

# $(\tau_1, \tau_2)^*$ -semi star generalized locally closed sets

M. Sundararaman and K. Chandrasekhara Rao

**Abstract**—The aim of this paper is to continue the study of  $(\tau_1, \tau_2)^*$ -semi star generalized closed sets by introducing the concepts of  $(\tau_1, \tau_2)^*$ -semi star generalized locally closed sets and study their basic properties in bitopological spaces.

**Keywords**— $(\tau_1, \tau_2)^*$ -semi star generalized locally closed sets,  $\tau_1\tau_2$ -semi star generalized closed sets,  $(\tau_1, \tau_2)^*$ -semi generalized locally closed sets,  $(\tau_1, \tau_2)^*$ -generalized locally closed sets,  $(\tau_1, \tau_2)^*$ -generalized semi locally closed sets.

## I. INTRODUCTION

**T**HE study of generalization of closed sets has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology. In recent years many generalizations of closed sets have been developed by various authors. K. Chandrasekhara Rao and K. Joseph [3] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in unital topological spaces.

Ganster and Reilly [14] introduced locally closed sets in topological spaces and Stone called locally closed sets as  $FG$  sets. H. Maki, P. Sundaram and K. Balachandran [26] introduced the concept of generalized locally closed sets and obtained seven different notions of generalized continuities.

Ganster, Arockiarani and Balachandran [13] introduced regular generalized locally closed sets and RGLC continuous functions and discussed some of their properties. K. Chandrasekhara Rao and K. Kannan [6], [7] introduced the concepts of semi star generalized locally closed sets and  $s^*g$ -submaximal spaces in unital topological spaces.

Mean while J.C. Kelly [21] introduced the study of bitopological spaces. M. Jelic [19] introduced locally closed sets and  $lc$ -continuity in bitopological settings. K. Chandrasekhara Rao and K. Kannan [4], [5] introduced the concepts of semi star generalized closed sets in bitopological spaces.

They [8] also introduced the concepts of  $\tau_1\tau_2$ -semi star generalized locally closed sets and pairwise  $s^*g$ -submaximal spaces with the help of  $s^*g$ -closed sets and studied their basic properties in bitopological spaces.

In this sequel the aim of this paper is to introduce the concepts of  $(\tau_1, \tau_2)^*$ -semi star generalized locally closed sets, pairwise  $s^*g$  submaximal spaces and study their basic properties in bitopological spaces. In the next section some prerequisites and abbreviations are established.

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## II. PRELIMINARIES

Let  $(X, \tau_1, \tau_2)$  or simply  $X$  denote a bitopological space. By  $\tau_1\tau_2$ - $S^*GO(X, \tau_1, \tau_2)$  {resp.  $\tau_1\tau_2$ - $S^*GC(X, \tau_1, \tau_2)$ }, we shall mean the collection of all  $\tau_1\tau_2$ - $s^*g$  open sets (resp.  $\tau_1\tau_2$ - $s^*g$  closed sets) in  $(X, \tau_1, \tau_2)$ . For any subset  $A \subseteq X$ ,  $\tau_i$ -int( $A$ ) and  $\tau_i$ -cl( $A$ ) denote the interior and closure of a set  $A$  with respect to the topology  $\tau_i$  respectively.  $A^c$  denotes the complement of  $A$  in  $X$  unless explicitly stated. We shall require the following known definitions.

**Definition 2.1:** A subset of a bitopological space  $(X, \tau_1, \tau_2)$  is called

- (a)  $\tau_1\tau_2$ -semi open if there exists a  $\tau_1$ -open set  $U$  such that  $U \subseteq A \subseteq \tau_2$ -cl( $U$ ).
- (b)  $\tau_1\tau_2$ -semi closed if  $X - A$  is  $\tau_1\tau_2$ -semi open. Equivalently, a subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called  $\tau_1\tau_2$ -semi closed if there exists a  $\tau_1$ -closed set  $F$  such that  $\tau_2$ -int( $F$ )  $\subseteq A \subseteq F$ .
- (c)  $\tau_1\tau_2$ -generalized closed ( $\tau_1\tau_2$ - $g$  closed) if  $\tau_2$ -cl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -open in  $X$ .
- (d)  $\tau_1\tau_2$ -generalized open ( $\tau_1\tau_2$ - $g$  open) if  $X - A$  is  $\tau_1\tau_2$ - $g$  closed.
- (e)  $\tau_1\tau_2$ -semi star generalized closed ( $\tau_1\tau_2$ - $s^*g$  closed) if  $\tau_2$ -cl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -semi open in  $X$ .
- (f)  $\tau_1\tau_2$ -semi star generalized open ( $\tau_1\tau_2$ - $s^*g$  open) if  $X - A$  is  $\tau_1\tau_2$ - $s^*g$  closed in  $X$ .
- (g)  $\tau_1\tau_2$ -semi generalized closed ( $\tau_1\tau_2$ - $sg$  closed) if  $\tau_2$ -scl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -semi open in  $X$ .
- (h)  $\tau_1\tau_2$ -semi generalized open ( $\tau_1\tau_2$ - $sg$  open) if  $X - A$  is  $\tau_1\tau_2$ - $sg$  closed in  $X$ .
- (i)  $\tau_1\tau_2$ -generalized semi closed ( $\tau_1\tau_2$ - $gs$  closed) if  $\tau_2$ -scl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1$ -open in  $X$ .
- (j)  $\tau_1\tau_2$ -generalized semi open ( $\tau_1\tau_2$ - $gs$  open) if  $X - A$  is  $\tau_1\tau_2$ - $gs$  closed in  $X$ .
- (k)  $(\tau_1, \tau_2)^*$ -generalized closed  $\{(\tau_1, \tau_2)^*$ - $g$  closed  $\}$  [23] if  $\tau_1\tau_2$ -cl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1\tau_2$ -open in  $X$ .
- (l)  $(\tau_1, \tau_2)^*$ -generalized open  $\{(\tau_1, \tau_2)^*$ - $g$  open  $\}$  [23] if  $X - A$  is  $(\tau_1, \tau_2)^*$ - $g$  closed.
- (m)  $(\tau_1, \tau_2)^*$ -semi generalized closed  $\{(\tau_1, \tau_2)^*$ - $sg$  closed  $\}$  [23] if  $(\tau_1, \tau_2)^*$ -scl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $(\tau_1, \tau_2)^*$ -semi open in  $X$ .
- (n)  $(\tau_1, \tau_2)^*$ -semi generalized open  $\{(\tau_1, \tau_2)^*$ - $sg$  open  $\}$  [23] if  $X - A$  is  $(\tau_1, \tau_2)^*$ - $sg$  closed.
- (o)  $(\tau_1, \tau_2)^*$ -generalized semi closed  $\{(\tau_1, \tau_2)^*$ - $gs$  closed  $\}$  if  $(\tau_1, \tau_2)^*$ -scl( $A$ )  $\subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1\tau_2$ -open in  $X$ .

- (p)  $(\tau_1, \tau_2)^*$ -generalized semi open  $\{(\tau_1, \tau_2)^*$ -gs open $\}$  if  $X - A$  is  $(\tau_1, \tau_2)^*$ -gs closed.
  - (q)  $(\tau_1, \tau_2)^*$ -semi star generalized closed  $\{(\tau_1, \tau_2)^*$ -s\* $g$  closed $\}$  if  $\tau_1\tau_2-cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\tau_1\tau_2$ -semi open in  $X$ .
  - (r)  $(\tau_1, \tau_2)^*$ -semi star generalized open  $\{(\tau_1, \tau_2)^*$ -s\* $g$  open $\}$  if  $X - A$  is  $(\tau_1, \tau_2)^*$ -s\* $g$  closed.
- Definition 2.2:** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is said to be a
- (a)  $\tau_1\tau_2$ -locally semi closed set if  $A = G \cap F$  where  $G$  is  $\tau_1$ -open and  $F$  is  $\tau_2$ -semi closed in  $X$ .
  - (b)  $\tau_1\tau_2$ -semi locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1$ -semi open and  $F$  is  $\tau_2$ -semi closed in  $X$ .
  - (c)  $\tau_1\tau_2$ - $g$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1$ - $g$  open and  $F$  is  $\tau_2$ - $g$  closed in  $X$ .
  - (d)  $\tau_1\tau_2$ - $sg$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1$ - $sg$  open and  $F$  is  $\tau_2$ - $sg$  closed in  $X$ .
  - (e)  $\tau_1\tau_2$ - $sg$  locally closed\* set if  $A = G \cap F$  where  $G$  is  $\tau_1$ - $sg$  open and  $F$  is  $\tau_2$ -closed in  $X$ .
  - (f)  $\tau_1\tau_2$ - $sg$  locally closed\*\* set if  $A = G \cap F$  where  $G$  is  $\tau_1$ -open and  $F$  is  $\tau_2$ - $sg$  closed in  $X$ .
  - (g)  $\tau_1\tau_2$ - $gs$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_2$ - $gs$  open and  $F$  is  $\tau_2$ - $gs$  closed in  $X$ .
  - (h)  $\tau_1\tau_2$ - $s^*g$  locally closed if  $A = G \cap F$  where  $G$  is a  $\tau_1$ - $s^*g$  open set and  $F$  is a  $\tau_2$ - $s^*g$  closed set in  $X$ ,
  - (i)  $(\tau_1, \tau_2)^*$ - $g$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $g$  open set and  $F$  is  $\tau_1\tau_2$ - $g$  closed set in  $X$ .
  - (j)  $(\tau_1, \tau_2)^*$ - $sg$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $sg$  open set and  $F$  is  $\tau_1\tau_2$ - $sg$  closed set in  $X$ .
  - (k)  $(\tau_1, \tau_2)^*$ - $gs$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $gs$  open set and  $F$  is  $\tau_1\tau_2$ - $gs$  closed set in  $X$ .

III.  $(\tau_1, \tau_2)^*$ -SEMI STAR GENERALIZED LOCALLY CLOSED SETS

**Definition 3.1:** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is said to be

- (a)  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed set if  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $s^*g$  open set and  $F$  is  $\tau_1\tau_2$ - $s^*g$  closed set in  $X$ .
- (b)  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\* if  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $s^*g$  open set and  $F$  is  $\tau_2$ -closed in  $X$ .
- (c)  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\*\* if  $A = G \cap F$  where  $G$  is  $\tau_1$ -open and  $F$  is  $\tau_1\tau_2$ - $s^*g$  closed in  $X$ .

**Remark 3.2:** (a) The class of all  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed sets in  $(X, \tau_1, \tau_2)$  is denoted by  $(\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .

- (b) The class of all  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\* sets in  $(X, \tau_1, \tau_2)$  is denoted by  $(\tau_1, \tau_2)^*$ - $S^*GLC^*(X, \tau_1, \tau_2)$ .
- (c) The class of all  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\*\* sets in  $(X, \tau_1, \tau_2)$  is denoted by  $(\tau_1, \tau_2)^*$ - $S^*GLC^{**}(X, \tau_1, \tau_2)$ .

**Example 3.3:** Let  $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b, c\}\}$ . Then  $\tau_1\tau_2$ - $s^*g$  open sets in  $(X, \tau_1, \tau_2)$  are  $\phi, X, \{a\}, \{b, c\}$  and  $\tau_1\tau_2$ - $s^*g$  closed sets in  $(X, \tau_1, \tau_2)$  are  $X, \phi, \{a\}, \{b, c\}$ . Then

- (a)  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed sets in  $(X, \tau_1, \tau_2)$  are  $\phi, X, \{a\}, \{b, c\}$ .
- (b)  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\* sets in  $(X, \tau_1, \tau_2)$  are  $\phi, X, \{a\}, \{b, c\}$ .

- (c)  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\*\* sets in  $(X, \tau_1, \tau_2)$  are  $\phi, X, \{a\}, \{b, c\}$ .

**Theorem 3.4:** In any bitopological space  $(X, \tau_1, \tau_2)$ ,

- (i)  $A \in (\tau_1, \tau_2)^*$ - $S^*GLC^*(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .
- (ii)  $A \in (\tau_1, \tau_2)^*$ - $S^*GLC^{**}(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .
- (iii)  $A \in \tau_1\tau_2$ - $S^*GC(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .
- (iv)  $A \in \tau_1\tau_2$ - $S^*GO(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .

**Proof:** (i) Since  $A$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\* subset in  $(X, \tau_1, \tau_2)$ , we have  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $s^*g$  open set and  $F$  is  $\tau_2$ -closed in  $X$ . Since every  $\tau_2$ -closed set is  $\tau_1\tau_2$ - $s^*g$  closed in  $(X, \tau_1, \tau_2), A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $s^*g$  open and  $F$  is  $\tau_1\tau_2$ - $s^*g$  closed in  $(X, \tau_1, \tau_2)$ . Therefore,  $A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .

(ii) Since  $A$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\*\* subset in  $(X, \tau_1, \tau_2)$ , we have  $A = G \cap F$  where  $G$  is  $\tau_1$ -open and  $F$  is  $\tau_1\tau_2$ - $s^*g$  closed in  $(X, \tau_1, \tau_2)$ . Since every  $\tau_1$ -open set is  $\tau_1\tau_2$ - $s^*g$  open in  $(X, \tau_1, \tau_2), A = G \cap F$  where  $G$  is  $\tau_1\tau_2$ - $s^*g$  open and  $F$  is  $\tau_1\tau_2$ - $s^*g$  closed in  $(X, \tau_1, \tau_2)$ . Therefore,  $A \in \tau_1\tau_2$ - $S^*GLC(X, \tau_1, \tau_2)$ .

(iii) Since  $A = A \cap X$  and  $A$  is  $\tau_1\tau_2$ - $s^*g$  closed and  $X$  is  $\tau_1\tau_2$ - $s^*g$  open in  $(X, \tau_1, \tau_2)$ , we have  $A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ .

(iv) Since  $A = A \cap X$  and  $A$  is  $\tau_1\tau_2$ - $s^*g$  open and  $X$  is  $\tau_1\tau_2$ - $s^*g$  closed in  $(X, \tau_1, \tau_2)$ , we have  $A \in (\tau_1, \tau_2)^*$ - $S^*GLC(X, \tau_1, \tau_2)$ . ■

**Remark 3.5:** The converses of (i), (ii), (iii) and (iv) of the above theorem are not true in general as can be seen from the following examples.

**Example 3.6:** Let  $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a\}, \{a, b\}\}, \tau_2 = \{\phi, X, \{b\}, \{a, c\}\}$ . Then  $\{a, b\}$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ , but not  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\* in  $(X, \tau_1, \tau_2)$ .

**Example 3.7:** In Example 3.3,  $\{b\}$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ , but not  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed\*\* in  $(X, \tau_1, \tau_2)$ .

**Example 3.8:** In Example 3.6,  $\{a\}$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ , but not  $\tau_1\tau_2$ - $s^*g$  open in  $(X, \tau_1, \tau_2)$  and  $\{a\}$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ , but not  $\tau_1\tau_2$ - $s^*g$  closed in  $(X, \tau_1, \tau_2)$ .

**Theorem 3.9:** If  $(X, \tau_1, \tau_2)$  is pairwise door space, then every subset of  $X$  is both  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed and  $(\tau_2, \tau_1)^*$ - $s^*g$  locally closed.

**Proof:** Since  $(X, \tau_1, \tau_2)$  is pairwise door space, every subset of  $(X, \tau_1, \tau_2)$  is either  $\tau_1$ -open or  $\tau_2$ -closed and  $\tau_2$ -open or  $\tau_1$ -closed. Since every  $\tau_1$ -open (resp.  $\tau_2$ -closed) subset of  $(X, \tau_1, \tau_2)$  is  $\tau_1\tau_2$ - $s^*g$  open (resp.  $\tau_1\tau_2$ - $s^*g$  closed), we have every subset of  $(X, \tau_1, \tau_2)$  is either  $\tau_1\tau_2$ - $s^*g$  open or  $\tau_1\tau_2$ - $s^*g$  closed. Since every  $\tau_1\tau_2$ - $s^*g$  open and  $\tau_1\tau_2$ - $s^*g$  closed subset of  $(X, \tau_1, \tau_2)$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed, we have every subset of  $X$  is  $(\tau_1, \tau_2)^*$ - $s^*g$  locally closed. Similarly we can prove that every subset of  $X$  is  $(\tau_2, \tau_1)^*$ - $s^*g$  locally closed. ■

**Theorem 3.10:** For a subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$ , the following are equivalent.



- (b) Every  $\tau_1\tau_2$ -locally closed set is  $(\tau_1, \tau_2)^*-s^*g$  locally closed,  $(\tau_1, \tau_2)^*-s^*g$  locally closed\* and  $(\tau_1, \tau_2)^*-s^*g$  locally closed\*\*.

**Remark 3.20:** But the converses of the assertions of above theorem are not true in general as can be seen in the following examples.

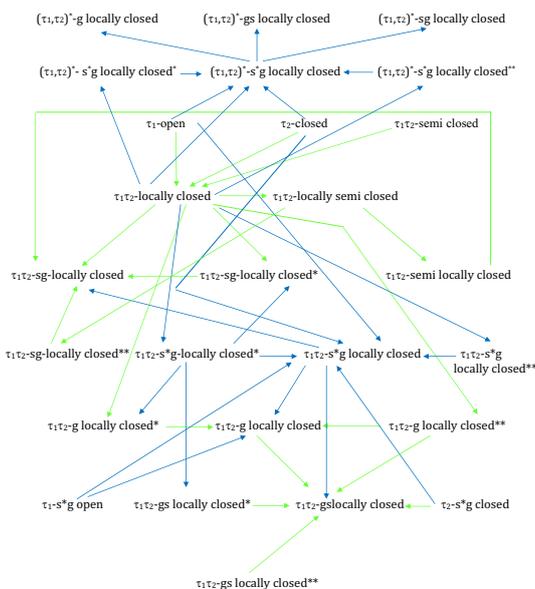
- Example 3.21:** (a) In Example 3.3,  $\{b, c\}$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed set in  $(X, \tau_1, \tau_2)$ , but  $\{b, c\}$  is not  $\tau_2$ -closed in  $(X, \tau_1, \tau_2)$ .
- (b) In Example 3.3,  $\{b, c\}$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed set in  $(X, \tau_1, \tau_2)$ , but  $\{b, c\}$  is not  $\tau_1$ -open in  $(X, \tau_1, \tau_2)$ .
- (c) Let  $X = \{a, b, c, d\}$ ,  $\tau_1 = \{\phi, X, \{a, b\}\}$ ,  $\tau_2 = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ . Then  $\{b\}$  is a  $(\tau_1, \tau_2)^*-s^*g$  locally closed set,  $(\tau_1, \tau_2)^*-s^*g$  locally closed\* set and  $(\tau_1, \tau_2)^*-s^*g$  locally closed\*\* set but not  $\tau_1\tau_2$ -locally closed in  $(X, \tau_1, \tau_2)$ .

**Remark 3.22:** Since every  $\tau_1\tau_2-s^*g$  closed set is  $\tau_1\tau_2-g$  closed,  $\tau_1\tau_2-sg$  closed and  $\tau_1\tau_2-gs$  closed, we conclude the following.

- Theorem 3.23:** (a) Every  $(\tau_1, \tau_2)^*-s^*g$  locally closed is  $(\tau_1, \tau_2)^*-g$  locally closed.
- (b) Every  $(\tau_1, \tau_2)^*-s^*g$  locally closed is  $(\tau_1, \tau_2)^*-sg$  locally closed.
- (c) Every  $(\tau_1, \tau_2)^*-s^*g$  locally closed is  $(\tau_1, \tau_2)^*-gs$  locally closed.

**Remark 3.24:** But none of the assertions of the above theorem are reversible in general as can be seen in the following example.

**Example 3.25:** Let  $X = \{a, b, c\}$ ,  $\tau_1 = \{\phi, X, \{a\}\}$ ,  $\tau_2 = \{\phi, X, \{a\}, \{a, c\}\}$ . Then  $\{a, b\}$  is a  $(\tau_1, \tau_2)^*-g$  locally closed set,  $(\tau_1, \tau_2)^*-sg$  locally closed set and  $(\tau_1, \tau_2)^*-gs$  locally closed set, but not  $(\tau_1, \tau_2)^*-s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ . From the above results we conclude the following.



**Fig 1.** Relationship between several locally closed sets

Since the finite intersection of  $\tau_1$ -open sets is  $\tau_1$ -open and the intersection of two  $\tau_1\tau_2-s^*g$  closed sets is  $\tau_1\tau_2-s^*g$  closed, we immediately get

**Theorem 3.26:** In any bitopological space  $(X, \tau_1, \tau_2)$ , intersection of two  $(\tau_1, \tau_2)^*-s^*g$  locally closed\*\* sets is  $(\tau_1, \tau_2)^*-s^*g$  locally closed\*\*.

In this sequel our next result exhibits the intersection of a  $(\tau_1, \tau_2)^*-s^*g$  locally closed set and a  $\tau_2$ -closed set in a bitopological space.

**Theorem 3.27:** If  $A \in (\tau_1, \tau_2)^*-S^*GLC(X, \tau_1, \tau_2)$  and  $B$  is  $\tau_2$ -closed in  $X$ , then  $A \cap B \in (\tau_1, \tau_2)^*-S^*GLC(X, \tau_1, \tau_2)$ .

**Proof:** It is obvious since every  $\tau_2$ -closed set is  $\tau_1\tau_2-s^*g$  closed and the intersection of two  $\tau_1\tau_2-s^*g$  closed sets is  $\tau_1\tau_2-s^*g$  closed. ■

Our next result is an immediate consequence of the above theorem.

**Theorem 3.28:** If  $A \in (\tau_1, \tau_2)^*-S^*GLC(X, \tau_1, \tau_2)$  and  $B$  is  $\tau_1\tau_2-s^*g$  closed in  $X$ , then  $A \cap B \in (\tau_1, \tau_2)^*-S^*GLC(X, \tau_1, \tau_2)$ .

**Remark 3.29:** The complement of a  $(\tau_1, \tau_2)^*-s^*g$  locally closed set in  $(X, \tau_1, \tau_2)$  is not  $(\tau_1, \tau_2)^*-s^*g$  locally closed in general and hence the finite union of  $(\tau_1, \tau_2)^*-s^*g$  locally closed sets need not be  $(\tau_1, \tau_2)^*-s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ . The next examples show the claim.

**Example 3.30:** In Example 3.6,  $\{a\}$  is a  $(\tau_1, \tau_2)^*-s^*g$  locally closed set, but its complement  $\{b, c\}$  is not  $(\tau_1, \tau_2)^*-s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ .

**Example 3.31:** In Example 3.6,  $A = \{b\}$ ,  $B = \{c\}$  are  $(\tau_1, \tau_2)^*-s^*g$  locally closed sets, but  $A \cup B = \{b, c\}$  is not  $(\tau_1, \tau_2)^*-s^*g$  locally closed in  $(X, \tau_1, \tau_2)$ .

**Theorem 3.32:** In a bitopological space  $(X, \tau_1, \tau_2)$ , the following are equivalent.

- (a)  $A$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed if and only if  $A^C$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed.
- (b)  $(\tau_1, \tau_2)^*-s^*g$  locally closed sets are closed under finite union.

**Proof:** (a)  $\Rightarrow$  (b) : Suppose that  $A$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed if and only if  $A^C$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed. Let  $A, B$  be  $(\tau_1, \tau_2)^*-s^*g$  locally closed. Then by our assumption,  $A^C, B^C$  are  $(\tau_1, \tau_2)^*-s^*g$  locally closed. Consequently,  $(A \cup B)^C = A^C \cap B^C$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed. Therefore,  $A \cup B$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed.

(b)  $\Rightarrow$  (a) : Suppose that  $(\tau_1, \tau_2)^*-s^*g$  locally closed sets are closed under finite union. Let  $A$  be  $(\tau_1, \tau_2)^*-s^*g$  locally closed. Then  $A = G \cap F$  where  $G$  is  $\tau_1\tau_2-s^*g$  open and  $F$  is  $\tau_1\tau_2-s^*g$  closed in  $X$ . Since  $G^C$  is  $\tau_1\tau_2-s^*g$  closed and  $F^C$  is  $\tau_1\tau_2-s^*g$  open in  $X$  and every  $\tau_1\tau_2-s^*g$  open is  $(\tau_1, \tau_2)^*-s^*g$  locally closed and  $\tau_1\tau_2-s^*g$  closed set is  $(\tau_1, \tau_2)^*-s^*g$  locally closed, we have  $A^C$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed by our assumption. Similarly, we can prove if  $A^C$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed then  $A$  is  $(\tau_1, \tau_2)^*-s^*g$  locally closed. ■

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