# The Accuracy of the Flight Derivative Estimates Derived from Flight Data

Jung-hoon Lee, Eung Tai Kim, Byung-hee Chang, In-hee Hwang, Dae-sung Lee

*Abstract*— The accuracy of estimated stability and control derivatives of a light aircraft from flight test data were evaluated. The light aircraft, named ChangGong-91, is the first certified aircraft from the Korean government. The output error method, which is a maximum likelihood estimation technique and considers measurement noise only, was used to analyze the aircraft responses measures. The multi-step control inputs were applied in order to excite the short period mode for the longitudinal and Dutch-roll mode for the lateral-directional motion. The estimated stability/control derivatives of ChanGong-91 were analyzed for the assessment of handling qualities comparing them with those of similar aircraft. The accuracy of the flight derivative estimates derived from flight test measurement was examined in engineering judgment, scatter and Cramer-Rao bound, which turned out to be satisfactory with minor defects..

*Keywords*—Light Aircraft, Flight Test, Accuracy, Engineering Judgment, Scatter, Cramer-Rao Bound

## I. INTRODUCTION

MATHEMATICAL analyses of flight dynamics and handling qualities of aircraft are required for experimental design, modeling and flight simulation of an airplane, and design and refinement of flight control systems. System identification has been adapted to improve the accuracy of flight parameter estimation. Aerodynamic parameters obtained from any source are by nature only estimates and not exact values. In order to make effective use of these estimates, it is necessary to have some gage of their reliability.

The accuracy of estimated aerodynamic parameters has been studied by many researchers. Maine and Iliff examined the use of the various measures of accuracy with flight data from both theoretical and the practical aspects [1]. They also suggested improved computations of the bound to correct large discrepancies caused by colored noise and modeling error [2].

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Morelli and Klein developed a different approach to parameter accuracy which removing the assumption of white measurement noise [3]. In spite of degradation of system identification caused by inaccuracy in transformation, the method for evaluating the finite Fourier transform using cubic interpolation of sampled time domain data for high accuracy was researched [4].

To make effective use of estimates, it is necessary to have some measure of their reliability. If highly accurate estimates cannot be distinguished from worthless estimates, all estimates would be rather worthless. Therefore, measures of the estimate accuracy are as valuable as the estimates themselves [1].

Engineering judgment is the oldest measure of estimate reliability. The process of applying engineering judgment can not be precisely and quantitatively described. When several maneuvers are made under a given flight condition, the scatter of the resulting estimates is an indication of accuracy. The scatter is a measure of actual performance of the theoretical measures. As the maximum likelihood estimation is asymptotically bias-free and efficient, the Fisher information matrix provides a good approximation to the parameter error covariance matrix. The diagonal elements of parameter error covariance matrix are indicators of accuracies of the estimates and are called Cramer-Rao bounds. Comparison of the Cramer-Rao bounds and the sample standard deviation obtained from the data scatter gives a good indication of the adequacy of the assumptions made in the theoretical development [1].

In this paper, the flight test data of ChangGong-91, the first certificated light airplane from the Korean government, were processed with the output error estimation method, which is a kind of the maximum likelihood estimation approach using measurement noise only [5]. The accuracy of the estimated flight parameters was evaluated by engineering judgment, scatter and Cramer-Rao bound. Several reasons are analyzed for inevitable inaccuracy in estimation results.

## II. TEST AIRCRAFT

The research aircraft for the accuracy of the flight derivative estimates is ChangGong-91, which is a light airplane developed through the cooperation of Korean Air and the Korea Aerospace Research Institute. It is the first type certified aircraft from the Korea Ministry of Construction and Transportation. ChangGong-91 is powered by a Textron

Lycoming IO-360-A1B6 engine, which is rated at 200hp(@2,700rpm) with an empty weight of 823kg (1,826lb) and a maximum allowable take-off weight of 1,225kg (2,700lb). The length, wingspan, and height are 7.74m (25.1ft), 10.20m (33.5ft), and 2.70m (8.87ft) respectively. More detailed specifications of this aircraft can be found in reference [6].
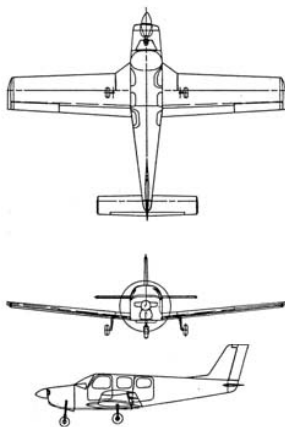


Fig. 1 Three View drawing of Test Airplane

### III. Flight Test Instrumentation

The accuracy of parameter estimation is directly dependent on the quality of the measured flight data. The instrumentation system is the most important aspect of a flight test program. The instrumentation system consisted of everything required to measure, process and record all the parameters of interest. Table 1 shows the various kinds of sensors used in the flight test in relation to data acquisition [7]. The data acquired from these sensors was recorded on a magnetic tape by a 20 channel data recorder with an son-board self-amplifier. On the ground, the data were converted to a digital signal using an A/D converter and reduced to a sampling rate of 50 Hz [8].

### IV. Flight Test

Flight tests were performed for two dynamic motions – longitudinal motion and lateral-directional motion, respectively. Generally, it is desirable to excite only the short period dynamic mode in contrast to suppress the phugoid mode for longitudinal motion. The longitudinal motion includes short period motion excited by rapid triple step inputs which were known to increase the accuracy of the estimated results to the stabilator during trimmed level flights. The detail procedure to excite short period motion is explained in the following [5];

1) Stabilize and trim carefully at the desired flight condition.
2) Start recording as the recorder power switches on.
3) Smoothly apply triple step input to the stabilator.
 - From the neutral stabilator position push the control wheel 3cm and maintain for 1 minute.
 - Pull the control wheel 6cm and maintain for 1.5 minutes.
 - Push the control wheel 6cm and maintain for 1 minute.

 - Return the control wheel to the trim position and fix (or free) control wheel.
4) After 20 minutes stop recording as the recorder power switches off.

The lateral-directional motion including Dutch-roll motion was excited by the rudder, and followed by input to the aileron. To excite Dutch-roll motion, a similar but more complicate procedure than that for the longitudinal motion was performed.

1) Stabilize and trim carefully at the desired flight condition.
2) Start recording as the recorder power switches on.
3) Smoothly apply triple step input to the rudder.
 - From neutral rudder position push the left rudder pedal 5cm and maintain for 1 minute.
 - Push the right rudder pedal 5cm and maintain for 1 minute.
 - Push the left rudder pedal and return to the neutral position.
4) Maintain trim flight for 3 minutes and smoothly apply triple step input to the aileron wheel
 - At the neutral control wheel position maintain trim flight for 3 minutes.
 - From the control wheel trim position rotate the control wheel to the left 5 degrees and maintain for 1.5 minutes.
 - Over the control wheel neutral position rotate the control wheel to the right 5 degrees and maintain for 1.5 minutes.
 - Over the control wheel neutral position rotate the control wheel to the left 5 degrees and maintain for 1 minute.
 - Return the control wheel to the trim position and fix (or free) control wheel.
5) After 20 minutes stop recording as the recorder power switches off.

During the flight tests, airspeed, pressure altitude, angle of attack, angle of sideslip, air temperature, control surface deflections, pitch and roll angles, and 3-axes rates and accelerations were recorded by the onboard recorder.

### V. Mathematical model

The mathematical model of the motion of an airplane can be formulated from 6 degree-of-freedom linear differential equations in translational and rotational motion. They are composed of forces (X, Y, Z) and moments (L, M, N) acting on the airplane in flight. They involve aerodynamic parameters such as aerodynamic coefficients, stability and control derivatives. We shall assume that the aircraft's motion consists of small deviations from its equilibrium condition and the motion of the airplane can be analyzed by separating the equations into two groups; longitudinal motion and lateral-directional motion.

To describe the motion of an aircraft, we need to rely on the following mathematical and physical assumptions: [7]

 - The aircraft is a solid body, neglecting the effects of aeroelasticity
 - The aircraft is assumed to be symmetric and the

asymmetry effect of the propeller is neglected

- The motions of aircraft are separated into longitudinal mode and lateral-directional mode, which are independent of each other

- Mass, center of gravity and moment of inertia are not varied during flight

- Aerodynamic coefficients and stability/control derivatives are constants which are not affected with angle of attack and flight condition

- Small perturbation theory is applicable to flight dynamic model

- There is no large amplitude aircraft behavior such as stall or spin

- The compressibility effect and Reynolds number variation are neglected throughout the flight

For longitudinal motion, the equations of motion can be obtained from the equilibriums of $x$-direction forces, $z$-direction forces, and the pitching moments. The state variables are velocity $u$, angle of attack $\alpha$, pitch rate $q$, and the pitch angle $\theta$, while the input is stabilator deflection $\delta_e$. After application of the small perturbation theory, the equations of the longitudinal motion are found in matrix form as follows. For the relations between the derivatives and coefficients, please refer to [5].

$$
\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{\alpha} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_\alpha & 0 & -g \\ Z^*_u & Z^*_\alpha & 1 & 0 \\ \tilde{M}_u & \tilde{M}_\alpha & \tilde{M}_g & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta\alpha \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z^*_{\delta_e} \\ \tilde{M}_{\delta_e} \\ 0 \end{bmatrix} [\Delta\delta_e] \quad (1)
$$

Following similar processes, the equations of the lateral-directional motion can be obtained from balances of $y$-direction forces, rolling moments, and yawing moments for lateral-directional motion. The state variables are angle of sideslip $\beta$, roll rate $p$, yaw rate $r$, and roll angle $\varphi$, while the inputs are aileron deflection $\delta_a$ and rudder deflection $\delta_r$.

$$
\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\phi} \end{bmatrix} = \begin{bmatrix} Y^*_\beta & Y^*_p & Y^*_r & \dfrac{g\cos\theta_o}{u_o} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\phi \end{bmatrix} + \begin{bmatrix} 0 & Y^*_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \quad (2)
$$

## VI. PARAMETER ESTIMATION

The output error method is applied to produce the stability/control derivatives of ChangGong-91. It is a nonlinear optimization method that has been the most commonly used

technique for deriving parameter estimates from dynamic flight data since its introduction around the 1970s. It does not account for any process noise. For the detailed theory and approaches for the output error method, please refer to reference [9]. The modified Newton-Raphson method was introduced by Taylor and Iliff. For a set of measured flight time histories of selected aircraft response variables, it determines the values of the parameters in the equations of motion by minimizing the difference between the flight response and the response computed from the system equations [9]. The equations of aircraft motion are formulated in the state space as

$$
x(t_0) = x_0 \quad (3)
$$
$$
\dot{x}(t) = A\,x(t) + B\,u(t) + F\,n(t) \quad (4)
$$
$$
z(t) = C\,x(t) + D\,u(t) + G\,\eta(t) \quad (5)
$$

If there is no state noise and the matrix G is known, then the maximum likelihood estimator minimizes the cost function.

$$
J(\xi) = \frac{1}{2}\sum_{i=1}^{N}[\tilde{z}_\xi(t_i) - z(t_i)]^*(GG^*)^{-1}[\tilde{z}_\xi(t_i) - z(t_i)] \quad (6)
$$

Use The cost function is a function of the difference between the measured and computed time histories. Eq. (6) is the negative logarithm of the likelihood function, which reduces to the output error cost function for a given $GG^*$. In order to minimize the cost function $J(\xi)$ the Gauss-Newton algorithm is applied, which is a preferred optimization algorithm. Let $L$ be the iteration number. Then the $L+1$ estimate of $\xi_L$ is obtained from the $L$ estimate as

$$
\tilde{\xi}_{L+1} = \tilde{\xi}_L + [\nabla_\xi^2 J(\tilde{\xi}_L)]^{-1}[\nabla_\xi^{*2} J(\tilde{\xi}_L)] \quad (7)
$$

, where the first and second gradients are defined as

$$
\nabla_\xi J(\xi) = -\sum_{i=1}^{N}[\tilde{z}_\xi(t_i) - z(t_i)]^*(GG^*)^{-1}[\nabla_\xi \tilde{z}_\xi(t_i)] \quad (8)
$$

$$
\nabla_\xi^2 J(\xi) = \sum_{i=1}^{N}[\nabla_\xi \tilde{z}_\xi(t_i)]^*(GG^*)^{-1}[\nabla_\xi \tilde{z}_\xi(t_i)] \quad (9)
$$

## VII. ACCURACY OF PARAMETER ESTIMATION RESULTS

Various means are currently used for measuring the accuracy of derivative estimates. The accuracy of the estimates can be precisely and quantitatively defined. Common accuracy measures are engineering judgment, bias of the estimates, scatter, sensitivity, correlation, and Cramer-Rao bound. The sensitivity is a reasonable measure of accuracy only when a single parameter is estimated, while correlation is suitable for two dimensional applications [1]. Since more than 10 parameter values have to be identified in this study, the utility of a tool restricted to two-dimensional subspace is limited. Thus engineering judgment, scatter, and Cramer-Rao bound remain valid for accuracy measurement of the estimates for aerodynamic parameters.

*A. Engineering Judgment*

Large The output error approach was successfully used to produce the aerodynamic parameter estimates for aircraft maneuvers. One of the most fundamental factors in judging the accuracy of the estimates is the anticipated accuracy. The basic criterion is the reasonability of the estimated derivatives. Engineering judgment is the quality of the fit of the measured and estimated time histories, and is used to assemble and weigh all of the available information about the estimates.

In Figs. 2 and 3 the actual flight data and the simulation results derived from estimated parameters are compared for longitudinal motion and lateral-directional motion, respectively. Longitudinal motion is excited by the stabilator control input during the level flight for 93.4 knots initial airspeed at 4,873 feet altitude. The top graph of Fig. 2 shows the triple step control input to the stabilator (solid line) to increase the accuracy of parameter estimated results. The rest of the graphs dashed lines represent flight test data and solid lines represent simulated results of estimated variables from the output error method. As can be seen, the simulation results for the parameter estimates match the flight test data of airspeed, angle of attack, pitch rate, and pitch angle with good accuracy.
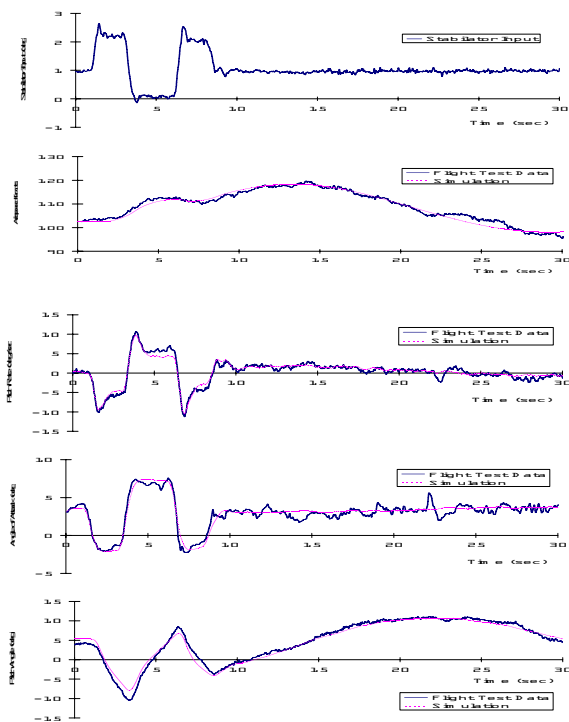


Fig. 2 Comparison of Longitudinal Motions Excited by Stabilator Control Inputs in the Short Period Mode

Fig. 3 represents the lateral-directional motions excited by rudder and aileron control inputs in the Dutch-roll mode. The top graph shows multi-step control input to the rudder (solid line) followed by the aileron input (dashed line). The rest of the graphs represent sideslip angle, yaw rate, roll rate, and roll angle. All the figures, in general, indicate good agreement between simulation results and measured data.
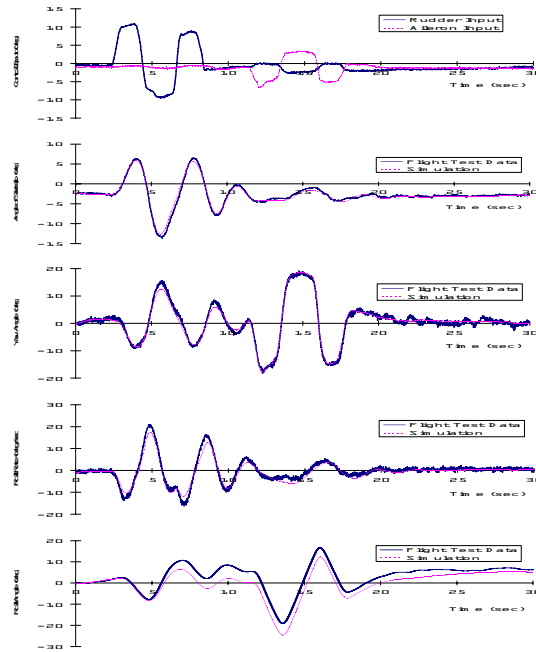


Fig. 3 Comparison of Lateral-Directional Motions Excited by Rudder and Aileron Control Inputs in the Dutch-Roll Mode

*B. Scatter*

The Plots in Fig. 4 through Fig. 9 show the comparison of the longitudinal stability/control derivatives estimated from flight test by stick-fixed (●) and stick-free (○), and from wind tunnel test (✕) of ChangGong-91 [10], to the derivatives of general aviation aircraft Navion (△) [11], while the dashed lines represent normal ranges of the relevant derivatives for the general aviation aircraft suggested by Smetana [12].

Fig. 4 shows a scatter diagram of the estimated pitching moment coefficient for the angle of attack, $C_{m_\alpha}$, which affects the frequency in short period motion, with respect to the angle of attack of the tested aircraft. One of the wind tunnel test data for ChanGong-91 is located outside of the normal range of general aviation aircraft derivatives (-0.5~-1.0 $rad^{-1}$), but all of the estimates of $C_{m_\alpha}$ of the tested aircraft are located inside the limits. Note the tendency of the absolute value of $C_{m_\alpha}$ to increase as the angle of attack increases.

Fig. 5 exhibits a scatter diagram of $C_{m_q}$ with respect to the angle of attack, called 'pitching damping'. This derivative is a very important coefficient in the determination of short period damping and phugoid frequency. The estimates of $C_{m_q}$ are

located around -15.0 $sec\,rad^{-1}$, which are slightly larger absolute values than the derivatives of general aviation aircraft.

Fig. 6 represents a scatter diagram of $C_{m_{\delta e}}$, called 'elevator effectiveness', with respect to the angle of attack. The figure shows that estimates of absolute values are much lager than the wind tunnel test measures of ChanGong-91 or the derivatives of Navion. The horizontal tail of ChangGong-91 is an all movable type with tab, which increase the effectiveness of horizontal tail, so $C_{m_{\delta e}}$ estimates are larger than other general aviation aircraft.
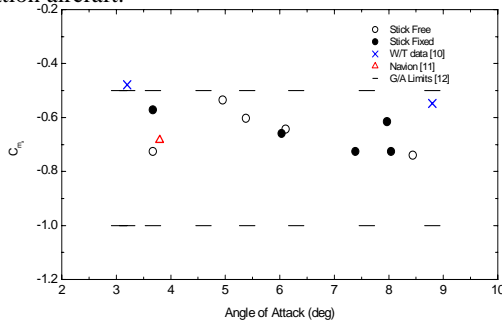


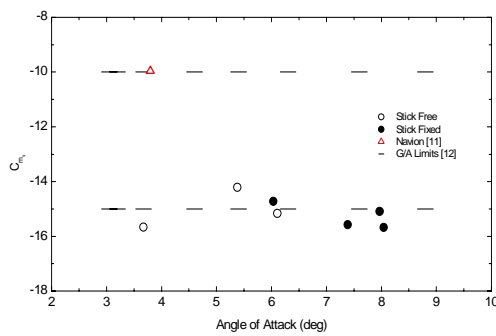Fig. 4 Scatter Diagram of Estimated $C_{m_\alpha}$ for AOA
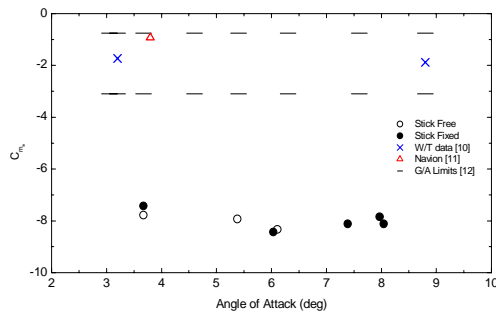


Fig. 5 Scatter Diagram of Estimated $C_{m_q}$ for AOA



Fig. 6 Scatter Diagram of Estimated $C_{m_{\delta e}}$ for AOA

Plots in Fig. 7 through Fig. 9 show the lateral-directional stability and control derivative estimates from the flight test. Fig. 7 presents the scatter diagram of estimated $C_{n_\beta}$ (the

change in yawing moment coefficient resulting from a change in sideslip angle) with respect to the angle of attack of the tested aircraft. The value of $C_{n_\beta}$ determines primarily the Dutch-roll natural frequency and affects the spiral stability of the aircraft. It is generally agreed that the value $C_{n_\beta}$ as high as practically possible are desirable for good flying qualities. Some of the estimates are located outside of the normal range of the general aviation aircraft, but the scatter of the estimates is small. This figure shows $C_{n_\beta}$ estimates of the tested aircraft larger than those of the general aviation aircraft. It seems to be caused by the fact that the vertical tail area of the tested aircraft is larger than that of the typical general aviation aircraft. The figure also reveals that the values of $C_{n_\beta}$ from the stick-free flight test are slightly larger than those from the stick-fixed flight test.
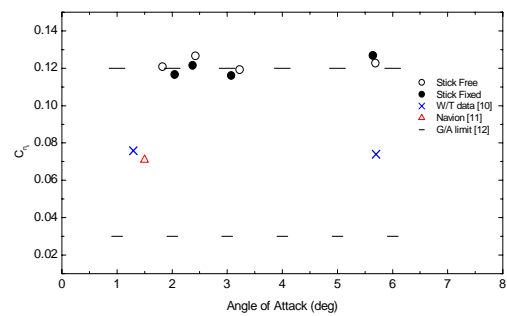


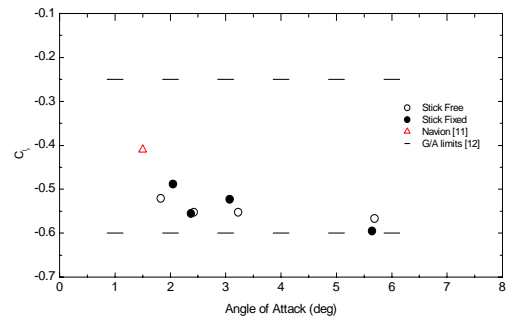Fig. 7 Scatter Diagram of Estimated $C_{n_\beta}$ with respect to AOA



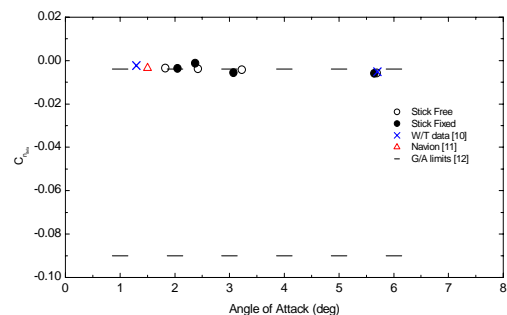Fig. 8 Scatter Diagram of Estimated $C_{l_p}$ with respect to AOA



Fig. 9 Scatter Diagram of Estimated $C_{n_{\delta a}}$ with respect to AOA

Fig. 8 shows the scatter diagram of the estimated stability derivative $C_{l_p}$, the roll damping derivative, which is the change in rolling moment coefficient due to variation in rolling velocity. $C_{l_p}$ is the principal determinant of the damping-in-roll characteristics of the aircraft. The $C_{l_p}$ estimates are located inside of typical range for general aviation aircraft from -0.25 per radian to -0.6 per radian. The increasing tendency for the estimates of absolute value is observed in the derivatives as the angle of attack is increased.

Fig. 9 exhibits the scatter diagram of estimated $C_{n_{\delta a}}$ (the change in yawing moment coefficient with variation in aileron deflection) with respect to the angle of attack of the tested aircraft. It results from the difference between drag on the up and down ailerons deflection for angle and direction. So, the desirable value of $C_{n_{\delta a}}$ is either zero or a very small positive value. Some estimates are located outside of the normal range of the general aviation aircraft, but the scatter of the estimates is small and is located around the wind tunnel data. No cumbersome tendencies can be observed in the derivatives estimates for control fixed or free condition. But it can be noted that the absolute values of $C_{n_{\delta a}}$ increases as the angle of attack is increased. In spite of the operating aileron, it is implied that the yawing motion of ChangGong-91 is small, since it is more stable than other similar aircraft. Also, ChangGong-91 is more stable under the higher airspeed conditions than for lower airspeed.

## C. Cramer-Rao Bound

The Cramer-Rao bound is known as the best theoretical measure for the estimates of the accuracy for Maximum Likelihood Estimation (MLE) approach. While the insensitivity is approximately the conditional standard deviation of the parameter estimate, given that all of the other parameters are known, the Cramer-Rao Bound is an approximation of the unconditional standard deviation, which is not same with the sampled standard deviation. The comparison of the Cramer-Rao bounds as the sample standard deviations obtained from the data scatter gives a good indication of the adequacy of the assumptions made in the theoretical development.

Table 1 and 2 comprise averages and standard deviations of aerodynamic parameter estimates in the scatter diagrams as well as their Cramer-Rao bounds in the longitudinal motion and lateral-directional motion respectively, which are obtained from flight tests. In longitudinal motion, the estimates for aerodynamic parameters, such that $C_{x_u}$, $C_{x_\alpha}$ can be converted to stability and control derivatives, such as $C_{D_0}$, $C_{D_\alpha}$. Since addition or subtraction is required to convert from some aerodynamic parameters to derivatives, it is difficult to convert from Cramer-Rao bounds for aerodynamic parameters to those for stability and control derivatives. The last column of Tables

2 and 3 indicate the differences between the Cramer-Rao

TABLE I
STANDARD DEVIATION AND CRAMER-RAO BOUND OF ESTIMATED PARAMETERS IN LONGITUDINAL MOTION

| Aerodynamic Parameters | Averages (1) | Standard Deviations (2) | Averages of C-R Bound (3) | S. D / C-R B (2) / (3) |
|---|---|---|---|---|
| $C_{x_u}$ | -0.0744 | 0.04269 | 0.0318 | 1.343582 |
| $C_{z_u}$ | -0.4206 | 0.225344 | 0.5037 | 0.447344 |
| $C_{z_\alpha}$ * | -4.3154 | 0.200451 | 0.50503 | 0.396907 |
| $C_{x_{\delta e}}$ | 0.0185 | 0.051126 | 0.1807 | 0.283005 |
| $C_{z_{\delta e}}$ | -0.3743 | 0.111588 | 0.0073 | 15.30834 |

bounds and standard deviations for longitudinal and lateral-directional aerodynamic estimates based on the flight test measurement, respectively.

TABLE II
STANDARD DEVIATION AND CRAMER-RAO BOUND OF ESTIMATED PARAMETERS IN LATERAL-DIRECTIONAL MOTION

| Stability / Control Derivatives | Averages (1) | Standard Deviations (2) | Averages of C-R Bound (3) | S. D / C-R B (2) / (3) |
|---|---|---|---|---|
| $C_{n_\beta}$ | 0.121323 | 0.004039 | 0.00027482 | 14.69823 |
| $C_{n_p}$ | -0.066170 | 0.009047 | 0.00240127 | 3.767443 |
| $C_{l_p}$ | -0.544438 | 0.032649 | 0.00116361 | 28.05836 |
| $C_{n_r}$ | -0.130425 | 0.013346 | 0.00130225 | 10.24858 |
| $C_{l_r}$ | 0.115525 | 0.016694 | 0.00096251 | 17.34424 |
| $C_{n_{\delta a}}$ | -0.004213 | 0.001522 | 0.01567526 | 0.097071 |
| $C_{l_{\delta a}}$ | 0.225104 | 0.019188 | 0.00830147 | 2.311379 |
| $C_{l_{\delta r}}$ | -0.071538 | 0.006439 | 0.01574283 | 0.409014 |

The For the flight test derivatives of ChangGong-91, the ratios of the standard deviations to the Cramer-Rao bounds in the last column of Table 2 and 3 varies between 0.0971 and 28.058. Table 2 and 3 show that the ratio of standard deviations to the Cramer-Rao bounds of the estimates for the control derivatives is larger than the ratio for the stability derivatives except $C_{z_{\delta e}}$. The ratio implies that the aerodynamic model structure is more accurate for the control derivatives than for the stability derivatives.

According to Cramer-Rao theory, the smaller the ratio is, the more accurate the estimates are. A typical range of this ratio, fudge factor, is known to lie between 5 and 10 for appropriate estimation [1]. For instance, it might be true that the individual estimates are as accurate as indicated by the Cramer-Rao bounds and that the scatter reflects actual changes in the coefficients.

There seems to be several reasons for unsatisfying estimation accuracy results. First, the mathematical model for the flight dynamics used in this study is not perfect. The evaluation of accuracy measures on actual flight data is complicated by the impossibility of establishing true values for comparison, and by the inevitable presence of unmodeled effects. Also, based on some assumptions the nonlinear equations of motion were simplified to linear versions. This seems to cause errors in parameter estimation which assumes accurate models. The scatter in the estimates results from unmodeled errors that would not be reflected in the Cramer-Rao bounds. Second, flight test data tends to involve both measurement and process noises, but the output error method considers only measurement noise. More accurately estimated results can be extracted if both measurement noise and process noise are considered. The third reason results from inevitable turbulence during flight test. It is a kind of measurement noise, so it is likely to affect the accuracy of estimated parameters. The last reason is the error derived from the input value of the aircraft and the flight condition. For the airplane, the moment of inertia was not measured but calculated, and the weight and the center of gravity were changed during the flight. The mathematical and physical assumptions, solid body aircraft with neglecting aeroelastic effect, symmetric aircraft for x-axis, neglected asymmetry effect of the propeller and etc., affect on the results of the derivatives.

## VIII. CONCLUSION

The flight test data of light aircraft were processed with the maximum likelihood estimation approach using measurement noise only. The reliability of the parameter estimates determined from flight data is examined in engineering judgment, scatter and Cramer-Rao bound, which turns out to be satisfactory with minor defects. For inevitable inaccuracy of the flight derivatives estimates, several reasons are analyzed.

The empirical and theoretical measures should all be taken to aid engineering judgments. For the engineering judgment, good agreement between the measured data and the simulation results, which is derived from the parameter estimates, shows the accuracy for flight derivative estimates. The stability and control derivatives estimates are compared with the wind tunnel data of ChangGong-91. The appropriateness of the stability and control derivatives estimates are also evaluated using normal ranges of the relevant derivatives for a general aviation aircraft as the criteria.

The scatter has a significant advantage over many theoretical measures of accuracy. The scatter diagrams of the stability and control derivatives estimates are analyzed for accuracy. The scatter diagrams of estimates, which are gathered within small range, imply reliable parameter estimated results. Also they allude the flying quality of ChagGong-91 compared with other general aviation aircraft.

The Cramer-Rao bound is known as the best of the theoretical and quantitative measures of accuracy. But it is significantly affected by the colored measurement noise and modeling error present in actual flight data. In this study, the ratios of the standard deviations to the Cramer-Rao bounds for control derivatives are larger than for stability derivatives, which are results from modeling error. The comparison of the scatter and the Cramer-Rao bounds provides a good check of empirical and theoretical measures. Modeling error, not considering process noises, inevitable turbulence and instrument error produce inaccurate results.

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