level of i at $r=4$ but the %Diff_ $AOQL$ values are different at the same level of i at $r=10$.

B. The Values of p

In this section, the AOQ values of MCSP-C and MCSP-2-C at $c=2$ for all sets of p for each set of i and r are shown in Fig 4 to 7. We saw that for all sets of i and r at $c=2$, for the low level of p , the AOQ values of MCSP-2-C are a little lower than MCSP-C. However at the high level of p , the AOQ values of MCSP-2-C are a little higher than MCSP-2-C and the AOQ values of MCSP-2-C are greater than the AOQ values of MCSP-C when p is at a moderate level. For all sets of r , the difference of the AOQ values between MCSP-C and MCSP-2-C are relatively small when the value of i increases. For all sets of i the difference of the AOQ values between MCSP-C and MCSP-2-C are relatively large when r increases.

In Table II, the values of p for the AOQ values of MCSP-C and MCSP-2-C are similar or different for all sets of i , r , and c values are shown. We found that the AOQ values of MCSP-C and MCSP-2-C are similar at the low or high level of p but the AOQ values of MCSP-C and MCSP-2-C are different at the moderate level of p .

TABLE I
THE $AOQL$ VALUES OF MCSP-2-C AND MCSP-C AND THE PERCENTAGE DIFFERENCES OF THE $AOQL$ VALUES BETWEEN MCSP-2-C AND MCSP-C (%Diff_ $AOQL$)

i, r, c	$AOQL$		% Diff_ $AOQL$
	MCSP-2-C	MCSP-C	

10, 4, 2	0.06981	0.06523	6.56
10, 4, 3	0.07350	0.06872	6.50
15, 4, 2	0.04801	0.04456	7.19
15, 4, 3	0.05062	0.04699	7.17
20, 4, 2	0.03659	0.03387	7.43
20, 4, 3	0.03860	0.03570	7.51
30, 4, 2	0.02479	0.02287	7.75
30, 4, 3	0.02617	0.02409	7.95
40, 4, 3	0.01980	0.01819	8.13
50, 4, 2	0.01507	0.01385	8.10
50, 4, 3	0.01592	0.01459	8.35
10, 10, 2	0.12991	0.10856	16.43
10, 10, 3	0.13682	0.11216	18.02
15, 10, 2	0.09021	0.07476	17.13
15, 10, 3	0.09517	0.07725	18.83
20, 10, 2	0.06909	0.05699	17.51
20, 10, 3	0.07295	0.05888	19.29
30, 10, 2	0.04702	0.03864	17.82
30, 10, 3	0.04970	0.03993	19.66
40, 10, 2	0.03564	0.02923	17.99
40, 10, 3	0.03770	0.03021	19.87
50, 10, 2	0.02871	0.02350	18.15
50, 10, 3	0.03033	0.02429	19.91

TABLE II

THE VALUES OF P FOR THE $AOQL$ VALUES OF MCSP-C AND MCSP-2-C ARE SIMILAR OR DIFFERENT

i	r	c	The values of p	
			similar	different
10	4	all	0 - 0.130 or 0.560 - 1	0.131 - 0.559
15	4	all	0 - 0.090 or 0.410 - 1	0.091 - 0.409
20	4	all	0 - 0.070 or 0.315 - 1	0.071 - 0.314
30	4	all	0 - 0.045 or 0.215 - 1	0.046 - 0.214
40	4	all	0 - 0.035 or 0.160 - 1	0.036 - 0.159
50	4	all	0 - 0.030 or 0.125 - 1	0.031 - 0.124
10	10	all	0 - 0.110 or 0.620 - 1	0.111 - 0.619
15	10	all	0 - 0.075 or 0.465 - 1	0.076 - 0.464
20	10	all	0 - 0.060 or 0.365 - 1	0.061 - 0.364
30	10	all	0 - 0.040 or 0.255 - 1	0.041 - 0.254
40	10	all	0 - 0.030 or 0.190 - 1	0.031 - 0.189
50	10	all	0 - 0.025 or 0.155 - 1	0.026 - 0.154

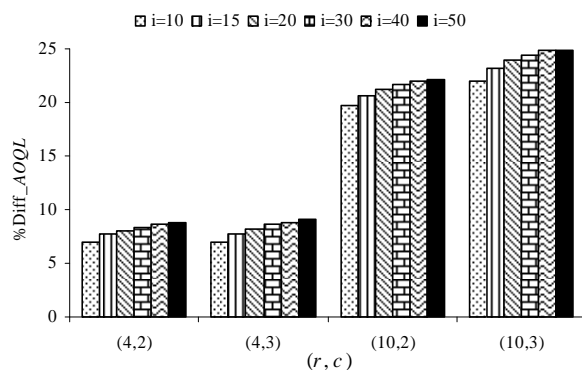


Fig. 1 The percentage differences of the $AOQL$ values ($\%Diff_AOQL$) between MCSP-2-C and MCSP-C for all sets of i .

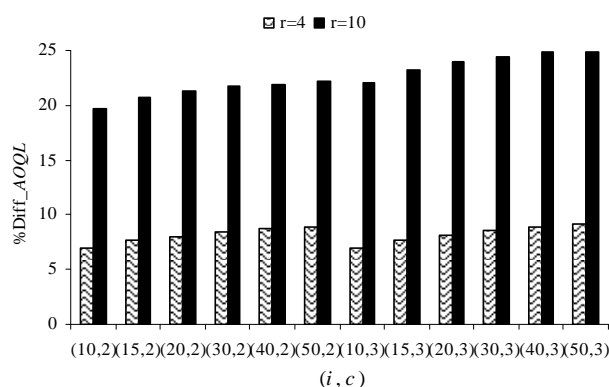


Fig. 2 The percentage differences of the $AOQL$ values ($\%Diff_AOQL$) between MCSP-2-C and MCSP-C for all sets of r .

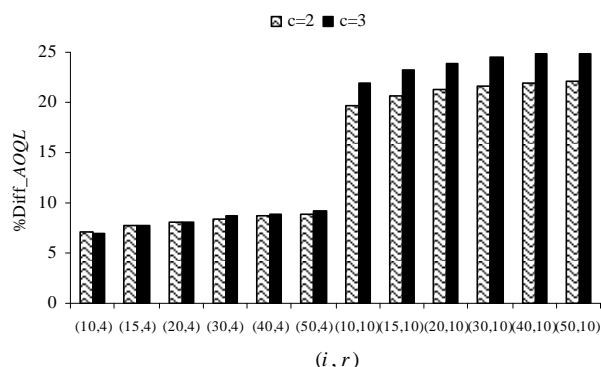


Fig. 3 The percentage differences of the $AOQL$ values ($\%Diff_AOQL$) between MCSP-2-C and MCSP-C for all sets of c .

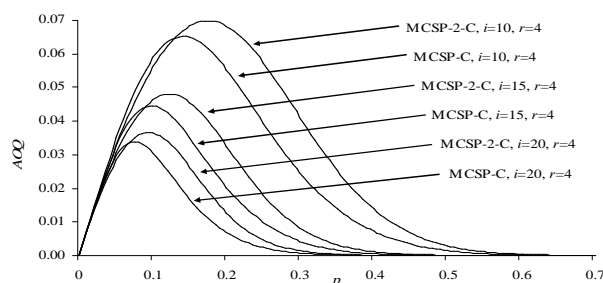


Fig. 4 The AOQ values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 4$ where $i = 10, 15$ and 20

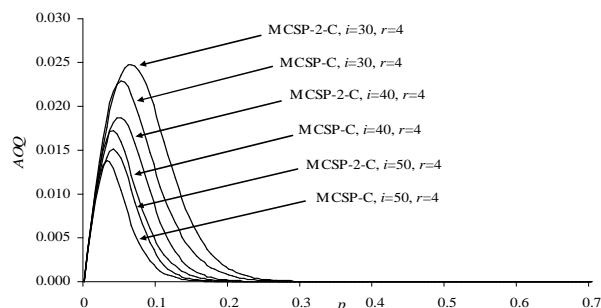


Fig. 5 The AOQ values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 4$ where $i = 30, 40$ and 50 .

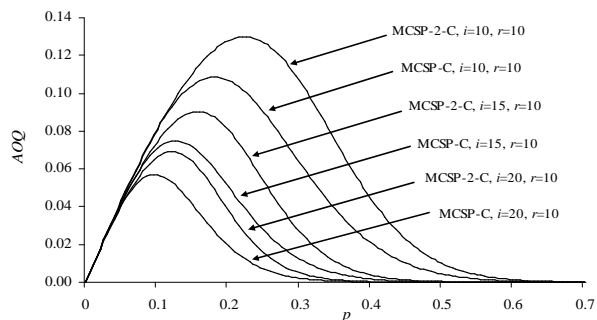


Fig. 6 The AOQ values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 10$ where $i = 10, 15$ and 20

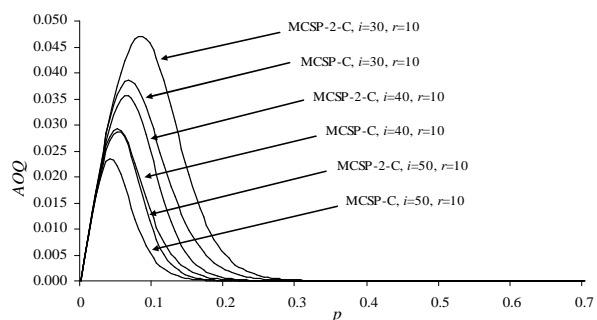


Fig. 7 The AOQ values of MCSP-C and MCSP-2-C at level of $c = 2$ for $r = 10$ where $i = 30, 40$ and 50

IV. DISCUSSIONS AND CONCLUSIONS

The average outgoing quality limit (AOQL) is one of the performance measures which is the primary index for choosing the continuous sampling plans. So when considering the results of the AOQL comparisons, the operators may choose to use MCSP-C because this plan gives a lower number of non-conforming units that passed inspection and an easier operating process of inspection than MCSP-2-C. If sampling plans give high values of AOQL then they give low number of units inspected. In case the operators want to reduce the number of units inspected, they may choose the MCSP-2-C plan. We also observed that for values of i , there was a small effect on the differences of the AOQL values between MCSP-2-C and MCSP-C. However, for values of r , there was a great influence on the differences of the AOQL values. For values of c , there was no effect on the differences of the AOQL values when $r = 4$ but there was influence when $r = 10$.

When considering the low or high level of p , the two plans give similar AOQ values and the operators can choose MCSP-C or MCSP-2-C. At the moderate level of p , MCSP-C gives lower values of AOQ than MCSP-2-C, so they may choose MCSP-C. For values of i and r , there are also effects on the levels of p values for choosing the sampling plan.

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