

# Study on the Variation Effects of Diverging Angle on Characteristics of Flow in Converging and Diverging Ducts by Numerical Method

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**Abstract**—The present paper develops and validates a numerical procedure for the calculation of turbulent combustive flow in converging and diverging ducts and through simulation of the heat transfer processes, the amount of production and spread of Nox pollutant has been measured. A marching integration solution procedure employing the TDMA is used to solve the discretized equations. The turbulence model is the Prandtl Mixing Length method. Modeling the combustion process is done by the use of Arrhenius and Eddy Dissipation method. Thermal mechanism has been utilized for modeling the process of forming the nitrogen oxides. Finite difference method and Genmix numerical code are used for numerical solution of equations. Our results indicate the important influence of the limiting diverging angle of diffuser on the coefficient of recovering of pressure. Moreover, due to the intense dependence of Nox pollutant to the maximum temperature in the domain with this feature, the Nox pollutant amount is also in maximum level.

**Keywords**—Converging and Diverging Duct, Combustion, Diffuser, Diverging Angle, Nox

## I. INTRODUCTION

THE necessity to decrease of velocity and preventing the pressure loss in most of the inside flow lead to the use of diffuser in various kinds [1]. In gas turbine, which is actually a thermal motor, and in which the air is used as the operating fluid for creating pressure, diffusers play a main role in converting the kinetic energy into pressure energy. Aerodynamic of diffusers also play important role in controlling the distribution of the mass flow, the stability of the combustion chamber system, and decreasing of the pressure loss of the turbine gas [2]. The air entering into the airplane engines is provided through channel to the form of the diffuser having the Mach number range number of 0.4 and the less, which in flight condition the high Mach number lead to increasing the pressure [3]. In these engines, the effect of low pressure drop and the uniformity of the flow to prevent of vibration are of great importance.

In flow of boundary layer inside the diffuser having flat walls, diverging angle has considerable importance. If this angle is very small, the length of diffuser will become long and the friction losses will increase and too much enlarging of this angle leads to separation of boundary layer and production of recirculation and consequently the losses of fluid power [4]-[5].

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The flow with proper diverging angle is a kind of boundary layer and the equations can be solved through using marching method. This method has significant advantages over other calculating methods with a view to the speed of doing calculations and also the memory volume for doing calculations [5]-[6]. In numerical simulations, the flow being inside the diffusers and also the grid conformity to solution zone and increasing its compression in the vicinity of the solid wall play a great role in accuracy of the gained results [7]. One of the approaches utilized for gaining the conformity of calculating grid to solution field, is the use of Von Misses coordinate, in which flow lines are chose as one of the independent coordinates [8].

The purpose of this paper is development of a numerical method to measure the parabolic flow inside and outside the diverging and converging ducts in combustive situation and numerical simulation of the Nox pollutant. The effects of the important parameters such as limiting diverging angle are also studied.

## II. GOVERNING EQUATIONS AND SELECTION THE COORDINATES SYSTEMS

Momentum, energy and chemical species conservation equations in coordinates set is on the basis of stream lines  $x \sim \psi$  [8] as follows:

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial \psi} \left( r^2 \rho U \mu_{eff} \frac{\partial U}{\partial \psi} \right) + \frac{1}{\rho U} \left( F_x - \frac{\partial p}{\partial x} \right) \quad (1)$$

$$\frac{\partial \tilde{h}}{\partial x} = \frac{\partial}{\partial \psi} \left( \frac{\partial}{\partial \psi} \left( r^2 \rho U \Gamma_{h,eff} \frac{\partial \tilde{h}}{\partial \psi} \right) \right) + \frac{\partial}{\partial \psi} \left[ (\mu_{eff} - \Gamma_{h,eff}) r^2 \rho U \frac{\partial (U^2/2)}{\partial \psi} \right] \quad (2)$$

$$\frac{\partial m_j}{\partial x} = \frac{\partial}{\partial \psi} \left( r^2 \rho U \Gamma_{j,eff} \frac{\partial m_j}{\partial \psi} \right) + \frac{R_j}{\rho U} \quad (3)$$

$$\frac{\partial (m_{ox} - m_{fus})}{\partial x} = \frac{\partial}{\partial \psi} \left( r^2 \rho U \Gamma_{eff} \frac{\partial (m_{ox} - m_{fus})}{\partial \psi} \right) \quad (4)$$

It should be noted that one of the independent variables of the coordinates set is stream function ( $\psi$ ) that considering its definition and satisfying the continuity equation, there is no need to solve the continuity equation.

In above equation the dependent variables ( $m_{ox} - m_{fus}$ ),  $m_j$ ,  $\tilde{h}$ ,  $U$ , are indicated with the general variable  $\phi$  and

through changing the variable  $\omega = \frac{\psi - \psi_E}{\psi_E - \psi_I}$  instead of  $\psi$  the above differential equations can be written in a total form as follows:

$$\frac{\partial \phi}{\partial x} + (a + b\omega) \frac{\partial \phi}{\partial \omega} = \frac{\partial}{\partial \omega} \left( c \frac{\partial \phi}{\partial \omega} \right) + d \quad (5)$$

Coefficients as following; a, b, c, d:

$$a = \frac{r_I \dot{m}_I}{(\psi_E - \psi_I)} \quad b = \frac{(r_E \dot{m}_E - r_I \dot{m}_I)}{(\psi_E - \psi_I)} \quad (6)$$

$$c = \frac{r^2 \rho U \Gamma_{eff}}{(\psi_E - \psi_I)^2} \quad d = \frac{1}{\rho U} S_\phi \quad (7)$$

$S_\phi$  source term for the variable  $\phi$ ,  $\dot{m}_I$ ,  $\dot{m}_E$  the rate of mass flow from the boundaries I, E and also  $\psi_I$ ,  $\psi_E$  the amount of the stream function are boundaries I, E. Turbulence effects has been measured by the use Prandtl mixing length method. to measure the features of the flow in the vicinity of the solid boundaries, The wall function provided by the help of the theory of the Couette flow have been utilized [9].

$$\tilde{h}^+ = \sigma_t (U^+ + P_h) + (1 - \sigma_t) W \frac{(U^+)^2}{2} \quad (8)$$

$$m_j^+ = \sigma_t (U^+ + P_j) \quad (9)$$

The sign (+) is indicators of the dimensions less in parameters. The values  $P_h$  and  $P_j$  are dependent upon the Prandtl- Schmidt number of laminar to Prandtl-Schmidt number of turbulence ratio.

Reacting rates are determined in the source term of the energy equation by the relation of Arrhenius rate [10] and Eddy dissipation method [11]. The rate of fuel consumption is measured by two calculating methods and the smallest of them with consideration of the effects of mixing is selected.

$$R_{fu} = -p P^2 m_{fu} m_{ox} \exp \left\{ -\frac{E}{RT} \right\} \quad (10)$$

$$R_{fu} = -C \rho m_{fu} \left| \frac{\partial U}{\partial y} \right| \quad (11)$$

### III. POLLUTANT FORMATION MODELS

Pollution arising from the nitrogen oxides has prejudicial effects on the health of human beings and the environment, and it also plays a great role on the forming the acid rains, fog, chemical smoke and perforation of the ozone layer. The annual production of these gasses has been raised six times since 1950.

The collection of the NO and NO<sub>2</sub> is called Nox that is formed through combustion processes and the chemical reaction of nitrogen present in combustion air and/or in-fuel nitrogen. Chiefly, for production of Nox in combustion of gas fuels, three mechanisms are brought into forth: thermal mechanism [12], prompt mechanism [12]-[13], and fuel mechanism [12].

Thermal mechanism is the most important mechanism recognized up to now. This mechanism was laid on the carpet for first time by Zeldovich in 1946. High temperature and concentration of the Oxygen are among the important factors in this mechanism, and are considered almost in the most combustion processes.

Fuel NO is formed through oxidation of nitrogenous composites of the fuel in flame part, and when the amount of the fuel Nitrogen is more than the one-tenth (0.1) of the fuel weight, its production is considerable. The prompt NO mechanism was set forth by Fenimore in 1971. This mechanism is created in the consequence of collision between Nitrogen molecules and Hydrocarbon radicals.

In order to predict the amount of the NO in this research, the Zeldovich Model is used which it measures successfully the NO concentration. Considering that in this model relatively small amount of computer memory is necessary, so it is economical and this fact is an important factor in selecting the model for NO modeling. To measure the intensity of NO production through combustion process, we should be able to solve the equation of the conservation of NO mass fraction. To measure the NO concentration that is enough to solve a differential equation with partial derivations.

### IV. GRID GENERATION AND SOLVING THE DIFFERENTIAL EQUATIONS

Grid generation in the field of calculation is done through consideration of the coordinates  $x$  and  $\omega$ . Considering the parabolic nature of the (5), it can be solved by the use of the marching method [14] along the flow. In this method, it is supposed that each node in the grid is influenced at the most by the 5 upstream nodes and neighbor nodes, which two points in its neighbor and three points in its upstream section are placed.

$$D_i \phi_{i,D} = A_i \phi_{i+1,D} + B_i \phi_{i-1,D} + E_i \phi_{i,U} + F_i \phi_{i+1,U} + G_i \phi_{i-1,U} + H_i \quad (12)$$

The index U and D are indicators of the upstream and downstream section of the flow which the upstream values are active and the downstream values are passive. Therefore, all the active values can be placed in  $c_i$ .

$$D_i \phi_{i,D} - A_i \phi_{i+1,D} - B_i \phi_{i-1,D} = C_i \quad (13)$$

The coefficients  $A_i, B_i, \dots$  also is produced through integration of the conservation equation on the computational element.

$$A_i = \left( T - \frac{1}{2} \dot{m}' \right)_{i+\frac{1}{2}} \quad (14) \quad B_i = \left( T + \frac{1}{2} \dot{m}' \right)_{i-\frac{1}{2}}$$

$$C_i = \phi_{i,D} P \left( \omega_{i+\frac{1}{2}} - \omega_{i-\frac{1}{2}} \right) + S_i \quad (15)$$

$$D_i = A_i + B_i + P \left( \omega_{i+\frac{1}{2}} - \omega_{i-\frac{1}{2}} \right) - S_i \quad (16)$$

Which in the above relations:

$$P = \frac{(\psi_E - \psi_I)_U}{\delta x} T_{i \pm \frac{1}{2}} = \frac{(r_{eff})_{i \pm \frac{1}{2}}}{\pm (y_{i \pm 1} - y_i)_U} \quad (17)$$

By expansion of the equations for the passive points of the each section, we will reach to an equation system having the matrix of three-diameter coefficients. This equation system is solved by TDMA algorithm.

#### V. CALCULATION OF THE PRESSURE FIELD

In fluid flow inside the converging and diverging ducts, the pressure changes due to the change of flow section is considerable, is a great role in calculation of flow field. The cross section between the two lines of flows (1&2) can be written as follows [14]:

$$A_{12} = \int_1^2 \frac{d\psi}{\rho U} \quad (18)$$

Through differentiation of the above relation in proportion to pressure and the use of the relation between the speed of sound and fluid thermodynamic properties, it can be written:

$$\frac{dP}{dA_{12}} = \frac{1}{\int_1^2 \frac{1}{\rho^2 U^3} (1 - M^2) d\psi} \quad (19)$$

In which M is the fluid mach number. Through knowing the changes of the pressure in proportion to cross section and the determination of the geometry of flow field (the changes of the section level along the flow), the pressure changes on the path of the flow can be calculated.

$$\frac{dP}{dx} = \frac{\frac{dA_{12}}{dx}}{\int_1^2 \frac{1}{\rho^2 U^3} (1 - M^2) d\psi} \quad (20)$$

#### VI. RESULTS AND DISCUSSION

In Fig. 1 the numerical calculations results are compared with Kline and Fox experimental results [15] regarding the flat diffuser. In this figure, the effect of the length to diameter ratio of the input section upon the limiting angle of the diffuser head has been showed. The results of two numerical and experimental methods show satisfactory agreement (particularly, in the proportionalities of length to entrance diameter of more than 10). The consequences of these two approaches indicate that if the length proportionalities to entrance diameter decrease from about 10, limiting diverging angle (largest angle before fluid separation from wall) increase quickly.

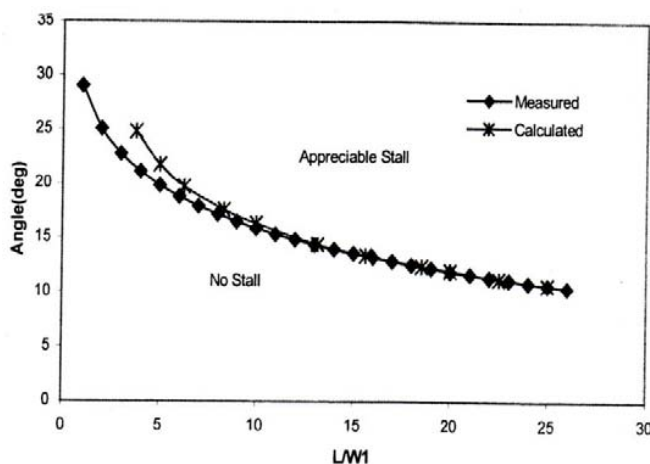


Fig. 1 Comparison of numerical calculations and experimental results for limiting diverging angles

Fig. 2 presents the results of the pressure recovery coefficient ( $C_{PR}$ ) defined as  $C_{PR} = 1 - (U_2/U_1)^2$  ideally by using Kline and Fox [15] (as  $U_1$ ,  $U_2$  are respectively the velocities of input and output sections). The results of the experimental measurements [15] and the results of the ideal conditions for the length to diameter ratio of the input equal to 10 have been compared. The ideal condition results and the numerical results are very close and a little more than the experimental results. The difference between the experimental results with two different methods can be due to not considering the effects of input and output sections and also the influence of the roughness of the diffuser experimental walls that it leads to wasting of energy. As a whole, these three methods have same changes process.

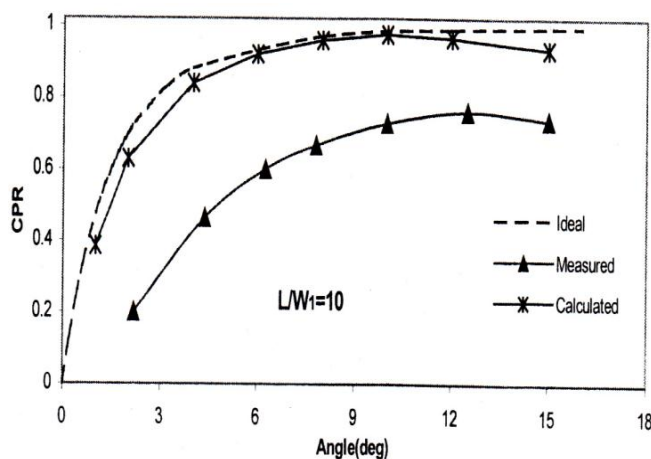


Fig. 2 Comparison of numerical calculations and experimental  $C_{PR}$  results for different diverging angles

In rockets and launch systems the propulsion force is driven by the thrust chamber which includes combustion chamber and diffuser. The Figs. (3, 4, 5) have presented velocity contours, isothermal lines and mass friction contours of Nox inside the combustion chamber (converging), diverging flat diffuser, and outside the diffuser (inside the static air).

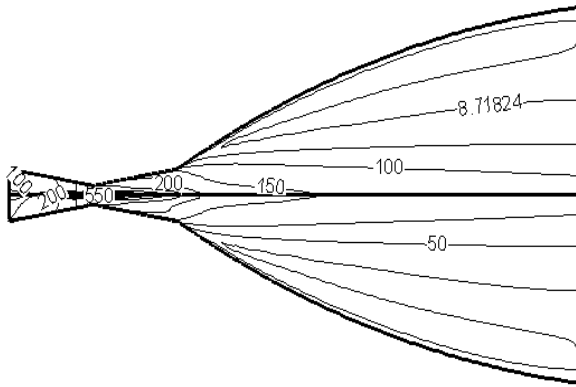


Fig. 3 Axial velocity contours inside and outside the converging and diverging duct (m/s)

Fig. 3 indicates that the maximum velocity of combustive gasses occurs in the location of the combustion chamber linking with diffuser and a desired symmetry for velocity distribution inside and outside of the thrust combustion chamber is observed which help can to the stability of the launch system.

In Fig. 4 the temperature reach to the maximum level around the symmetry axis, that is in the place of mixing the fuel and the ingoing air. The calculations indicated that if the combustion occurs in rich conditions, some quantity of the fuel is combusted in the output place of the diffuser. And the in this zone the temperature increases.

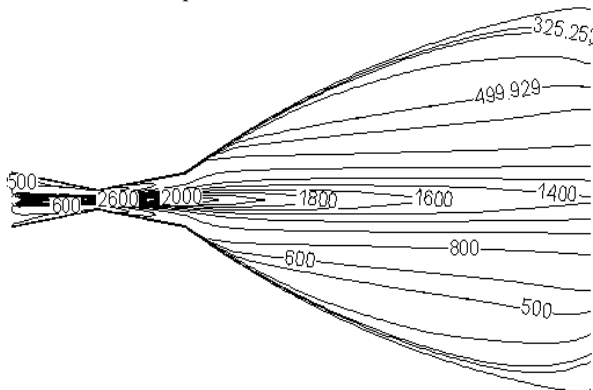


Fig. 4 Isothermal lines inside and outside the converging and diverging duct (K)

Due to the intense dependence of Nox pollutant to the maximum temperature in the domain with this feature, the Nox pollutant amount is also in maximum level (Fig. 5).

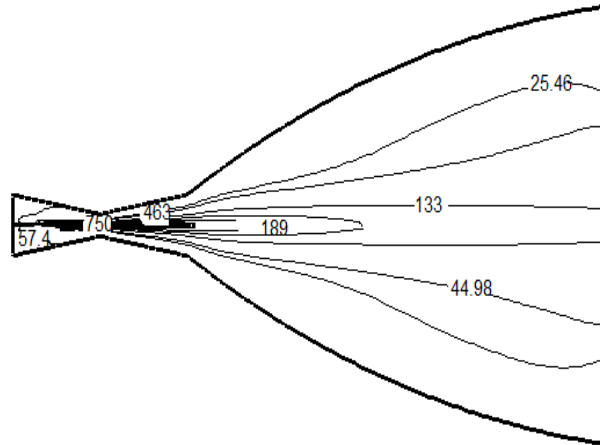


Fig. 5 Mass fraction of Nox inside and outside the converging and diverging duct (ppm)

The effect of the variation of diverging angle on velocity in converging and diverging duct is shown in Fig. 6. The result show that increasing diverging angle led to increasing of velocity in the combustion chamber (converging) and reducing of velocity in the flat diffuser. Also with increasing of diverging angle, combustion rate will increase (Fig. 7).

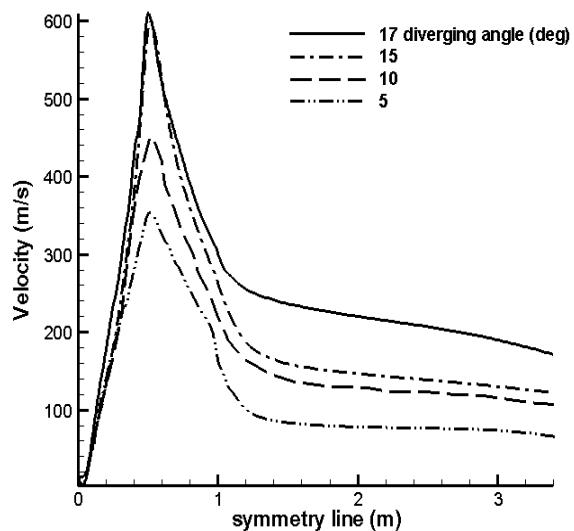


Fig. 6 The effect of the variation of diverging angle on velocity in converging and diverging duct

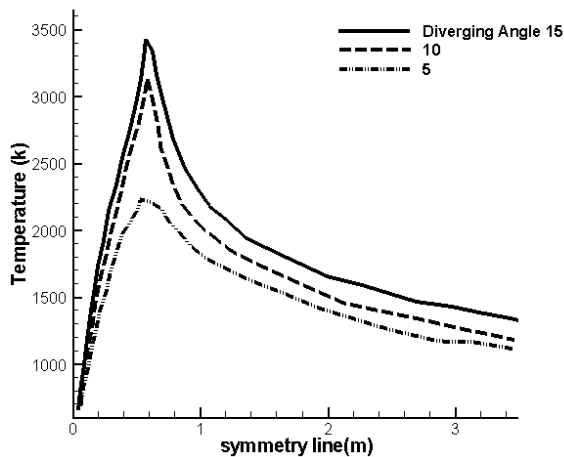


Fig. 7 The effect of the variation of diverging angle on temperature in converging and diverging duct

As was mentioned above, due to the intense dependence of Nox pollutant to the maximum temperature, with increasing of combustion rate, mass fraction of Nox pollutant will increase (Fig. 8).

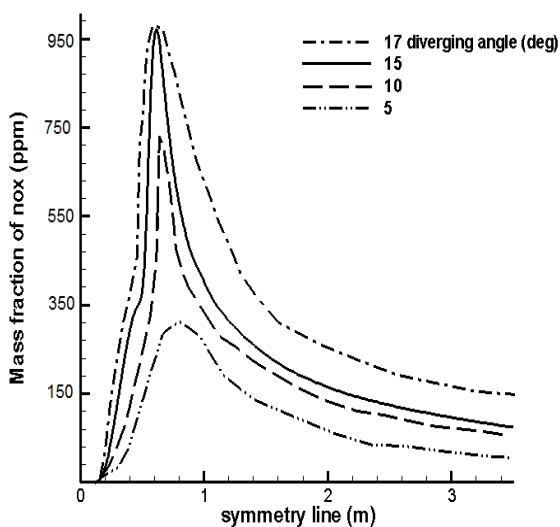


Fig. 8 The effect of the variation of diverging angle on mass fraction of nox in converging and diverging duct

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