

Survival of Neutrino Mass Models in Non-thermal Leptogenesis

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Abstract—The Constraints imposed by non-thermal leptogenesis on the survival of the neutrino mass models describing the presently available neutrino mass patterns, are studied numerically. We consider the Majorana CP violating phases coming from right-handed Majorana mass matrices to estimate the baryon asymmetry of the universe, for different neutrino mass models namely quasi-degenerate, inverted hierarchical and normal hierarchical models, with tribimaximal mixings. Considering two possible diagonal forms of Dirac neutrino mass matrix as either charged lepton or up-quark mass matrix, the heavy right-handed mass matrices are constructed from the light neutrino mass matrix. Only the normal hierarchical model leads to the best predictions of baryon asymmetry of the universe, consistent with observations in non-thermal leptogenesis scenario.

Keywords—Thermal leptogenesis, Non-thermal leptogenesis.

I. INTRODUCTION

THE existence of heavy right-handed Majorana neutrinos in some of the left-right symmetric GUT models, not only gives small but non vanishing neutrino masses through the celebrated seesaw mechanism [1], it also plays an important role in explaining the baryon asymmetry of the universe [2] $Y_B = (6.1_{-0.2}^{+0.3}) \times 10^{-10}$. Such an asymmetry can be dynamically generated if the particle interaction rate and the expansion rate of the universe satisfy Sakharov's three famous conditions [3]: (i) Baryon number violation, (ii) C and CP violation, and (iii) Thermal out of equilibrium decay. Majorana right-handed neutrino satisfy the second condition i.e., C and CP violation as they can have asymmetric decay to lepton and Higgs particles, and the process occurs at different rates for particles and antiparticles. The lepton asymmetry is then partially converted to baryon asymmetry through the non-perturbative electroweak sphaleron effects [4], [5]. In such thermal leptogenesis the right-handed neutrinos can be generated thermally after inflation, if their masses are comparable to or below the reheating temperature $M_1 \leq T_R$. This allows high scale reheating temperature $T_R \geq 10^9$ GeV [6]. In non-thermal leptogenesis [7] it is

possible to produce lepton asymmetry by using the low reheating temperature, where the right-handed neutrinos are produced through the direct non-thermal decays of inflaton. This is particularly important for supersymmetric models where gravitino problem [8] can be avoided provided the reheating temperature after inflation is bounded from above in a certain way, namely $T_R \leq (10^6 - 10^7)$ GeV.

In order to calculate the baryon asymmetry from a given neutrino mass model, one usually starts with the light neutrino mass matrices m_{LL} and then relates it with the heavy Majorana neutrino mass matrix M_{RR} and the Dirac neutrino mass matrix m_{LR} through inverse seesaw mechanism. We consider the Dirac neutrino mass matrix as either charged lepton or up quark mass matrix for phenomenological analysis. The complex CP violating phases are usually derived from the MNS leptonic mixing matrix. In the present work we are interested to consider the complex Majorana phases which are derived from the right-handed Majorana mass M_{RR} in the estimation of baryon asymmetry of the universe. We wish to consider the left-handed light Majorana neutrino mass matrices m_{LL} which obey the μ - τ symmetry [9] where tribimaximal mixings [10] are realized, for all possible patterns of neutrino masses, e.g., degenerate, inverted hierarchical and normal hierarchical mass patterns. We first parametrised the light left-handed Majorana neutrino mass matrices which are subjected to correct predictions of neutrino mass and mixing angles. The calculation of baryon asymmetry of the universe in the light of thermal as well as nonthermal leptogenesis, may serve as an additional information to further discriminate the correct pattern of neutrino mass models and also shed light on the structure of Dirac neutrino mass matrix. In section II we briefly mention the formalism for estimating the lepton asymmetry in thermal leptogenesis scenario, followed by numerical calculation and results. Section III is devoted to nonthermal leptogenesis and numerical predictions. Finally in section IV we conclude with a summary and discussion. Expressions related to m_{LL} are relegated to Appendix.

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II. BARYON ASYMMETRY OF THE UNIVERSE IN THERMAL LEPTOGENESIS

The canonical seesaw formula (Known as type-I) [1] relates the light left-handed Majorana neutrino mass matrix m_{LL} , heavy right-handed Majorana mass matrix M_{RR} and the Dirac neutrino mass matrix m_{LR} in an elegant way:

$$m_{LL} = -m_{LR} M_{RR}^{-1} m_{LR}^T \quad (1)$$

We consider the model [5],[11], where the lightest of the heavy Majorana neutrinos can have asymmetric decay to lepton and Higgs, and their CP conjugate states, thus producing the CP asymmetry. The CP asymmetry ε is traced to the Complex Yukawa coupling h , which is calculated from the interference of tree level with one loop vertex and self energy corrections for the decay of M_1 and it is defined as

$$[5], [12] \quad \varepsilon_1 = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}. \quad \text{Here } \Gamma = \Gamma(N_1 \rightarrow l_j \phi^\dagger) \quad \text{and}$$

$\bar{\Gamma} = \Gamma(N_1 \rightarrow \bar{l}_j \phi)$ are the decay rates for particles and their CP conjugate states. For standard model scenario, in terms of Yukawa coupling and heavy Majorana masses the expression for CP asymmetry can be expressed as [13]

$$\varepsilon_1 = -\frac{1}{8\pi} \sum_{j=2,3} \frac{\text{Im}(h^\dagger h)_{1j}^2}{(h^\dagger h)_{11}} [f(x_j) + g(x_j)] \quad (2)$$

Where, $x_j = M_j^2 / M_1^2$. The functions $f(x_j)$ and $g(x_j)$ take care th vertex and self energy corrections respectively. For hierarchical structure of heavy neutrinos, $f(x_j) + g(x_j)$ can be approximated to $3/2\sqrt{x_j}$ and in terms of light neutrino mass matrix the eq.(2) can be written as

$$\varepsilon_1 \approx -\frac{3M_1}{16\pi v^2} \frac{\text{Im}(h^\dagger m_{LL} h^*)_{11}}{(h^\dagger h)_{11}} \quad (3)$$

For quasi-degenerate spectrum i.e., $M_1 \approx M_2 < M_3$, the asymmetry is largely enhanced by a resonance factor [14]

$$R = \frac{M_2^2(M_2^2 - M_1^2)}{(M_1^2 - M_2^2) + \Gamma_2^2 M_1^2} \quad \text{where } \Gamma_2 = \frac{(h^\dagger h)_{22} M_2}{8\pi}. \quad \text{In}$$

supersymmetric case, $f(x_j) + g(x_j) \approx 3/\sqrt{x_j}$, and ε_1 will be enhanced by a factor of 2.

The CP asymmetric parameter ε_1 is related to leptonic asymmetry parameter Y_L by $Y_L = \sum_i \frac{\varepsilon_i \kappa_i}{g_{*i}}$. Here, κ_i is the dilution factor and g_* is the effective numbers of degrees of freedom at temperature $T = M_i$. For standard model case we have $g_* = 106.75$ and for Minimal supersymmetric case $g_* = 228.75$. The baryon asymmetry n_B produced through the sphaleron transmutation of Y_L , while $B-L$ remains

conserved is given by[15] $n_B / s = C Y_{B-L} = \frac{C}{C-1} Y_L$, where

$$C = \frac{8N_F + 4N_H}{22N_F + 13N_H}. \quad \text{The number of fermion families and}$$

number of Higgs doublet are represented by N_F and N_H respectively. The entropy density s is related to photon number density by $s = 7.04 n_\gamma$ and the observed baryon asymmetry of the universe is expressed as [16]

$$Y_B^{SM} = (n_B / n_\gamma)^{SM} \approx 0.98 \times 10^{-2} \kappa_1 \varepsilon_1 \quad (4)$$

In case of MSSM, there is no major numerical change, but one expect an approximate enhancement factor of $\sqrt{2}(2\sqrt{2})$ for strong (weak) washout regime [2].

The factor κ_1 describes the washout of the lepton asymmetry due to various lepton number violating processes. This efficiency factor (also known as dilution factor) mainly depends on the effective neutrino mass $\tilde{m}_1 = (h^\dagger h)_{11} v^2 / M_1$, where v is the electroweak Higgs expectation values. For $10^{-2} eV < \tilde{m}_1 < 10^3 eV$, the washout factor can be well approximated by [12],[17]

$$\kappa_1(\tilde{m}_1) = 0.3(10^{-3} / \tilde{m}_1)[\log(\tilde{m}_1 / 10^{-3})]^{-0.6} \quad (5)$$

We adopt a single expression for dilution factor valid only for the given range of effective neutrino mass [17],[18],[19].

A. Numerical calculations and results

To compute the numerical result, we first choose the light, left handed Majorana mas matrices proposed in Appendix A. These mass matrices obey the $\mu-\tau$ symmetry [9], which guarantees the tribimaximal mixings [10].The input parameters are fixed at the stage of predictions of neutrino mass parameters and mixings given in table I.

For the calculation of baryon asymmetry, we first translate the light left handed neutrino mass matrices to heavy right handed neutrino mass matrix via the inversion of the seesaw relation: $M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. We choose a basis U_R where

the M_{RR} is diagonal with real and positive eigenvalues. We transform the diagonal Dirac neutrino mass matrix m_{LR} to the U_R basis and add the CP violating Majorana phases

$Q = \text{diag}(1, e^{i\alpha}, e^{i\beta})$: $m_{LR} \rightarrow m'_{LR} = m_{LR} U_R Q$. In terms of Wolfenstein parameter $\lambda = 0.3$, the diagonal Dirac neutrino mass matrix $m_{LR} = \text{diag}(\lambda^m, \lambda^n, 1)v$, where v is the Higgs vacuum

expectation values. The choice $(m, n) = (6, 2)$ and $(m, n) = (8, 4)$ represent charged lepton and up-quark mass hierarchy respectively. The Yukawa coupling matrix h' becomes complex and hence the term $\text{Im}(h^\dagger h)_{1j} \neq 0$. A straight forward simplification [20] shows that

$(h^\dagger h)_{1j}^2 = (Q_{11}^*)^2 Q_{22}^2 R_2 + (Q_{11}^2)^2 Q_{33}^2 R_3$ where $R_{2,3}$ are real parameters. After inserting the values of phases the above expression leads to $\text{Im}(h^\dagger h)_{1j} = -[R_2 \sin(\alpha - \beta) + R_3 \sin 2\alpha]$, which imparts nonzero CP-asymmetry for particular choice of (α, β) .

In our estimation of lepton asymmetry, we choose some arbitrary values of α and β other than $\pi/2$ and 0. For example, light neutrino masses $(m_1, -m_2, m_3)$ leads to $M_{RR}^{diag} = \text{diag}(M_1, -M_2, M_3)$, and we thus fix the Majorana phase as $Q = \text{diag}(1, e^{i\alpha}, e^{i\beta}) = \text{diag}(1, e^{i(\pi/2+\pi/4)}, e^{i\pi/4})$. The extra phase $\pi/2$ in α absorbs the negative sign before heavy Majorana mass M_2 . In our search programme such choice of the phases leads to highest numerical estimations of lepton asymmetry.

In Table I we give the solar and atmospheric mass squared differences for different neutrino mass models mentioned in Appendix. They obey $\mu - \tau$ symmetry and predict tribimaximal mixings in addition. In Table II the three heavy right-handed masses are mentioned for two choice of the Dirac neutrino mass matrix. We get degenerate spectrum of heavy Majorana masses for normal hierarchical model and this allows us to use the resonant leptogenesis formula. The produced baryon asymmetry are mentioned in Table III, which shows that only normal hierarchical model predicts reasonable value whereas inverted hierarchical model (IIB) nearly misses the observational bound. Degenerate models predict very small baryon asymmetry.

Our estimated baryon asymmetry for normal hierarchical model (IIIA, IIIB) lies between 9.27×10^{-9} and 7.28×10^{-11} respectively for Dirac neutrino mass matrix taken as charged lepton mass matrix and up-quark mass matrix. This hints for some other choice of Dirac neutrino mass matrix [21]. As mentioned earlier, our starting point is the neutrino mass matrix which satisfies the observed neutrino mass parameters and mixings. The values of input parameters are fixed at this level.

III. NONTHERMAL LEPTOGENESIS

We next consider the neutrino mass models discussed in section II (Tables I-III) to nonthermal leptogenesis scenario [7] where the right-handed neutrinos are produced through the direct nonthermal decay of the inflaton. We follow the standard procedure outlined in [22] where nonthermal leptogenesis and baryon asymmetry in the universe had been studied in different neutrino mass models whereby some mass models were excluded using bounds from below and from above on the inflaton mass and reheating temperature after inflaton. Though we adopt similar analysis, the texture of the neutrino mass models considered here are different and hence the conclusions are also expected to be different.

We start with the inflaton decay rate given by

$$\Gamma_\phi = \Gamma(\phi \rightarrow N_i N_i) = \frac{|\lambda_i|^2}{4\pi} M_I \quad (5)$$

where λ_i are the Yukawa coupling for the interaction of three heavy right handed neutrinos N_i with inflaton of mass M_I . The reheating temperature after inflation is given by the expression

$$T_R = \left(\frac{45}{2\pi^2 g_*}\right)^{1/4} (\Gamma_\phi M_P)^{1/2} \quad (6)$$

Where $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass [23].

If the inflaton dominantly couples to N_i , the branching ratio of this decay process is taken as unity, and the produced baryon asymmetry of the universe can be calculated by the following relation [24]

$$Y_B = C Y_L = C \frac{3T_R}{2M_I} \epsilon_1 \quad (7)$$

Here Y_L is the lepton asymmetry generated by CP violating decay of N_1 and T_R is the reheating temperature. The value of C is $-28/79$ for SM case and it is $-8/15$ for MSSM. The above expression (7) of baryon asymmetry is supplemented by two more boundary conditions [22]: (i) lower bound on inflaton mass $M_I \geq 2M_1$ coming from allowed kinematics of inflaton decay, and (ii) an upper bound for reheating temperature $T_R \leq 0.01M_1$ coming from out of equilibrium decay of N_1 . Using the observed central value [2] of the baryon asymmetry $Y_B = 8.7 \times 10^{-11}$ and theoretical prediction of CP asymmetry ϵ_1 for a particular mass model, one can establish a relation between the reheating temperature and inflaton mass using (7). The right handed neutrino mass M_1 from Table II and CP asymmetry from Table III for all the neutrino mass models under consideration, are used to calculate the bounds: $T_R^{\min} < T_R \leq T_R^{\max}$ and

$M_I^{\min} < M_I \leq M_I^{\max}$. Only those models, for which the predicted maximum reheating temperature is always greater than the minimum reheating temperature could survive in nonthermal leptogenesis. These models are identified as IA, IIB, and III(A, B) with Dirac neutrino mass matrix taken as charged lepton mass matrices and III(A, B) with Dirac neutrino mass matrix taken as upquark mass matrix. From Table IV it is seen that inflationary models in which $M_I \sim 10^{13}$ GeV are compatible only with normal hierarchical model III(A,B). In fact with $T_R = 10^{16}$ GeV, we get $M_I \sim 2.8 \times 10^{13}$ GeV, $\Gamma_\phi = 2.85 \times 10^{-6}$ GeV, and $|\lambda_I| = 1.13 \times 10^{-8}$ which are compatible with chaotic

inflationary model. In supersymmetric models, the gravitino problem can be avoided provided that the reheating temperature after inflation is bounded from above in a certain way, namely $T_R \leq (10^6 - 10^7) \text{ GeV}$. In fact the reheating temperature $T_R = 10^6 \text{ GeV}$ is relevant in order to realize the weak scale gravitino mass $m_{3/2} \sim 100 \text{ GeV}$ without causing gravitino problem. Even this reheating temperature is relaxed for two order $T_R = 10^7 \text{ GeV}$, we would have $M_I \sim 10^{11} \text{ GeV}$ in normal hierarchical type III (A,B) with (8, 4). We conclude that the only surviving model in this analysis is the normal hierarchical model (III).

IV. SUMMARY AND DISCUSSION

To summarise, we first parametrise the light left-handed Majorana neutrino mass matrices describing the possible pattern of neutrino masses, e.g., degenerate, inverted hierarchical and normal hierarchical, which obey the $\mu\text{-}\tau$ symmetry having tribimaximal mixings. As a first test these mass matrices predict the neutrino mass parameters and mixings consistent with data, and all the input parameters are fixed at this stage. In the next stage these mass matrices are employed to estimate the baryon asymmetry in both thermal as well as nonthermal scenario. We use CP violating Majorana phases derived from right-handed Majorana mass matrix and two possible forms of Dirac neutrino mass matrix. The overall analysis shows that normal hierarchical mass model appears to be the most favorable choice in nature. The present analysis though phenomenological may serve as an additional criteria to discard some of the presently available neutrino mass models. There are some suggestions in literature [26] for inverted hierarchical model to enhance the estimation of baryon asymmetry if m_3 is increased. The present investigation has taken care of the maximum allowed non-zero value of $m_3 \sim 0.033 \text{ eV}$ in case of inverted hierarchy type IIB model. Our results also differs from a recent study in nonthermal leptogenesis with strongly hierarchical right-handed neutrinos [27] where the mass of the lightest right-handed neutrino $M_1 \leq 10^6 \text{ GeV}$. There are some propositions [28],[29] for probing the reheating temperature at the LHC and this hopefully decides the validity of thermal leptogenesis.

TABLE I REPRESENTS THE MASS SQUARED DIFFERENCES. FOR ALL THE

MODELS $\tan^2 \theta_{12} = 0.5, \sin^2 2\theta_{23} = 1.0$ AND $\sin \theta_{13} = 0$.

Type	$\Delta m_{21}^2 [10^{-5} \text{ eV}^2]$	$\Delta m_{23}^2 [10^{-3} \text{ eV}^2]$
Deg (IA)	7.8	2.6
Deg (IB)	7.9	2.5
Deg (IC)	7.9	2.5
Inh (IIA)	7.3	2.5
Inh (IIB)	8.5	2.3
Nh (IIIA)	7.1	2.1
Nh (IIIB)	7.5	2.4

TABLE II HEAVY MASSES FOR DIFFERENT NEUTRINO MASS MODELS

Type	(m,n)	$M_1 (\text{GeV})$	$M_2 (\text{GeV})$	$M_3 (\text{GeV})$
Deg(IA)	(6,2)	1.22×10^8	-6.01×10^{11}	2.59×10^{13}
Deg (IA)	(8,4)	9.86×10^5	-5.03×10^9	2.51×10^{13}
Deg (IB)	(6,2)	4.05×10^7	6.16×10^{11}	7.60×10^{13}
Deg (IB)	(8,4)	3.28×10^5	4.99×10^9	7.60×10^{13}
Deg (IC)	(6,2)	4.05×10^7	-6.69×10^{12}	6.99×10^{12}
Deg (IC)	(8,4)	3.28×10^5	-4.83×10^{11}	7.84×10^{11}
Inh (IIA)	(6,2)	3.29×10^8	9.73×10^{12}	6.25×10^{16}
Inh (IIA)	(8,4)	2.63×10^6	7.94×10^{10}	6.21×10^{16}
Inh (IIB)	(6,2)	-9.97×10^8	2.63×10^{12}	5.59×10^{14}
Inh (IIB)	(8,4)	-8.10×10^6	2.14×10^{10}	5.47×10^{14}
Nh (IIIA)	(6,2)	3.93×10^{11}	-4.09×10^{11}	2.87×10^{14}
Nh (IIIA)	(8,4)	3.19×10^9	-3.22×10^9	2.85×10^{14}
Nh (IIIB)	(6,2)	3.85×10^{11}	-3.99×10^{11}	2.99×10^{14}
Nh (IIIB)	(8,4)	3.13×10^9	-3.25×10^9	2.97×10^{14}

TABLE III PRESENTS THE PREDICTED VALUES OF CP ASYMMETRY AND BARYON ASYMMETRY FOR DIFFERENT MASS MODELS

Type	(m,n)	$\tilde{m}_1 (GeV)$	$(h^\dagger h)_{11}$	κ_1	ϵ_1	Y_B
Deg (IA)	(6,2)	1.19×10^{-9}	4.78×10^{-6}	9.3×10^{-5}	1.53×10^{-7}	1.55×10^{-13}
Deg (IA)	(8,4)	1.19×10^{-9}	3.87×10^{-8}	9.3×10^{-5}	4.14×10^{-9}	4.16×10^{-15}
Deg (IB)	(6,2)	3.97×10^{-10}	5.31×10^{-7}	2.83×10^{-4}	4.46×10^{-16}	1.36×10^{-21}
Deg (IB)	(8,4)	3.97×10^{-10}	4.30×10^{-9}	2.83×10^{-4}	3.26×10^{-18}	1.10×10^{-23}
Deg (IC)	(6,2)	3.97×10^{-10}	5.31×10^{-7}	2.83×10^{-4}	2.49×10^{-15}	7.62×10^{-21}
Deg (IC)	(8,4)	3.97×10^{-10}	4.30×10^{-9}	2.83×10^{-4}	2.16×10^{-16}	6.62×10^{-22}
Inh (IIA)	(6,2)	4.95×10^{-11}	5.31×10^{-7}	2.95×10^{-3}	1.56×10^{-12}	4.98×10^{-17}
Inh (IIA)	(8,4)	4.95×10^{-11}	4.30×10^{-9}	2.95×10^{-3}	1.26×10^{-14}	4.04×10^{-19}
Inh (IIB)	(6,2)	1.08×10^{-12}	5.01×10^{-6}	8.83×10^{-4}	2.69×10^{-7}	2.57×10^{-12}
Inh (IIB)	(8,4)	1.52×10^{-10}	4.06×10^{-8}	8.83×10^{-4}	2.18×10^{-9}	2.07×10^{-14}
Nh (IIIA)	(6,2)	5.80×10^{-10}	7.51×10^{-3}	1.82×10^{-4}	4.59×10^{-3}	9.27×10^{-9}
Nh (IIIA)	(8,4)	5.80×10^{-10}	6.13×10^{-5}	1.82×10^{-4}	3.62×10^{-5}	7.28×10^{-11}
Nh (IIIB)	(6,2)	5.93×10^{-10}	7.51×10^{-3}	1.83×10^{-4}	4.91×10^{-3}	9.66×10^{-9}
Nh (IIIB)	(8,4)	5.93×10^{-10}	6.13×10^{-5}	1.83×10^{-4}	3.88×10^{-5}	7.59×10^{-11}

TABLE IV CONTAINS THE THEORETICAL BOUND S ON REHEATING TEMPERATURE

Type	(m,n)	$T_R^{\min} < T_R \leq T_R^{\max} (GeV)$	$M_I^{\min} < M_I \leq M_I^{\max} (GeV)$
Deg (IA)	(6,2)	$2.66 \times 10^5 < T_R \leq 1.22 \times 10^6$	$2.44 \times 10^8 < M_I \leq 1.14 \times 10^9$
Deg (IA)	(8,4)	$7.14 \times 10^4 < T_R \leq 19.86 \times 10^3$	$1.97 \times 10^6 < M_I \leq 2.49 \times 10^5$
Deg (IB)	(6,2)	$3.40 \times 10^{13} < T_R \leq 4.05 \times 10^5$	$8.10 \times 10^7 < M_I \leq 1.10$
Deg (IB)	(8,4)	$3.02 \times 10^{13} < T_R \leq 3.28 \times 10^3$	$6.56 \times 10^5 < M_I \leq 7.20 \times 10^{-5}$
Deg (IC)	(6,2)	$5.29 \times 10^{13} < T_R \leq 4.05 \times 10^5$	$8.10 \times 10^7 < M_I \leq 6.16$
Deg (IC)	(8,4)	$5.06 \times 10^{11} < T_R \leq 3.28 \times 10^3$	$6.56 \times 10^5 < M_I \leq 4.35 \times 10^{-3}$
Inh (IIA)	(6,2)	$6.95 \times 10^{10} < T_R \leq 3.29 \times 10^6$	$6.58 \times 10^8 < M_I \leq 3.14 \times 10^4$
Inh (IIA)	(8,4)	$6.80 \times 10^{10} < T_R \leq 2.63 \times 10^4$	$5.26 \times 10^6 < M_I \leq 2.03$
Inh (IIB)	(6,2)	$1.24 \times 10^6 < T_R \leq 9.97 \times 10^6$	$1.99 \times 10^9 < M_I \leq 1.64 \times 10^{10}$
Inh (IIB)	(8,4)	$1.22 \times 10^6 < T_R \leq 8.10 \times 10^4$	$1.62 \times 10^7 < M_I \leq 1.08 \times 10^6$
Nh (IIIA)	(6,2)	$2.86 \times 10^4 < T_R \leq 3.93 \times 10^9$	$7.86 \times 10^{11} < M_I \leq 1.10 \times 10^{17}$
Nh (IIIA)	(8,4)	$2.94 \times 10^4 < T_R \leq 3.19 \times 10^7$	$6.38 \times 10^9 < M_I \leq 7.06 \times 10^{12}$
Nh (IIIB)	(6,2)	$2.57 \times 10^4 < T_R \leq 3.85 \times 10^9$	$7.70 \times 10^{11} < M_I \leq 1.15 \times 10^{17}$
Nh (IIIB)	(8,4)	$2.69 \times 10^4 < T_R \leq 3.13 \times 10^7$	$6.26 \times 10^9 < M_I \leq 7.42 \times 10^{12}$

and the solar mixing angle is fixed at tribimaximal mixings.

APPENDIX

Left-handed Majorana neutrino mass matrices which obey $\mu - \tau$ symmetry [10],[25] have the following form

$$m_{LL} = \begin{pmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{pmatrix} m_0$$

This predicts an arbitrary solar mixing angle $\tan 2\theta_{12} = \frac{2\sqrt{2}}{(X-Z-W)}$, while the predictions on atmospheric mixing angle is maximal ($\theta_{23} = \pi/4$) and Chooz angle is zero. The following mass matrices consist of two parameters

1. Degenerate Type A (IA) ($m_i = m_1, -m_2, m_3$)

$$m_{LL} = \begin{pmatrix} \delta_1 - 2\delta_2 & -\delta_1 & -\delta_1 \\ -\delta_1 & 1/2 - \delta_2 & -1/2 - \delta_2 \\ -\delta_1 & -1/2 - \delta_2 & 1/2 - \delta_2 \end{pmatrix} m_0$$

with $\delta_1 = 0.66115$, $\delta_2 = 0.16535$ and $m_0 = 0.4eV$.

2. Degenerate Type B (IB) ($m_i = m_1, m_2, m_3$)

$$m_{LL} = \begin{pmatrix} 1-\delta_1-2\delta_2 & \delta_1 & \delta_1 \\ \delta_1 & 1-\delta_2 & -\delta_2 \\ \delta_1 & -\delta_2 & 1-\delta_2 \end{pmatrix} m_0$$

with $\delta_1 = 8.314 \times 10^{-5}$, $\delta_2 = 0.00395$ and $m_0 = 0.4eV$.

3. Degenerate Type C (IC) ($m_i = m_1, m_2, -m_3$)

$$m_{LL} = \begin{pmatrix} 1-\delta_1-2\delta_2 & \delta_1 & \delta_1 \\ \delta_1 & -\delta_2 & 1-\delta_2 \\ \delta_1 & 1-\delta_2 & -\delta_2 \end{pmatrix} m_0$$

with $\delta_1 = 8.314 \times 10^{-5}$, $\delta_2 = 0.00395$ and $m_0 = 0.4eV$.

4. Inverted hierarchical (Inh) mass matrix with $m_3 \neq 0$

$$m_{LL} = \begin{pmatrix} 1-2\varepsilon & -\varepsilon & -\varepsilon \\ -\varepsilon & 1/2 & 1/2-\eta \\ -\varepsilon & 1/2-\eta & 1/2 \end{pmatrix} m_0$$

Inh (IIA): ($m_i = m_1, m_2, m_3$) with $\eta/\varepsilon = 1.0$, $\eta = 0.0048$,

$m_0 = 0.05eV$. Inh (IIB): ($m_i = m_1, -m_2, m_3$) with $\eta/\varepsilon = 1.0$,

$\eta = 0.6607$, $m_0 = 0.05eV$

5. Normal hierarchical mass matrix (IIIA): $m_{LL}(1,1) \neq 0$

$$m_{LL} = \begin{pmatrix} -\eta & -\varepsilon & -\varepsilon \\ -\varepsilon & 1-\varepsilon & -1 \\ -\varepsilon & -1 & 1-\varepsilon \end{pmatrix} m_0$$

with $\eta/\varepsilon = 0.0$, $\varepsilon = 0.175$ and $m_0 = 0.029eV$.

6. Normal hierarchical mass matrix (IIIB) $m_{LL}(1,1) = 0$

$$m_{LL} = \begin{pmatrix} 0 & -\varepsilon & -\varepsilon \\ -\varepsilon & 1-\varepsilon & -1+\eta \\ -\varepsilon & -1+\eta & 1-\varepsilon \end{pmatrix} m_0$$

with $\eta/\varepsilon = 0.0$, $\varepsilon = 0.164$ and $m_0 = 0.028eV$.

The textures of mass matrices for inverted hierarchy (IIA, IIB) as well as normal hierarchy (IIIA, IIIB) have the potential to decrease the solar mixing angle from the tribimaximal value without sacrificing $\mu - \tau$ symmetry. This is possible through the identification of 'flavor twister' [25] $\eta/\varepsilon \neq 0$.

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