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Supply Air Pressure Control of HVAC System Using MPC Controller

P. Javid, A. Aeenmehr, J. Taghavifar

Abstract**—**In this paper, supply air pressure of HVAC system has been modeled with second-order transfer function plus dead-time. In HVAC system, the desired input has step changes, and the output of proposed control system should be able to follow the input reference, so the idea of using model based predictive control is proceeded and designed in this paper. The closed loop control system is implemented in MATLAB software and the simulation results are provided. The simulation results show that the model based predictive control is able to control the plant properly.

Keywords**—**Air conditioning system, GPC, dead time, Air supply control.

I. Introduction

In this paper, the speed control of supply air fan in an air conditioning system (HVAC) using a model based predictive controller has been studied. Typically, the PID controller is used in HVAC system because it is simple to understand and implement in industrial processes [1]. In this point of view, PID controllers are more interesting and applicable than other advanced controllers unless the evidences show that the PID controller cannot satisfy the conditions. On the other hand, tuning of coefficients of the PID controller is time-consuming, expensive and difficult task, and in some cases, after a certain time, the re-tuning of the controller coefficients is needed because of changes in operating and environmental conditions [2]. Also, in tuning of controller coefficients when the controller is not auto-tuned and the classical methods are used to adjust the coefficients there is no guarantee that the coefficients are selected optimal which can affect efficiency. This defect is more pronounced, when in most cases, the control engineer does not have complete knowledge of HVAC system [2]. However, it should be emphasized that since the PID controllers are easy to understand and implemented, usually used in HVAC control system. In this paper, the idea of using MPC to control the speed of supply air fan is proposed, because it is able to cover the lack of knowledge of process and existing uncertainties [3]. In addition, MPC Algorithms inherently compensate the dead time that is convenient and practical for the system discussed in this paper. Also by replacement of MPC instead of PID controller in HVAC system, the need for adjustment of coefficients will be disappeared. The model which is used in this paper, is second-order system plus dead time, which describes the dynamic of real process properly [2]. The closed loop control system is implemented in MATLAB software and the results are presented.

P. Javid, A. Aeenmehr, and J. Taghavifar are with the Bahman Gostaresh Kian Industrial Group, BGK, CO Iran (phone: +98-611-4439066-7; fax: +98-611-4433283; e-mail: parvinjavid@yahoo.com, aaeinmehr@yahoo.com, jahantaghavifar@ yahoo.com).

II. Heating and Ventilation Air Conditioning System

In Fig. 1, a typical air conditioning system is shown. As you can see in Fig. 1, the fresh air is entering and passing through the filter and cooling (heating) coil of HVAC system to the target zone and the discharged air temperature is decreased (increased). This change directly depends on the rate of cool (hot) water flow through cooling (heating) coil [5]. Also in HVAC systems, the supply air pressure depends on the fan speed, where the higher speed leads to greater pressure and vice versa. Thus, by feeding the control signal to fan drive, the fan speed can be adjusted on a specific constant value [2].

As the real process is very complicated, determining the precise models for control purpose is neither practical nor necessary. Usually, we represent the real processes by low-order plus dead time model and after the test; the model for the supply air pressure loop is obtained as [2]:

$$G_{sap}=\frac{1}{0.11s^2+17.9s+2.55}e^{-0.44s} \quad (1)$$

III. Model Based Predictive Control

Model-based predictive control refers to controller that uses the model of process and minimizes the cost function to gain control signal [6]. In model based predictive control approach, an explicit model is used to predict the process output in future. To create a control signal, a quadratic cost function is minimized and optimal control signal is obtained at any given moment [6]. An important advantage of predictive control is that the system can apply to a wide range of processes such as delayed, none minimum phase and unstable systems [6]. So in this paper it is expected that predictive control can control the process which is described in Section II.

Many studies have been done on the control of HVAC systems in advance. Most single input- single output (SISO) plants, when considering operation around a particular set point and after linearization can be described by

$$A(z^{-1})y(t)=z^{-d}B(z^{-1})u(t-1)+C(z^{-1})e(t) \quad (2)$$

where u(t) and y(t) are the control and output sequence of the plant and e(t) is a zero mean white noise [7]. A and B are the following polynomials in the backward shift operator z^{-1}:

$$A(z^{-1})=1+a_1z^{-1}+a_2z^{-2}+\ldots.. \quad (3)$$

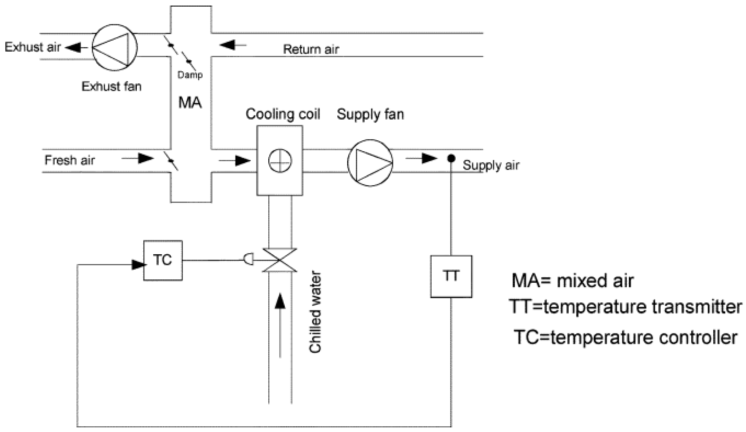

Fig. 1 Typical HVAC system [4]

$$B(z^{-1})=1+b_1z^{-1}+b_2z^{-2}+\ldots \quad (4)$$

where d is the dead time of the system. This model is known as Auto-Regressive Moving Average (CARMA) model.

The generalized predictive control algorithm consists of applying a control sequence that minimizes a multi stage cost function of the form

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j|t) - w(t+j)]^2 + \sum_{J=1}^{Nu} \lambda_j\,[\Delta u(t+j-1)]^2 \quad (5)$$

where $\hat{y}(t+j|t)$ is an optimum j-step ahead prediction of the system output on data up to time t, N_1 and N_2 are the minimum and maximum costing horizons of N_u is the control horizon, δ(j)andλ(j) are weighting sequences and w(t+j) is the future reference trajectory, which can be calculated.

δ(j) is considered to be 1 and λ(j) is considered to be constant. The objective of predictive control is to compute the future control sequence u(t) , u(t+1) , …

In such a way that the future plant output y(t+j) is driven close to w(t+j). This is accomplished by minimizing $J(N_1, N_2, N_u)$.

In order to optimize the cost function the optimal prediction of y(t+j) for $N_1 \leq j \leq N_2$ will be obtained. Consider the following Diophantine equation:

$$1 = E_j(z^{-1})\tilde{A}(z^{-1}) + z^{-j}F_j(z^{-1}) \quad (6)$$

where$\tilde{A}(z^{-1}) = \Delta A(z^{-1})$ and $\Delta = 1 - z^{-1}$

As the degree of polynomial $E_j(z^{-1})$ the best prediction of y(t+j) is therefore :

$$\tilde{y}(t+j|t) = G_j(z^{-1})\Delta u(t+j-d-1) + F_j(z^{-1})y(t) \quad (7)$$

where $G_j(z^{-1})=E_j(z^{-1})B(z^{-1})$.

The polynomials E_j and F_j are uniquely defined with degrees j-1 and n_a are respectively. They can be obtained dividing 1 by $\tilde{A}(z^{-1})$ until the reminder can be factorized as $z^{-j}F_j(Z^{-1})$ [7]. The quotient of the division is the polynomial $E_j(z^{-1})$.

$$F_j(z^{-1})=f_{j,0}+f_{j,1}z^{-1}+\ldots+f_{j,na}z^{-na} \quad (8)$$

$$E_j(z^{-1})=e_{j,0}+e_{j,1}z^{-1}+\ldots+e_{j,j-1}z^{-(j-1)} \quad (9)$$

Suppose that the same procedure is used to obtain E_{j+1} and F_{j+1}, that is, dividing 1 by$\tilde{A}(z^{-1})$ until the reminder of division can be factorized as $z^{-(j+1)}F_{j+1}(z^{-1})$.

$$F_{j+1}(z^{-1})=f_{j+1,0}+f_{j+1,1}z^{-1}+\ldots+f_{j+1,na}z^{-na} \quad (10)$$

It is clear that only another step of the division performed to obtain the polynomials E_j and F_j has to be taken in order to obtain the polynomials E_{j+1} and F_{j+1}. The polynomials E_{j+1} will be given by:

$$E_{j+1}(z^{-1})=E_j(z^{-1})+e_{j+1,j}z^{-j} \quad (11)$$

with $e_{j=1,j}=f_{j,0}$[7].

The coefficients of polynomials F_{J+1} can then be expressed as [7]:

$$F_{j+1,i}=f_{j,i+1}-f_{j,0}\,a_{i+1} \quad i=0\ldots na-1 \quad (12)$$

The polynomial G_{j+1} can be obtained recursively as follows [7]:

$$G_{j+1}=E_{j+1}B=(E_j+f_{j,0}z^{-j}B) \quad (13)$$

$$G_{j+1}=G_j+f_{j,0}z^{-j}B \quad (14)$$

To solve the GPC problem the set of control signals u(t), u(t+1), …, u(t+n) has to be obtained in order to optimize (5). As the system considered has a dead time of d sampling periods, the output of the system will be influenced by signal u(t) after sampling period d+1. The values N_1, N_2, N_u defining the horizon can be defined by N_1=d+1, N_2=d+N, and N_u=N. Notice that there is no point in making, N_1<d+1 as terms

added to (5) will only depend on the past control signals. On the other hand if N_1>d+1 the first points in the reference sequence, being the ones guessed with most certainty, will not be taken into account.

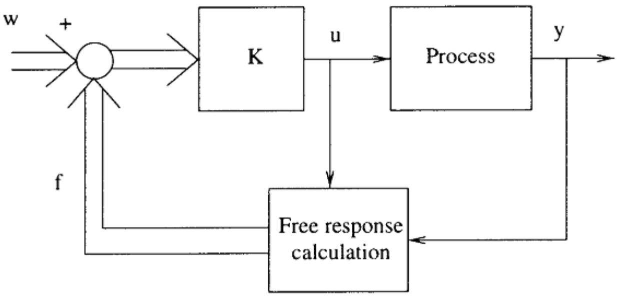

Fig. 2 MPC Law [7]

Now consider the following set of j ahead optimal predictions [7]:

$$\hat{y}(t+d+1|t) = G_{d+1}\Delta u(t) + F_{d+1}y(t) \quad (15)$$

$$\hat{y}(t+d+2|t) = G_{d+2}\Delta u(t+1) + F_{d+2}y(t) \quad (16)$$

$$\hat{y}(t+d+N|t) = G_{d+N}\Delta u(t+N-1) + F_{d+N}y(t) \quad (17)$$

which can be written as [7] :

$$y=Gu+F(z^{-1})y(t)+G'(z^{-1})\Delta u(t-1) \quad (18)$$

where [7]:

$$y = \begin{bmatrix} \hat{y}(t+d+1|t) \\ \hat{y}(t+d+2|t) \\ \vdots \\ \hat{y}(t+d+n|t) \end{bmatrix} \quad (19)$$

$$u = \begin{bmatrix} \Delta u(t) \\ \Delta u(t+1) \\ \vdots \\ \Delta u(t+N-1) \end{bmatrix} \quad (20)$$

$$G = \begin{bmatrix} g_0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ g_{N-1} & \cdots & g_0 \end{bmatrix} \quad (21)$$

$$F(z^{-1}) = \begin{bmatrix} F_{d+1}(z^{-1}) \\ F_{d+2}(z^{-1}) \\ \vdots \\ F_{d+N}(z^{-1}) \end{bmatrix} \quad (22)$$

Notice that the last two terms in (18) only depend on the past and can be grouped into f leading to [7]:

$$Y=Gu+f$$

Notice that if all initial conditions are zero, the free response f is also zero. if unit step is applied to the input at time t; that is

$$\Delta u(t)=1,\ \Delta u(t+1)=o\ldots.,\ \Delta u(t+N-1)=0;$$

The excepted output sequence $[\hat{y}(t+1), \hat{y}(t+2), \ldots, \hat{y}(t+N)]$ is equal to the first column of matrix G. that is, the first column of matrix G can be calculated as the step response of the plant when a unit step is applied to the manipulated variable. The free response term can be calculated recursively by [7]:

$$f_{j+1}=z(1-\tilde{A}(z^{-1}))f_j+B(z^{-1})\Delta u(t-d+j) \quad (23)$$

with f_0=y(t) and $\Delta u(t+j)=0$ for $j\geq 0$.

Expression (5) can be written as

$$J=(Gu+f-w)^T(Gu+f-w)+\lambda u^T u \quad (24)$$

where [7]:

$$W=[w(t+d+1) \quad w(t+d+2) \ldots.. \quad w(t+d+N)]^T$$

Equation (24) can be written as [7]:

$$J=0.5u^T Hu+b^T u \quad (25)$$

where:

$$H=2(G^T G+\lambda I)$$

$$b^T=2(f-w)^T G$$

$$f_0=(f-w)^T (f-w)$$

The minimum of J, assuming there are no constraints on the control signals, can be found by making the gradient of J equal to zero, which leads to [7]:

$$U=(G^T G+\lambda I)^{-1}G^T(w-f) \quad (26)$$

Notice that the control signal that is actually send to the process is the first element of vector u, that is given by [7]:

$$\Delta u(t)=k(w-f) \quad (27)$$

where k is the first row of matrix $(G^T G+\lambda I)^{-1}G^T$.

IV. Simulation

Model based predictive control with 10 control horizon for HVAC system is applied and implemented and the results are presented in Figs. 3 to 5.

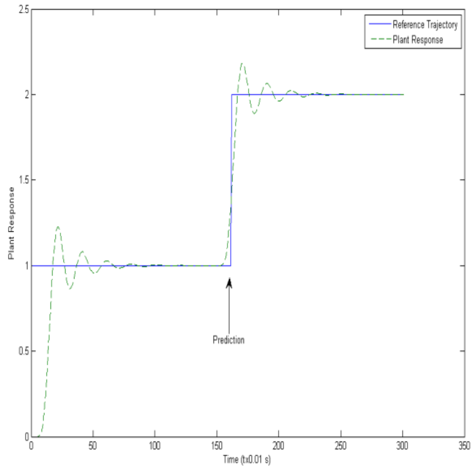

Fig. 3 Reference input and HVACAir Supply Pressure

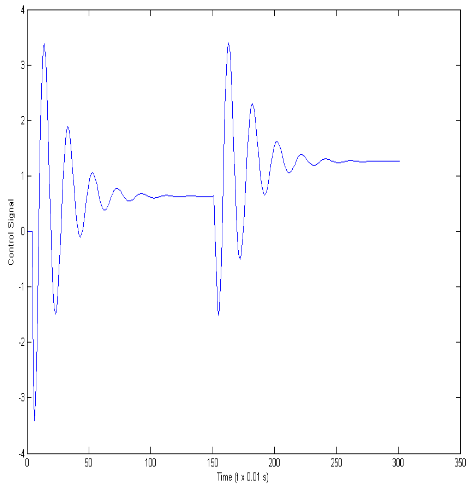

Fig. 4 Control signal

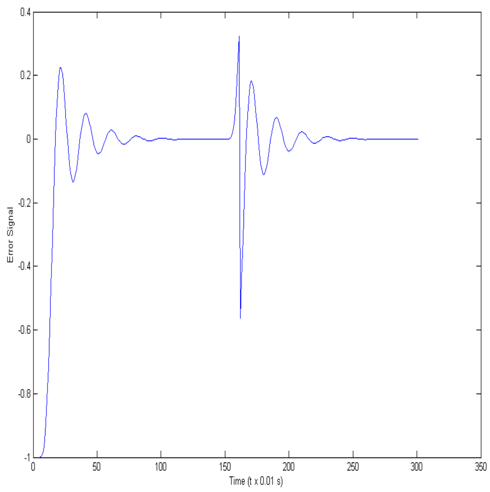

Fig. 5 Error Signal

In Fig. 3 the reference input signal and the output signal of the closed loop control system is shown. Also the control signal generated by the model based predictive controller is shown in Fig. 4 and the error signal is shown in Fig. 5. As seen in Fig. 3, at time t=0 to t=1550 ms the input reference value is reached from 0 to 1 and at t= 1550 ms is increased from 1 to 2 and as shown the output of the process is tracked the input reference properly.

V. Conclusion

In this Paper, the adjustment of supply air pressure in HVAC system, proposed and controlled with model based predictive control. A second order system plus dead time was considered as a process model. Then one of predictive control algorithm is introduced and applied to the process. The results of the simulation are shown that the closed loop system successfully followed the changes in reference input with MPC.

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