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Takagi-Sugeno Fuzzy Control of Induction Motor

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Abstract—This paper deals with the synthesis of fuzzy state feedback controller of induction motor with optimal performance. First, the Takagi-Sugeno (T-S) fuzzy model is employed to approximate a non linear system in the synchronous d-q frame rotating with electromagnetic field-oriented. Next, a fuzzy controller is designed to stabilise the induction motor and guaranteed a minimum disturbance attenuation level for the closed-loop system. The gains of fuzzy control are obtained by solving a set of Linear Matrix Inequality (LMI). Finally, simulation results are given to demonstrate the controller's effectiveness.

Keywords—Rejection disturbance, fuzzy modelling, open-loop control, Fuzzy feedback controller, fuzzy observer, Linear Matrix Inequality (LMI)

I. Introduction

THE great interest to the induction motor is linked to its large industrial application field as well as its properties of reliability, robustness and low cost. In the last two decades, many studies have been developed around fuzzy law control [1], [2], [3], [4], [5]. Control of induction motor is well known to be difficult owing to the fact that the dynamic model is nonlinear, its electrical parameters drift with temperature and the flux which must be controlled was not available for feedback. We try in this paper to design a fuzzy feedback control that guarantee both stability and disturbance rejection. We suppose that all the state variables susceptible to be controlled are available for feedback. Several contributions on the design of feedback control have been presented [6], [7], [8], [9]. Paper [10] deal with methods based in induction model linearization around a working point in order to apply linear control theory. This solution limited the effectiveness of the feedback controller. To overcome this problem a fuzzy tracking control design based on Takagi-Sugeno (T-S) model is developed. First, the TS fuzzy model is used to represent a nonlinear system. A tracking performance, which is related to tracking error for all bounded known load torque inputs, is formulated. The global linear fuzzy model is composed of set linear models witch are interpolated by membership functions. The stability conditions of the induction motor with the fuzzy controller are converted in terms of LMI problem which can solved very efficiently using the optimisation techniques. The premise variables are assumed to be measurable.

This paper is organised as follows: in section 2, an open-loop control strategy is presented which include the physical model expression. Section 3, deal with the synthesis of a fuzzy tracking control with optimal performance. The gains of fuzzy control are obtained by solving a set of Linear Matrix Inequality (LMIs). In section 4, simulation results and are given to highlight the effectiveness of the proposed control law. The last section gives a conclusion on the main works developed in this paper.

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II. Open-Loop Control Strategy

A. Physical model of induction motor

Under the assumptions of linearity of the magnetic circuit, the electromagnetic dynamic model of the induction motor in the synchronously d-q reference frame can be described as:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + v(t) \tag{1}$$

where

$$f(x(t)) = \begin{bmatrix} -\gamma i_{sd} + w_s i_{sq} + \frac{K_s}{\tau_r}\Psi_{rd} + K_s n_p w_m \Psi_{rq} \\ -w_s i_{sd} - \gamma i_{sq} - K_s n_p w_m \Psi_{rd} + \frac{K_s}{\tau_r}\Psi_{rq} \\ \frac{M}{\tau_r} i_{sd} - \frac{1}{\tau_r}\Psi_{rd} + (w_s - n_p w_m)\Psi_{rq} \\ \frac{M}{\tau_r} i_{sq} - (w_s - n_p w_m)\Psi_{rd} - \frac{1}{\tau_r}\Psi_{rq} \\ \frac{n_p M}{J L_r}(\Psi_{rd} i_{sq} - \Psi_{rq} i_{sd}) - \frac{f}{J} w_m \end{bmatrix},$$

$$g(x(t)) = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \end{bmatrix}^T,$$

$$x(t) = [i_{sd}\ i_{sq}\ \Psi_{rd}\ \Psi_{rq}\ w_m]^T$$

$$u(t) = [u_{sd}\ u_{sq}]^T,$$

$$v(t) = \left[0\ 0\ 0\ 0\ -\frac{C_r}{J}\right]^T,$$

$$\gamma = \left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r}\right),\ K_s = \frac{M}{\sigma L_s L_r},\ \tau_r = \frac{L_r}{R_r},$$

$$\tau_s = \frac{L_s}{R_s},\ \sigma = 1 - \frac{M^2}{L_s L_r}$$

In which w_m is rotor speed, w_s is the electrical speed of stator, (Ψ_{rd}, Ψ_{rq}) are the rotor fluxes, (i_{sd}, i_{sq}) are the stator currents and (u_{sd}, u_{sq}) are the stator voltages. The load torque C_r is a known step disturbance. The motor parameters are: moment of inertia J, rotor and stator resistances (R_s, R_r), inductances (L_s, L_r) , mutual inductance M, friction coefficient f and number of poles pairs n_p .

B. Open-loop control

In this paragraph, we explore the structure of the open-loop control which will then be used in the next section to construct the T-S fuzzy model of the induction motor. After replacing the state variables $(i_{sd}, i_{sq}, \Psi_{rd}, \Psi_{rq}, w_m)$ of the induction motor by the reference signals $x_r(t) =$

$(i_{sdc}, i_{sqc}, \Psi_{rdc}, 0, w_{mc})$ in (1), we obtain the following model [6].

$$\begin{cases} \frac{d}{dt}i_{sdc} = -\gamma i_{sdc} + w_s i_{sqc} + \frac{K_s}{\tau_r}\Psi_{rdc} + \frac{1}{\sigma L_s}u_{sdc} \\ \frac{d}{dt}i_{sqc} = -w_s i_{sdc} - \gamma i_{sqc} - K_s n_p w_{mc}\Psi_{rdc} + \frac{1}{\sigma L_s}u_{sqc} \\ 0 = -(w_s - n_p w_{mc})\Psi_{rdc} + \frac{M}{\tau_r}i_{sqc} \\ \frac{d}{dt}\Psi_{rdc} = \frac{M}{\tau_r}i_{sdc} - \frac{1}{\tau_r}\Psi_{rdc} \\ \frac{d}{dt}w_{mc} = \frac{n_p M}{JL_r}(\Psi_{rdc}i_{sqc}) - \frac{f}{J}w_{mc} - \frac{1}{J}C_r \end{cases} \quad (2)$$

The tow last equations in (2) lead to the stator current reference:

$$\begin{cases} i_{sdc} = \frac{\Psi_{rc}}{M} + \frac{\tau_r}{M}\frac{d}{dt}\Psi_{rc} \\ i_{sqc} = \frac{JL_r}{n_p M\Psi_{rc}}(\frac{C_r}{J} + \frac{f}{J}w_{mc} + \frac{d}{dt}w_{mc}) \end{cases} \quad (3)$$

From the first two equations in (2), we obtain the expression of the open-loop control:

$$\begin{cases} u_{sdc} = \sigma L_s(\frac{d}{dt}i_{sdc} + \gamma i_{sdc} - w_{sc}i_{sqc} - \frac{K_s}{\tau_r}\Psi_{rdc}) \\ u_{sqc} = \sigma L_s(\frac{d}{dt}i_{sqc} + \gamma i_{sqc} + w_{sc}i_{sdc} + K_s n_p w_m \Psi_{rdc}) \end{cases} \quad (4)$$

According to (2), the electrical speed reference of the stator is:

$$w_{sc} = n_p w_{mc} + \frac{M}{\tau_r \Psi_{rdc}} i_{sqc} \quad (5)$$

III. FUZZY CONTROLLER DESIGN

It is known that the effect of external disturbance will deteriorate the control performance of the fuzzy control system and even lead to instability of the control system. Therefore, several works deal with the synthesis of a fuzzy control law with H_∞ performance [11], [12], [13], [14]. In this section, a feedback control law with H_∞ attenuation is proposed. Lyapunov approach is used to derive quadratic stability condition. The problem is expressed in an LMI form by using some matrix proprieties.

A. T-S Fuzzy model of induction model

The Takagi-Sugeno (T-S) fuzzy model is employed here to approximate the induction model system in the synchronous d-q frame rotating with electromagnetic field-oriented. The principle behind field oriented control is that the machine flux and torque are controlled independently, in a similar fashion to a separately exited DC machine. In consequently the rotor flux vector is (Ψ_{rd}, Ψ_{rq}) aligned to the d-axis and the following results can be obtained:

$$\begin{cases} \Psi_{rd} = \Psi_{rdc} \\ \Psi_{rq} = 0 \end{cases} \quad (6)$$

The electrical speed of the stator defined in the synchronous d-q frame rotating with electromagnetic field-oriented is obtained from (5)) with replacing w_{mc} by w_m and i_{sqc} by $i_s q$ the current measurement as follows:

$$w_s = n_p w_m + \frac{M}{\tau_r \Psi_{rdc}} i_{sq} \quad (7)$$

If we replace in (1) the electrical speed of the stator by the expression defined by (7), the nonlinear model of the induction motor can be written as the state space form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dw(t) \\ y = C\,x(t) \end{cases} \quad (8)$$

where

$$A = \begin{bmatrix} -\gamma & \begin{pmatrix} n_p w_m + \\ \frac{M}{\tau_r \Psi_{rdc}} i_{sq} \end{pmatrix} & \frac{K_s}{\tau_r} & K_s n_p w_m & 0 \\ -\begin{pmatrix} n_p w_m + \\ \frac{M}{\tau_r \Psi_{rdc}} i_{sq} \end{pmatrix} & -\gamma & -K_s n_p w_m & \frac{K_s}{\tau_r} & 0 \\ \frac{M}{\tau_r} & 0 & -\frac{1}{\tau_r} & \frac{M}{\tau_r \Psi_{rdc}} i_{sq} & 0 \\ 0 & \frac{M}{\tau_r} & -\frac{M}{\tau_r \Psi_{rdc}} i_{sq} & -\frac{1}{\tau_r} & 0 \\ 0 & 0 & \frac{n_p M}{JL_r} i_{sq} & -\frac{n_p M}{JL_r} i_{sd} & -\frac{f}{J} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \end{bmatrix}^T, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & -\frac{1}{J} \end{bmatrix}^T, \; w = C_r.$$

Considering the sector of nonlinearities of the terms $z_j = x_j \in [z_{j\min}, z_{j\max}]$ of the matrix $A(x(t)$ with $j = 1, 2, 3$:

$$\begin{cases} z_1(t) = i_{sd}(t) \\ z_2(t) = i_{sq}(t) \\ z_3(t) = w_m(t) \end{cases} \quad (9)$$

Thus, we can transform the non linear terms under the following shape:

$$z_j(t) = F_{1j}(t).z_{j\max} + F_{2j}(t).z_{j\min};\; j = \{1, 2, 3\} \quad (10)$$

where

$$\begin{cases} F_{1j}(t) = \frac{z_j(t) - z_{j\min}}{z_{j\max} - z_{j\min}} \\ F_{2j}(t) = \frac{z_{j\max} - z_j(t)}{z_{j\max} - z_{j\min}} \end{cases} \quad (11)$$

The fuzzy model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem for the induction motor. The ith rule of the fuzzy model for the nonlinear system is of the following form:

Rule Ri
If $(z_1(t)$ is F_{i1}) and $(z_2(t)$ is F_{i2}) and $(z_3(t)$ is F_{i3})
Then $\dot{x}(t) = A_i x(t) + Bu(t) + w(t),\; i = 1, 2, \cdots, 8$
The global fuzzy model is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{8} h_i(z(t))(A_i x(t) + B_i u(t) + v(t)) \quad (12)$$

where

$$\lambda_i(z(t)) = \prod_{j=1}^{3} F_{ij}(z_j(t)), \quad (13)$$

$$h_i(z(t)) = \frac{\lambda_i(z(t))}{\sum_{i=1}^{8} \lambda_i(z(t))} \quad (14)$$

for all $t > 0$, $h_i(z(t)) \geq 0$ and $\sum_{i=1}^{8} h_i(z(t)) = 1$.
In our work we consider B and D constant matrixes.

B. Error state model

Considering the global fuzzy linear model of the induction motor structured in the following forms:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{8} h_i(z(t))(A_i x(t) + Bu(t) + Dw(t)) \\ y(t) = C\, x(t) \end{cases} \quad (15)$$

Define the state reference vector as

$$x_c = (i_{sdc}\,, i_{sqc},\, \Psi_{rdc},\, 0,\, w_{mc})^T$$

Denoting by $e(t) = x(t) - x_c$ the state tracking errors. The main different for the ordinary PDC controller in [11], [12] is to have a term for the feedback signal of x_c. The new PDC fuzzy controller is constructed as follows [15], [16]:

$$u(t) = -\sum_{i=1}^{8} h_i(z(t)) K_i e(t) \quad (16)$$

The resulting fuzzy error model can be described as

$$\dot{e}(t) = \sum_{i=1}^{8} h_i(z(t))(A_i e(t) + Bu(t) + A_i x_c + Dw(t)) \quad (17)$$

In order to avoid static errors, an integral action is added to the new PDC fuzzy controller [14], [17], [18].

$$\begin{aligned} u(t) &= -\sum_{i=1}^{8} h_i(z(t)) \begin{bmatrix} K_i & F_i \end{bmatrix} \begin{bmatrix} e(t) \\ e_I(t) \end{bmatrix} \\ &= -\sum_{i=1}^{8} h_i(z(t)) \overline{K}_i \overline{e} \end{aligned} \quad (18)$$

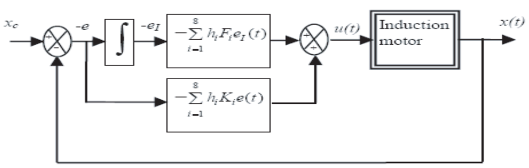

Fig. 1: Control scheme using an integral part

The TS model of the induction motor with an integral action related to the tracking error state can be written in the augmented form

$$\dot{\overline{e}}(t) = \sum_{i=1}^{8} h_i(t)\ [\overline{G}_i \overline{e}(t) + \overline{D}_i \overline{w}(t)] \quad (19)$$

Such as

$$\overline{G}_i = \overline{A}_i - \overline{BK}_i \quad (20)$$

$$\overline{A}_i = \begin{bmatrix} A_i & 0 \\ I & 0 \end{bmatrix}, \overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \overline{D}_i = \begin{bmatrix} A_i & D \\ 0 & 0 \end{bmatrix}, \quad \overline{w}(t) = \begin{bmatrix} x_c \\ w(t) \end{bmatrix} \quad (21)$$

Fig (1) shows the implementation scheme of the considered fuzzy control law.

C. LMI formulation for H_∞ performance

The presence of disturbances $\overline{w}(t)$ will deteriorate the tracking control performance of the fuzzy control system. In order to minimize the effect of $\overline{w}(t)$ on the control system, H_∞ tracking performance related to tracking error has been considered [19], [20], [21].

$$\int_0^\infty \overline{e}^T(t)\, \overline{e}(t)\ dt \leq\ \gamma^2 \int_0^\infty \overline{w}^T(t) \overline{w}(t) dt \quad (22)$$

Where γ is a prescribed value which denotes the worst case effect of $\overline{w}(t)$ on $e(t)$ and P is some symmetric positive-definite weighting matrix. The following lemma gives the results of the H_∞ norm bounded.

Lemma 1. *If there exists a symmetric and positive definite $X = X^T$ such that the following matrix inequality are satisfied*

$$\begin{bmatrix} \overline{A}_i X + X\overline{A}_i^T - \overline{B} Y_i - Y_i^T \overline{B}^T & \overline{D_i} & X \\ \overline{D}_i^T & -\gamma^2 I & 0 \\ X & 0 & -I \end{bmatrix} < 0 \quad (23)$$

Then for a prescribed γ, H_∞ tracking control performance in (22) is guaranteed via the TS fuzzy model-based state feedback controller (23). The control gain are given by

$$\overline{K}_i = Y_i X^{-1} \quad (24)$$

Proof .

Consider the Lyapunov function $V(\overline{e}(t)) = \overline{e}^T(t) P\, \overline{e}(t)$, where $P = P^T > 0$ the common positive matrix. In order to establish to the asymptotic stability of the inequality (22), the time derivative of $V(\overline{e}(t))$ has to satisfy the following condition:

$$\dot{V}(\overline{e}(t))\ < 0 \quad (25)$$

In order to achieve the H_∞ tracking performance related to the tracking error, for $x_c(t)$, (25) becomes:

$$\dot{V}(\overline{e}(t)) + \overline{e}^T(t)\, \overline{e}(t) - \gamma^2\, \overline{w}^T(t)\, \overline{w}(t) < 0 \quad (26)$$

Replacing $V(\bar{e}(t))$ by its value $\overline{e}^T P \overline{e}$ in (26), the last equation can be written as the following form LMI:

$$\begin{aligned} LMI_1 = &\sum_{i=1}^{8} h_i(z(t))\, \overline{e}(t)^T\ [\overline{G}_i P + P\overline{G}_i^T]\, \overline{e}(t) \\ &+ \sum_{i=1}^{8} h_i(z(t)) \overline{w}(t)^T \left[\overline{D}_i^T\ P\right] \overline{e}(t) \\ &+ \sum_{i=1}^{8} h_i(z(t)) \overline{e}(t)^T \left[P\ \overline{D}_i\right]\ \overline{w}(t) - \gamma^2 \overline{w}^T \overline{w} < 0 \end{aligned} \quad (27)$$

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$$LMI_1 = \sum_{i=1}^{8} h_i(z(t)) \left[\overline{e}^T(t) \;\; \overline{w}^T(t) \right] \begin{bmatrix} \overline{G}_{ij}^T P + P\overline{G}_{ij} + I & P\overline{D}_i \\ \overline{D}_i^T P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \overline{e}(t) \\ \overline{w}(t) \end{bmatrix} < 0 \quad (28)$$

This main:

$$LMI_1 = \begin{bmatrix} \sum_{i=1}^{8} h_i(z(t)) \left[\overline{G}_i^T P + P\overline{G}_i\right] + I & P \sum_{i=1}^{8} h_i(z(t))\overline{D}_i \\ \sum_{i=1}^{8} h_i(z(t))\overline{D}_i^T P & -\gamma^2 I \end{bmatrix} < 0 \quad (29)$$

$$LMI_1 = \begin{bmatrix} \sum_{i=1}^{8} h_i(z(t)) \left[\overline{G}_i^T P + P\overline{G}_i\right] & P \sum_{i=1}^{r} h_i(z(t))\overline{D}_i \\ \sum_{i=1}^{r} h_i(z(t))\overline{D}_i^T P & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad (30)$$

$$LMI_1 = \begin{bmatrix} \sum_{i=1}^{8} h_i(z(t)) \left[\overline{G}_i^T P + P\overline{G}_i\right] & P \sum_{i=1}^{r} h_i(z(t))\overline{D}_i \\ \sum_{i=1}^{r} h_i(z(t))\overline{D}_i^T P & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} I \\ 0 \end{bmatrix} \begin{bmatrix} I & 0 \end{bmatrix} < 0 \quad (31)$$

Using the Schur's complement, (31) is equivalent to:

$$LMI_1 = \begin{bmatrix} \sum_{i=1}^{8} h_i(z(t)) \left[\overline{G}_i^T P + P\overline{G}_i\right] & P \sum_{i=1}^{r} h_i(z(t))\overline{D}_i & I \\ \sum_{i=1}^{r} h_i(z(t))\overline{D}_i^T P & -\gamma^2 I & 0 \\ I & 0 & -I \end{bmatrix} < 0 \quad (32)$$

$$LMI_1 = \sum_{i=8}^{8} h_i(z(t)) \begin{bmatrix} \left[\overline{G}_i^T P + P\overline{G}_i\right] & P\overline{D}_i & I \\ \overline{D}_i^T P & -\gamma^2 I & 0 \\ I & 0 & -I \end{bmatrix} < 0 \quad (33)$$

Therefore

$$LMI_1 = \begin{bmatrix} \left[\overline{G}_i^T P + P\overline{G}_i\right] & P\overline{D}_i & I \\ \overline{D}_i^T P & -\gamma^2 I & 0 \\ I & 0 & -I \end{bmatrix} < 0 \quad (34)$$

After congruence with $diag\left[P^{-1}, I, I\right]$, inequality (34) becomes:

$$\begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \left[\overline{G}_i^T P + P\overline{G}_i\right] & P\overline{D}_i & I \\ \overline{D}_i^T P & -\gamma^2 I & 0 \\ I & 0 & -I \end{bmatrix} \times \begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \quad (35)$$

Developed the last equation, (35) can be written as:

$$\begin{bmatrix} P^{-1}\left[\overline{G}_i^T P + P\overline{G}_i\right] P^{-1} & P^{-1}P\overline{D}_i & P^{-1} \\ \overline{D}_i^T P P^{-1} & -\gamma^2 I & 0 \\ P^{-1} & 0 & -I \end{bmatrix} < 0 \quad (36)$$

Considering $X = P^{-1}$ and $Y = \overline{K}_i P^{-1} = \overline{K}_i X$, we obtain the same matrix as in (23):

$$\begin{bmatrix} \overline{A}_i X + X\overline{A}_i^T - \overline{B}Y_i - Y_i^T\overline{B}^T & \overline{D}_i & X \\ \overline{D}_i^T & -\gamma^2 I & 0 \\ X & 0 & -I \end{bmatrix} < 0 \quad (37)$$

IV. SIMULATION RESULTS

The proposed feedback control law has been first tested in simulation. The three-phase 1.1Kw induction motor is characterized by the following parameters:
Mutual inductance $M = 447.5mH$
Moment of inertia $J = 0.0293\ Kg.m^2$
Stator resistance $R_S = 9.65\Omega$
Rotor resistance $R_r = 4.3047\Omega$
Stator inductance $L_S = 471.8mH$
Rotor inductance $L_r = 471.8mH$
The closed-loop simulation results are shown in Figs.2-6. Using the LMI approach, the application of lemma.1 to calculate the control gains of the fuzzy control law (18) gives the followings result:

$$K_1 = \begin{bmatrix} -13.4136 & -19.8574 & 703.1823 & -543.6161 & 14.3120 \\ 19.8574 & -13.4137 & 540.7978 & 703.1823 & 149.2613 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -13.4136 & -19.8574 & 703.1819 & -540.6516 & 14.3117 \\ 19.8574 & -13.4137 & 543.7727 & 703.1819 & 149.2616 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} -13.4136 & -17.5642 & 703.1703 & -540.6107 & 14.3118 \\ 17.5642 & -13.4137 & 543.7732 & 703.1703 & 140.0616 \end{bmatrix}$$

$$K_4 = \begin{bmatrix} -13.4136 & -17.5642 & 702.9864 & -543.5943 & 14.3118 \\ 17.5642 & -13.4137 & 540.7934 & 702.9864 & 150.0128 \end{bmatrix}$$

$$K_5 = \begin{bmatrix} -13.4136 & 17.5642 & 702.8862 & -540.6516 & 14.3118 \\ -17.5612 & -13.4137 & 543.7732 & 702.9862 & 150.0135 \end{bmatrix} \quad (38)$$

$$K_6 = \begin{bmatrix} -13.4136 & 17.5642 & 702.8865 & 543.5943 & 14.3110 \\ -17.5642 & -13.4137 & -540.7935 & 702.7865 & 150.0136 \end{bmatrix}$$

$$K_7 = \begin{bmatrix} -13.4136 & 19.8574 & 700.2301 & 543.6161 & 14.2910 \\ -19.8574 & -13.4137 & -540.7978 & 700.2301 & 150.0137 \end{bmatrix}$$

$$K_8 = \begin{bmatrix} -13.4136 & 19.8574 & 700.2302 & 540.6107 & 13.9656 \\ -19.8574 & -13.4137 & -543.7727 & 700.2302 & 150.0131 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 1.3634 \times 10^7 & -2.2269 \times 10^3 & -9.6190 \times 10^6 & -7.8656 \times 10^7 & 111.9748 \\ 2.2154 \times 10^3 & 1.3621 \times 10^7 & 7.8731 \times 10^7 & -9.7006 \times 10^6 & -111.8061 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 1.3633 \times 10^7 & -2.2146 \times 10^3 & -9.6184 \times 10^6 & -7.8655 \times 10^7 & 111.9835 \\ 2.2175 \times 10^3 & 1.3621 \times 10^7 & 7.8731 \times 10^7 & -9.7013 \times 10^6 & 112.1654 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 1.3633 \times 10^7 & -2.2038 \times 10^3 & -9.6246 \times 10^6 & 7.8659 \times 10^7 & -111.9836 \\ 2.2166 \times 10^3 & 1.3621 \times 10^7 & -7.8735 \times 10^7 & -9.7014 \times 10^6 & -111.7529 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 1.3634 \times 10^7 & -2.2159 \times 10^3 & -9.6252 \times 10^6 & 7.8659 \times 10^7 & -111.9749 \\ 2.2044 \times 10^3 & 1.3621 \times 10^7 & -7.8735 \times 10^7 & -9.7068 \times 10^6 & 112.1741 \end{bmatrix}$$

$$F_5 = \begin{bmatrix} 1.3633 \times 10^7 & 2.2038 \times 10^3 & -9.6246 \times 10^6 & -7.8659 \times 10^7 & 111.9836 \\ 2.2166 \times 10^3 & 1.3621 \times 10^7 & 7.8735 \times 10^7 & -9.7074 \times 10^6 & -111.7929 \end{bmatrix}$$

$$F_6 = \begin{bmatrix} 1.3634 \times 10^7 & 2.2159 \times 10^3 & -9.6252 \times 10^6 & -7.8659 \times 10^7 & 111.9749 \\ -2.2044 \times 10^3 & 1.3621 \times 10^7 & 7.8735 \times 10^7 & -9.7068 \times 10^6 & 112.1741 \end{bmatrix}$$

$$F_7 = \begin{bmatrix} 1.3634 \times 10^7 & 2.2269 \times 10^3 & -9.6190 \times 10^6 & -7.8656 \times 10^7 & -111.9748 \\ -2.2154 \times 10^3 & 1.3621 \times 10^7 & -7.8731 \times 10^7 & -9.7006 \times 10^6 & -111.8016 \end{bmatrix}$$

$$F_8 = \begin{bmatrix} 1.3633 \times 10^7 & 2.2146 \times 10^3 & -9.6184 \times 10^6 & 7.8655 \times 10^7 & -111.9835 \\ -2.2275 \times 10^3 & 1.3621 \times 10^7 & -7.8731 \times 10^7 & -9.7013 \times 10^6 & 112.1654 \end{bmatrix} \quad (39)$$

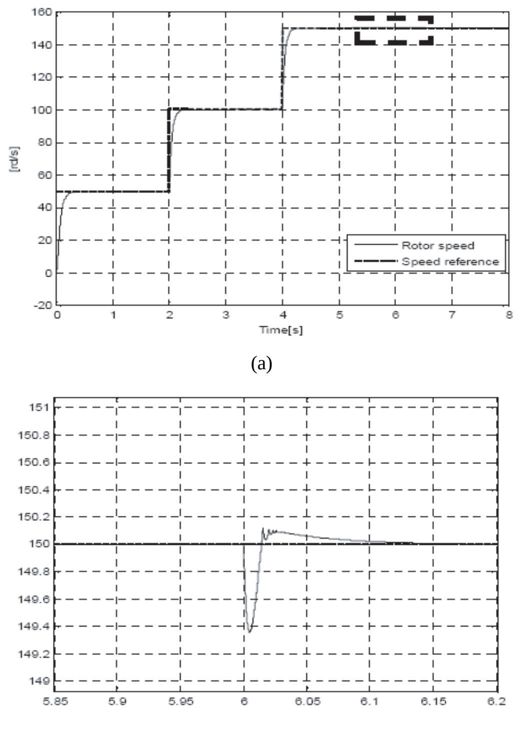

Fig. 2: (a)Rotor speed,(b)Zoom window on rotor speed

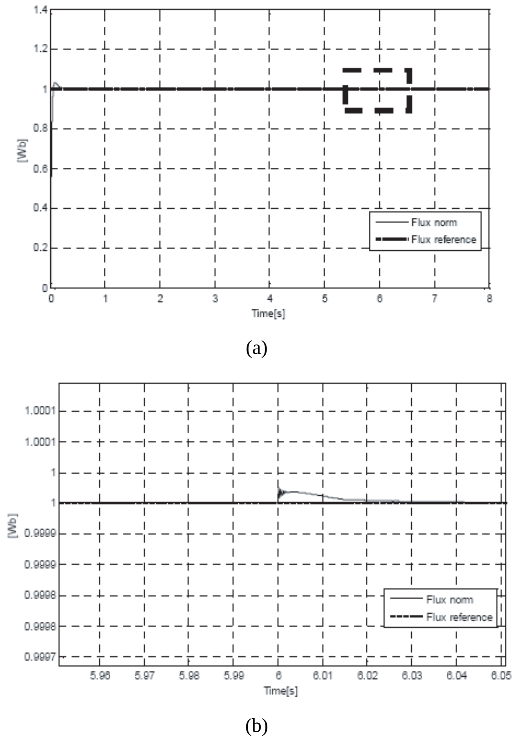

Fig. 5: (a)Flux norm,(b)Zoom window on Flux norm

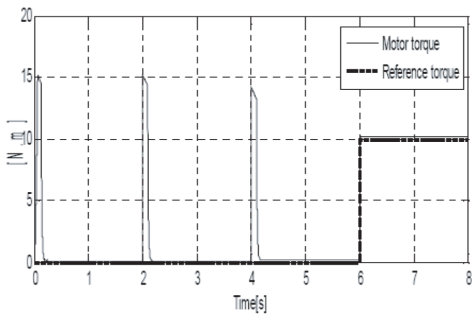

Fig. 3: Motor torque

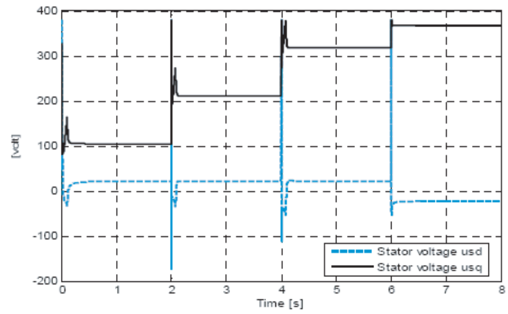

Fig. 6: Stator voltage

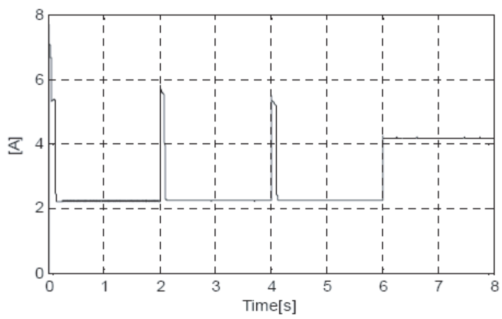

Fig. 4: Stator current norm

By analysis the simulation results, a less tracking errors is observed for the speed and the rotor flux norm in spite of the load torque is applied (Fig.2 a-b and Fig.5 a-b) that show the good performance of the fuzzy control law proposed in terms of pursuit (track) and disturbance rejection. It can be noticed that when the rotor speed reference change of value, the rotor flux norm undergo a weak fluctuation and remains close to its reference value (Fig.5). These results demonstrate and confirm the highlight effectiveness of the proposed control law. The stator current bounded by the saturation function presents an overshoot due to the variation of the rotor speed reference signal.

V. Conclusion

In this paper, we have applied a fuzzy control law of induction machine with H_∞ guaranteed performance. The shape of the used control law is similar to the one of a PDC control, except that this one is expressed with the error state. The T-S fuzzy model is used to represent the induction motor by sets of local state space models witch are interpolated by membership functions. A global fuzzy controller is designed by blending all such local state feedback controllers. The stability of the fuzzy closed-loop systems has been analysed using Lyapunov theory combined with an LMI approach. The effectiveness of the proposed controller is demonstrated through numerical simulations.

APPENDIX

The premise variables are bounded as:

$$w_{m_{\min}} = -200\ (rd/s), \qquad w_{m_{\max}} = 200\,(rd/s),$$
$$i_{sd_{\min}} = -6\text{A}, \qquad i_{sd_{\max}} = 6\text{A},$$
$$i_{sq_{\min}} = -6\text{A}, \qquad i_{sq_{\max}} = 6\text{A}.$$

The sets of the matrixes A_i defined in the 2^3 If-then fuzzy rules are given:

$$A_1 = \begin{bmatrix} -289.2 & -424.5 & -185.4 & -8115.5 & 0 \\ 424.5 & -289.2 & 8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & 24.5 & -9.1 & 0 \\ 0 & 0 & -583 & 583 & -0.0341 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -289.2 & -424.5 & -185.4 & -8115.5 & 0 \\ 424.5 & -289.2 & 8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & 24.5 & -9.1 & 0 \\ 0 & 0 & -583 & -583 & -0.0341 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -289.2 & -375.5 & -185.4 & -8115.5 & 0 \\ 375.5 & -289.2 & 8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & 24.5 & -9.1 & 0 \\ 0 & 0 & 583 & 583 & -0.0341 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -289.2 & -375.5 & 185.4 & -8115.5 & 0 \\ 375.5 & -289.2 & 8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & 24.5 & 0 \\ 0 & 4.1 & -24.5 & -9.1 & 0 \\ 0 & 0 & 583 & -583 & -0.0341 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} -289.2 & 375.5 & 185.4 & 8115.5 & 0 \\ -375.5 & -289.2 & -8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & 24.5 & -9.1 & 0 \\ 0 & 0 & -583 & 583 & -0.0341 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} -289.2 & 375.5 & 185.4 & 8115.5 & 0 \\ -375.5 & -289.2 & -8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & 24.5 & -9.1 & 0 \\ 0 & 0 & -583 & -583 & -0.0341 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} -289.2 & 424.5 & 185.4 & 8115.5 & 0 \\ -424.5 & -289.2 & -8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & -24.5 & -9.1 & 0 \\ 0 & 0 & 583 & 583 & -0.0341 \end{bmatrix},$$

$$A_8 = \begin{bmatrix} -289.2 & 424.5 & 185.4 & 8115.5 & 0 \\ -424.5 & -289.2 & -8115.5 & 185.4 & 0 \\ 4.1 & 0 & -9.1 & -24.5 & 0 \\ 0 & 4.1 & -24.5 & -9.1 & 0 \\ 0 & 0 & 583 & -583 & -0.0341 \end{bmatrix}$$

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