

T-DOF PID Controller Design using Characteristic Ratio Assignment Method for Quadruple Tank Process

Tianchai Suksri, U-thai Sritheeravirojana, Arjin Numsonran, Viriya Kongrattana, and Thongchai Werataweemart

Abstract—A control system design with Characteristic Ratio Assignment (CRA) is proven that effective for SISO control design. But the control system design for MIMO via CRA is not concrete procedure. In this paper presents the control system design method for quadruple-tank process via CRA. By using the decentralized method for both minimum phase and non-minimum phase are made. The results from PI and PID controller design via CRA can be illustrated the validity of our approach by MATLAB.

Keywords—CRA, Quadruple-Tank.

I. INTRODUCTION

GENERALLY, the performance analysis of control system focuses on time domain response such as percent overshoot, rise time, setting time and steady state error. Although there are many methods to design the controller, it is a few approach can be achieved the satisfied response. The characteristic ratio assignment (CRA) is one techniques based on defined parameter of characteristic equation that is a famous method at present [1] [2].

To design controller by CRA method [3], adjustment of speed response and the damping ration can be done by only one parameter. Therefore, this technique is convenient and suitable for tuning controller under the requirement of the system.

In this paper, the quadruple-tank process, the interactive process is controlled by PI controller for case of minimum phase and PID controller for case of non-minimum phase base on CRA method. The coefficient of characteristic (α_i) and time constant (τ) are the parameters to determine the characteristic equation of the control system that is necessary to design the time domain system.

II. QUADRUPLE-TANK PROCESS

Consider the quadruple-tank process shown in Fig. 1. This laboratory process has been used to illustrate many issues in multivariable control [4]. The target is to control the level in the lower two tanks with two pumps. The process inputs are

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u_1 and u_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltage from level measurement devices). The linearised dynamics for the process is given as:

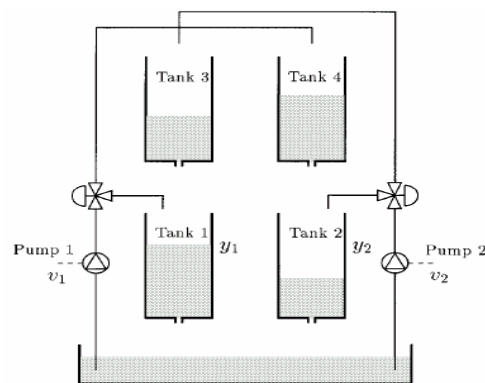


Fig. 1 Schematic diagram of the quadruple-tank process

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1 + sT_1} & \frac{(1 - \gamma_1)c_2}{(1 + sT_3)(1 + sT_2)} \\ \frac{(1 - \gamma_2)c_1}{(1 + sT_3)(1 + sT_1)} & \frac{\gamma_2 c_2}{1 + sT_2} \end{bmatrix} \quad (1)$$

$$T_i = \frac{A_i}{\beta_i a_i} \sqrt{\frac{2h_i^0}{g}}, \quad i = 1, \dots, 4 \quad (2)$$

and $c_1 = T_1 k_1 k_c / A_1$, $c_2 = T_2 k_2 k_c / A_2$. Here A_i is the cross-sectional area of tank i , a_i is the cross-sectional area of the outlet of tank i , β_i is value ratio at the outlet of tank i , h_i^0 is the steady-state water level, k_i is the gain of the pump i , k_c is the measurement gain, and g is the acceleration of gravity. The parameters $\gamma_1, \gamma_2 \in (0, 1)$ are determined from how the valves are prior set to an experiment; the flow to tank 1 is proportional to γ_1 and the flow to tank 4 is proportional to $(1 - \gamma_1)$, and similarly for γ_2 with respect to tank 2 and tank 3. Since

$$\det G(s) = \frac{c_1 c_2}{\gamma_1 \gamma_2 \prod_{i=1}^4 (1 + sT_i)} \times \left[(1 + sT_3)(1 + T_4) - \frac{(1 - \gamma_1)(1 - \gamma_2)}{\gamma_1 \gamma_2} \right] \quad (3)$$

and transfer matrix G has two finite zeros for $\gamma_1, \gamma_2 \in (0, 1)$.

The system is non-minimum phase for $0 < \gamma_1 + \gamma_2 < 1$, and minimum phase for $1 < \gamma_1 + \gamma_2 < 2$. Hence, by changing a single valve we can make the multivariable level control problem more or less difficult.

The relative gain (RGA) was introduced by Bristol [5] as a measure of interaction in multivariable control systems. The RGA Λ is defined as $\Lambda = G(0) * G^{-T}(0)$, where the asterisk denotes the schur product (element-by-element matrix multiplication) and $-T$ inverse transpose. It is possible to show that the elements of each row and column of Λ sum up to one, so for a 2×2 system the RGA is determined by the scalar $\lambda = \Lambda_{11}$. The RGA is used as a tool mainly in the process industry to decide on control structure issues such as input-output pairing for decentralized controllers [6]. The RGA of the Quadruple-Tank Process is given by the simple expression

$$\lambda = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1}, \text{ where } \Lambda = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix}$$

III. STRUCTURE OF THE CONTROL SYSTEM WITH PI AND PID CONTROLLER

From RGA analysis suggests that input-output pairing for decentralized control be chosen. In case of minimum phase system, transfer function G_{11} and G_{22} are used for design the controller, but the case of non-minimum phase system the transfer function G_{12} and G_{21} are instead used. The structure of MIMO control system using PI and PID controller for minimum phase and non-minimum phase are shown in Fig. 2 and Fig. 3 respectively, for case of minimum phase

$$B_{c1}(s) = K_{p1}s + K_{i1}, A_{c1}(s) = s, B_{d1}(s) = K_{i1}$$

$$B_{c2}(s) = K_{p2}s + K_{i2}, A_{c2}(s) = s, B_{d2}(s) = K_{i2}$$

and case of non-minimum phase

$$B_{c1}(s) = K_{d1}s^2 + K_{p1}s + K_{i1}, A_{c1}(s) = s, B_{d1}(s) = K_{i1}$$

$$B_{c2}(s) = K_{d2}s^2 + K_{p2}s + K_{i2}, A_{c2}(s) = s, B_{d2}(s) = K_{i2}$$

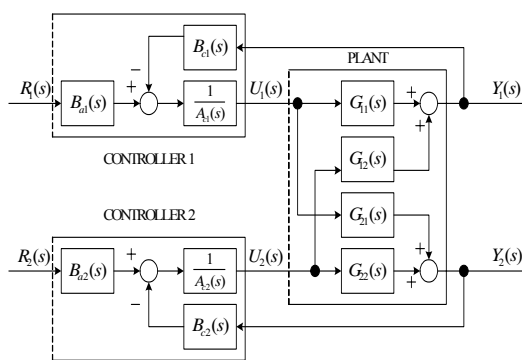


Fig. 2 Structure of minimum phase control system

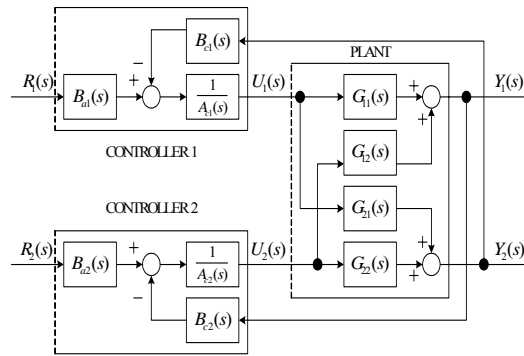


Fig. 3 Structure of non-minimum phase control system

and transfer matrix of system is

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{B_{p11}(s)}{A_{p11}(s)} & \frac{B_{p12}(s)}{A_{p12}(s)} \\ \frac{B_{p21}(s)}{A_{p21}(s)} & \frac{B_{p22}(s)}{A_{p22}(s)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{K_{11}}{a_1s + a_0} & \frac{K_{12}}{b_2s^2 + b_1s + b_0} \\ \frac{K_{21}}{c_2s^2 + c_1s + c_0} & \frac{K_{22}}{d_1s + d_0} \end{bmatrix}$$

The transfer functions used for design are:

A. Case of Minimum Phase

Loop 1 (Y1-R1),

$$\frac{Y_1(s)}{R_1(s)} = \frac{B_{c1}(s)B_{p11}(s)}{A_{c1}(s)A_{p11}(s) + B_{c1}(s)B_{p11}(s)}$$

Characteristic equation is

$$P_{11} = A_{c1}(s)A_{p11}(s) + B_{c1}(s)B_{p11}(s) = a_1s^2 + (a_0 + K_{p1}K_{11})s + K_{i1}K_{11} \quad (4)$$

Loop 2 (Y2-R2),

$$\frac{Y_2(s)}{R_2(s)} = \frac{B_{c2}(s)B_{p22}(s)}{A_{c2}(s)A_{p22}(s) + B_{c2}(s)B_{p22}(s)}$$

Characteristic equation is

$$P_{22} = A_{c2}(s)A_{p22}(s) + B_{c2}(s)B_{p22}(s) = d_1s^2 + (d_0 + K_{p2}K_{22})s + K_{i2}K_{22} \quad (5)$$

B. Case of Non-Minimum Phase

Loop 1 (Y1-R2),

$$\frac{Y_1(s)}{R_2(s)} = \frac{B_{c2}(s)B_{p12}(s)}{A_{c2}(s)A_{p12}(s) + B_{c2}(s)B_{p12}(s)}$$

Characteristic equation is

$$P_{12} = A_{c2}(s)A_{p12}(s) + B_{c2}(s)B_{p12}(s) = b_2s^3 + (b_1 + K_{d2}K_{21})s^2 + (b_0 + K_{p2}K_{12})s + K_{i2}K_{12} \quad (6)$$

Loop 2 (Y2-R1),

$$\frac{Y_2(s)}{R_1(s)} = \frac{B_{c1}(s)B_{p21}(s)}{A_{c1}(s)A_{p21}(s) + B_{c1}(s)B_{p21}(s)}$$

Characteristic equation is

$$P_{+21} = A_{c1}(s)A_{p21}(s) + B_{c1}(s)B_{p21}(s) \tag{7}$$

$$= c_2s^3 + (c_1 + K_{d1}K_{21})s^2 + (c_0 + K_{p1}K_{21})s + K_{i1}K_{21}$$

IV. THE CRA METHOD

Naslin [7] has studied the problems to optimize a damping ratio of control systems, and he found that the damping ratio relates with characteristic ratio. The relation of characteristic equation can be illustrated as follow.

$$p(s) = a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0, \forall a_i > 0 \tag{8}$$

Where, the characteristic ratio is given by,

$$\alpha_1 = \frac{a_1^2}{a_0 a_2}, \alpha_2 = \frac{a_2^2}{a_1 a_3}, \dots, \alpha_{n-1} = \frac{a_{n-1}^2}{a_{n-2} a_n} \tag{9}$$

and the inverse of characteristic equation is given as

$$b_0 = \frac{a_0}{a_1}, b_1 = \frac{a_1}{a_2}, \dots, b_{n-1} = \frac{a_{n-1}}{a_n} \tag{10}$$

Otherwise, it is able to depict the relation of coefficient ration and characteristic pulsantances as,

$$\alpha_1 = \frac{b_1}{b_0}, \alpha_2 = \frac{b_2}{b_1}, \dots, \alpha_{n-1} = \frac{b_{n-1}}{b_{n-2}} \tag{11}$$

Thus the time constant is given by

$$\tau = \frac{a_1}{a_0} \tag{12}$$

Given equation (9)-(10), it is able to describe in another form by the coefficient of characteristic equation

$$A = [a_n \ a_{n-1} \ \dots \ a_1 \ a_0] \tag{13}$$

$$B = [b_{n-1} \ b_{n-2} \ \dots \ b_1 \ b_0] \tag{14}$$

$$C = [c_{n-2} \ c_{n-3} \ \dots \ c_1 \ c_0] \tag{15}$$

Where

$$b_i = \frac{a_i}{a_{i+1}} \ i = 0, 1, 2, \dots, n-1 \tag{16}$$

$$c_i = \frac{b_{i+1}}{b_i} \ i = 0, 1, 2, \dots, n-2 \tag{17}$$

Design controller by CRA method, assigns value of time constant (τ) and characteristic pulsantances (α_i) up to system requirement, To define the value of α_i , it is under the rule of Lipatov and Sokolov [8] in order to retain the system stability which is given by,

$$\sqrt{\alpha_i} \alpha_{i+1} > 1.4656, i = 1, 2, \dots, n-2 \tag{18}$$

$$\alpha_i \geq 1.12374 \alpha_i^*, i = 2, 3, \dots, n-2 \tag{19}$$

$$\alpha_i^* = \frac{1}{\alpha_{i+1}} + \frac{1}{\alpha_i - 1}, \alpha_n = \alpha_0 = \infty \tag{20}$$

A. Adjustment Speed Response of Control System

To adjust a speed response of control system, the CRA method can be tuned by changing a value of time constant as shown in the next equation. Assume, the transfer function is given as

$$G(s) = \frac{a_0}{a_n s^n + a_{n-1} s + \dots + a_1 s + a_0} \tag{21}$$

Then it is arranged in new format

$$G(s) = \frac{a_0/a_n}{s^n + \frac{a_{n-1}}{a_n} s + \dots + \frac{a_1}{a_n} s + \frac{a_0}{a_n}} \tag{22}$$

From equation (22) is able to construct the coefficient as inverse form of characteristic equation

$$A = \left[1 \ \prod_{i=n-1}^{n-1} b_i \ \dots \ \prod_{i=2}^{n-1} b_i \ \prod_{i=1}^{n-1} b_i \right] \tag{23}$$

When increasing a gain with equivalent ration by k , it is obtained by

$$A = \left[1 \ k \prod_{i=n-1}^{n-1} b_i \ \dots \ k^{n-1} \prod_{i=2}^{n-1} b_i \ k^n \prod_{i=1}^{n-1} b_i \right] \tag{24}$$

Equation (23) and (24), coefficient ratio is unchanged, but the time constant is able to change as shown in equation (25) (26)

$$G_k(s) = \frac{k^n a_0}{a_n s^n + k a_{n-1} s^{n-1} + \dots + k^{n-1} a_1 s + k^n a_0} \tag{25}$$

$$\tau = \frac{1}{k} \left(\frac{a_1}{a_0} \right) \tag{26}$$

Where $0 < k < 1$

B. Adjustment Damping Ratio of Control System

To adjust the damping ratio, if a system is high order then there are many parameters (α_i) to be adjusted. Thus the CRA method is designed for tuning only one parameter that follows as,

$$G(s) = \frac{1}{\frac{a_n}{a_0} s^n + \frac{a_{n-1}}{a_0} s^{n-1} + \dots + \frac{a_1}{a_0} s + 1} \tag{27}$$

$$A = \left[\frac{1}{\prod_{i=0}^{n-1} b_i} \ \frac{1}{\prod_{i=0}^{n-2} b_i} \ \dots \ \frac{1}{\prod_{i=0}^1 b_i} \ \frac{1}{\prod_{i=0}^0 b_i} \right] \tag{28}$$

$$A = \left[\frac{1}{\left[\prod_{i=0}^{n-2} c_i \prod_{i=0}^{n-3} c_i \dots \prod_{i=0}^0 c_i \right] b_0^n} \ \frac{1}{\left[\prod_{i=0}^{n-3} c_i \prod_{i=0}^{n-4} c_i \dots \prod_{i=0}^0 c_i \right] b_0^{n-1}} \ \dots \ \frac{1}{b_0} \ 1 \right] \tag{29}$$

When the coefficient ratio is increased by k then the new characteristic equation is given by

$$A = \left[\frac{1}{k^{\frac{1}{2}n^2 - \frac{1}{2}} \left[\prod_{i=0}^{n-2} c_i \prod_{i=0}^{n-3} c_i \dots \prod_{i=0}^0 c_i \right] b_0^n} \ \frac{1}{k^{\frac{1}{2}n^2 - \frac{1}{2} - 1} \left[\prod_{i=0}^{n-3} c_i \prod_{i=0}^{n-4} c_i \dots \prod_{i=0}^0 c_i \right] b_0^{n-1}} \ \dots \ \frac{1}{b_0} \ 1 \right] \tag{30}$$

Finally, equation (29) and (30) are formulated to the equation (31) when $k > 1$; damping ration will be increased

then $0 < k < 1$; damping ratio will be decreased.

$$G_k(s) = \frac{k^{\frac{1}{2}n^2 - \frac{1}{2}n} a_0}{a_n s^n + k^{n-1} a_{n-1} s^{n-1} + \dots + k^{\frac{1}{2}n^2 - \frac{1}{2}n - 1} a_1 s + k^{\frac{1}{2}n^2 - \frac{1}{2}n} a_0} \quad (31)$$

V. THE SIMULATION RESULTS

In this paper, the simulation results of the quadruple-tank process for case of minimum and non-minimum phase, the interactive process is given by MATLAB. The experiment of PI and PID control based on CRA illustrates the adjustment of speed response and damping ration. Parameters and operating point show detail in Table I and Table II.

TABLE I
PARAMETERS OF QUADRUPLE-TANK PROCESS

$A_1, A_2, A_3, A_4; \text{cm}^2$	$a_1, a_2, a_3, a_4; \text{cm}^2$	$g; \text{cm} / \text{s}^2$
69.3978	0.1963	981

TABLE II
OPERATING POINT OF QUADRUPLE-TANK PROCESS

Operating Point	Minimum Phase	Non-Minimum Phase
$(h_1^0, h_2^0); \text{cm}$	(11.1, 11.6)	(10.8, 11.5)
$(h_3^0, h_4^0); \text{cm}$	(0.68, 0.39)	(7.33, 3.99)
$(u_1^0, u_2^0); \text{V}$	(5, 5)	(5, 5)
$(k_1, k_2); \text{cm}^3 / \text{V} \cdot \text{s}$	(2.6972, 2.3949)	(2.7887, 2.3950)
(γ_1, γ_2)	(0.791, 0.772)	(0.328, 0.237)
(β_1, β_2)	(0.4759, 0.4124)	(0.4759, 0.4124)
(β_3, β_4)	(0.3837, 0.5388)	(0.3837, 0.5388)

V.I Case of Minimum Phase

A. Step Response of Nominal Parameter

Loop 1 (Y1-R1)

$$G_{-11}(s) = \frac{3.4347}{111.7228s + 1} \quad (32)$$

According to equation (32), design of PI controller based on CRA, is assigned parameter follow as $\alpha_1 = 2$ and $\tau = 20$

$$P_{-11}(s) = 111.7228s^2 + 11.1722s + 0.5586$$

and the parameter of PI controller is given by

$$K_{p1} = 2.9616, \quad K_{i1} = 0.1626$$

Loop 2 (Y2-R2)

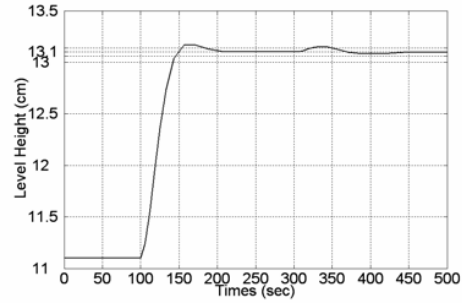
$$G_{-22}(s) = \frac{3.5113}{131.7973s + 1} \quad (33)$$

In the same manner, the $P_{-22}(s)$ is

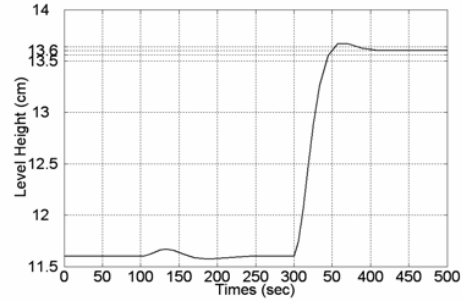
$$P_{-22}(s) = 131.7973s^2 + 13.1797s + 0.659$$

and the parameter of PID controller is given by

$$K_{p2} = 3.4687, \quad K_{i2} = 0.1877$$



(a)



(b)

Fig. 4 The step response of nominal parameter

The step response in Fig. 4 (a) is for the tank 1 and Fig. 4 (b) is for tank 2. For tank 1 the overshoot is 3.5 percent, settling time is 83 sec and interaction is 5.5 percent. For tank 2 the overshoot is 3.5 percent, settling time is 83 sec and interaction is 6.5 percent.

B. The Adjustment Speed of Response

By using CRA method, the adjustment of speed response can be produced by adjusting value of k according to equation (25).

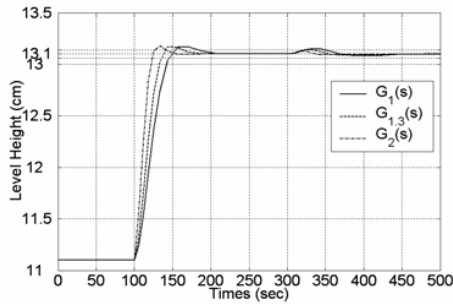
Loop 1 (Y1-R1)

$$G_k(s) = \frac{k^2 0.5586}{111.7228s^2 + k11.1722s + k^2 0.5586}$$

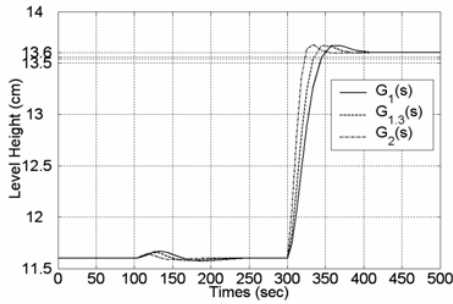
Loop 2 (Y2-R2)

$$G_k(s) = \frac{k^2 0.659}{131.7973s^2 + k13.1797s + k^2 0.659}$$

When k is adjust from 1.3 to 2 respectively, settling time is decrease from 64 sec to 42 sec for tank 1 and tank 2, while interaction for tank 1 is decrease from 4.3 percent to 3 percent and interaction for tank 2 is decrease from 5.5 percent to 4 percent, therefore the system response is faster and interaction is decrease.



(a)



(b)

Fig. 5 The result adjustment speed of step response by k

C. The Adjustment of Damping Ratio

By using CRA method, the adjustment of damping ratio can be produced by adjusting value of k according to equation (31).

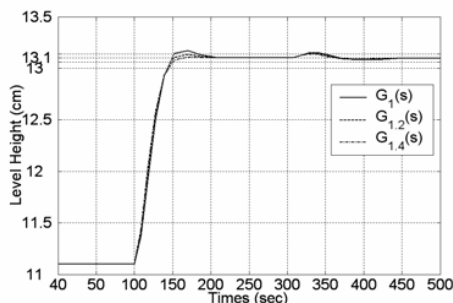
Loop 1 (Y1-R1)

$$G_k(s) = \frac{k \cdot 0.5586}{111.7228s^2 + k11.1722s + k0.5586}$$

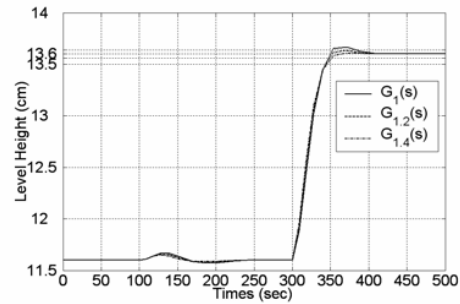
Loop 2 (Y2-R2)

$$G_k(s) = \frac{k \cdot 0.659}{131.7973s^2 + k13.1797s + k0.659}$$

When k is changed from 1.2 to 1.4 respectively then the damping ratio is also increased but the overshoot is decreased from 1.7 percent to 0.5 percent and interaction is decreased from 4.5 percent to 4 percent for tank 1 and decreased from 1.5 percent to 0.6 percent and interaction is decreased from 5.5 percent to 5 percent for tank 2.



(a)



(b)

Fig. 6 The response when adjustment damping ratio by k

V.II Case of Non-Minimum Phase

Loop 1 (Y1-R2)

$$G_{+12}(s) = \frac{2.9019}{12409.705s^2 + 228.8107s + 1} \tag{34}$$

According to equation (34), design of PID controller based on CRA, is assigned parameter follow as $\alpha_1 = 2.5, \alpha_2 = 2$ and $\tau = 200$

$$P_{+12}(s) = 12409.705s^3 + 310.2417s^2 + 3.878s + 0.0194$$

and the parameter of PID controller is given by

$$K_{d2} = 30.1292, K_{p2} = 0.9918, K_{i2} = 0.0067$$

Loop 2 (Y2-R1)

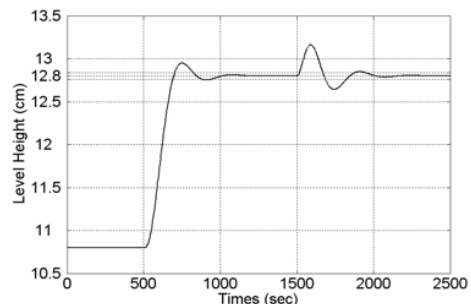
$$G_{+12}(s) = \frac{3.5437}{7760.889s^2 + 190.3685s + 1} \tag{35}$$

In the same manner, the $P_{+21}(s)$ is

$$P_{+21}(s) = 7760.889s^3 + 194.0221s^2 + 2.4253s + 0.0121$$

and the parameter of PID controller is given by

$$K_{d1} = 1.031, K_{p1} = 0.4022, K_{i1} = 0.0034$$



(a)

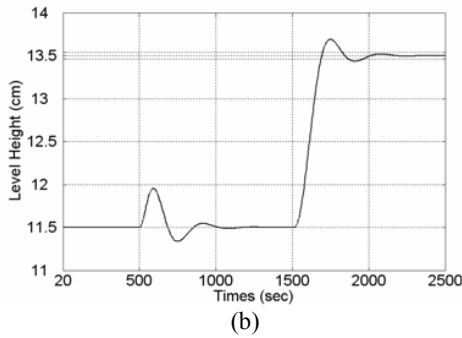


Fig. 7 The step response of nominal parameter

The step response in Fig. 4(a) is for the tank 1 and Fig. 4(b) is for tank 2. For tank 1 the overshoot is 7.5 percent, settling time is 880 sec and interaction is 36 percent. For tank 2 the overshoot is 9.5 percent, settling time is 920 sec and interaction is 44 percent.

B. The Adjustment Speed of Response

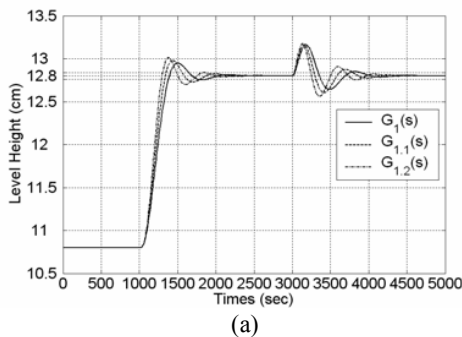
By using CRA method, the adjustment of speed response can be produced by adjusting value of k according to equation (25).

Loop 1 (Y1-R2)

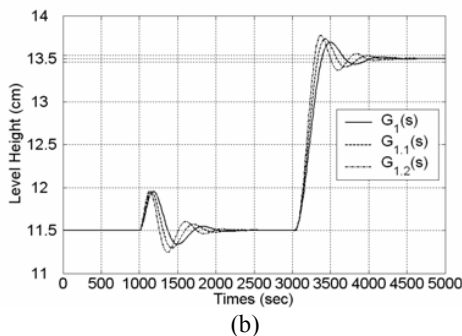
$$G_k(s) = \frac{k^3 0.0194}{12409.705s^3 + k^2 310.2417s^2 + k^3 3.878s + k^3 0.0194}$$

Loop 2 (Y2-R1)

$$G_k(s) = \frac{k^3 0.0121}{7760.889s^3 + k^2 194.0221s^2 + k^2 2.4253s + k^3 0.0121}$$



(a)



(b)

Fig. 8 The result adjustment speed of step response by k

When k is adjust from 1.1 to 1.2 respectively, settling time

is decrease from 790 sec to 700 sec for tank 1, overshoot is increase from 9 percent to 10 percent for tank 1, while interaction f is increase from 36.5 percent to 37 percent

For tank 2 settling time is decrease from 810 sec to 710 sec, overshoot is increase from 11.5 percent to 13.5 percent while interaction is increase from 45 percent to 45.5percent,

Therefore the system response is faster, overshoot is increased and interaction is increase.

C. The Adjustment of Damping Ratio

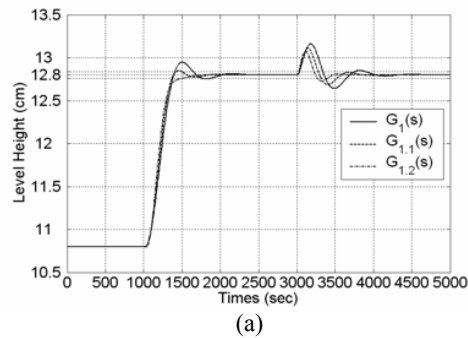
By using CRA method, the adjustment of damping ration can be produced by adjusting value of k according to equation (31).

Loop 1 (Y1-R2)

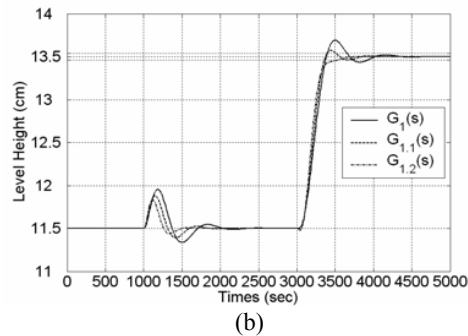
$$G_k(s) = \frac{k^3 0.0194}{12409.705s^3 + k^2 310.2417s^2 + k^3 3.878s + k^3 0.0194}$$

Loop 2 (Y2-R1)

$$G_k(s) = \frac{k^3 0.0121}{7760.889s^3 + k^2 194.0221s^2 + k^3 2.4253s + k^3 0.0121}$$



(a)



(b)

Fig. 9 The response when adjustment damping ration by k

When k is changed from 1.1 to 1.2 respectively then the damping ration is also increased but the overshoot is decreased from 2.5 percent to 0 percent and interaction is decreased from 31 percent to 27.5 percent for tank 1 and decreased form 3.5 percent to 0 percent and interaction is decreased from 38 percent to 33 percent for tank 2.

VI. CONCLUSION

In this paper, the design of PI and PID controller using CRA for quadruple-tank is presented; the simulation results

from MATLAB for both minimum phase and non-minimum phase are able to illustrate the advantage which only one parameter is to be adjusted. Thus, our scheme is convenient and suitable for designing and tuning the controller.

REFERENCES

- [1] S.Jayasuriya and J.W. Song, "On the Synthesis of Compensators for Non Overshooting Step Response," Proceedings American Control Conference, 1990, pp. 683-684
- [2] R.H. Middleton and S.F. Grabe, "Slow Stable Open-Loop Poles: To Cancel or not to Cancel," *Automatica*, Vol.35, 1999, pp. 877-886
- [3] Y.C. Kim, L.H. Keel and S.P. Bhattacharyya, "Transient Response Control via Characteristic Ratio Assignment," *IEEE Transactions On Automatic Control*, Vol.48, No.12, 2003, pp.2238-2244
- [4] K.H. Johansson, "The Quadruple-tank process a multivariable with an adjust zero," *IEEE Trans Control Syst.Technol.*, 8, (3), 2000, pp. 456-465
- [5] E.Bristol, "On a new measure of interaction for multivariable process control", *IEEE Transactions on Automatic Control*, 11:133 1966
- [6] M.Morari and E.Zafiriou, *Robust Process Control*, Prentice-Hall, Eaglewood cliffs, NJ, 1989
- [7] P. Naslin 1969. *Essentials of optimal control*, London Boston Technical Publishers Inc, Cambridge, MA, pp. 30-51
- [8] A.V. Lipatov and N.I. Sokolov, "Some Sufficient Conditions for Stability and Instability of Continuous Linear Stationary Systems," Translated from *Automatika*, No.9, 1978, pp. 30-37