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Study on Electrohydrodynamic Capillary Instability with Heat and Mass Transfer

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Abstract—The effect of an axial electric field on the capillary instability of a cylindrical interface in the presence of heat and mass transfer has been investigated using viscous potential flow theory. In viscous potential flow, the viscous term in Navier-Stokes equation vanishes as vorticity is zero but viscosity is not zero. Viscosity enters through normal stress balance in the viscous potential flow theory and tangential stresses are not considered. A dispersion relation that accounts for the growth of axisymmetric waves is derived and stability is discussed theoretically as well as numerically. Stability criterion is given by critical value of applied electric field as well as critical wave number. Various graphs have been drawn to show the effect of various physical parameters such as electric field, heat transfer capillary number, conductivity ratio, permittivity ratio on the stability of the system. It has been observed that the axial electric field and heat and mass transfer both have stabilizing effect on the stability of the system.

Keywords—Capillary instability, Viscous potential flow, Heat and mass transfer, Axial electric field.

I. Introduction

Capillary instability arises when a fluid cylinder in an infinite fluid collapses under the action of capillary forces due to surface tension [16], [19]. The capillary instability occurs in various situations such as film boiling, Breaking of liquid jet and in many Chemical and Metallurgical processes. The study of heat and mass transfer across the interface is very important in many situations such as boiling heat transfer in chemical engineering and in geophysical problems. The general formulation of the interfacial flow problem of two inviscid incompressible fluids with heat and mass transfer for Rayleigh-Taylor and Kelvin-Helmholtz instabilities in plane geometry was established by Hsieh [7], [8]. Hsieh [8] found that when the vapour layer is hotter than the liquid layer, the effect of heat and mass transfer tends to inhibit the growth of instability. Nayak and Chakraborty [5] established the formulation of Kelvin-Helmholtz instability of the cylindrical interface between the liquid and vapour phases with heat and mass transfer.

Viscous potential flow theory has played an important role in studying various stability problems. In viscous potential flow, we consider irrotational flow, so the viscous term i.e.$\mu\nabla^2\mathbf{u}$ in the Navier-Stokes equation is identically zero when the vorticity is zero but the viscous stresses are not zero, where μ denotes viscosity and $\mathbf{u}$ denotes velocity of fluid flow. Tangential stresses are not considered in viscous potential theory and viscosity enters through normal stress balance [6]. Funada and Joseph [20] studied the viscous potential flow analysis of capillary instability and observed that viscous potential flow is better approximation of the exact solution than the inviscid model. Funada and Joseph [21] extended their work of capillary instability for viscoelastic fluids and observed that the growth rates are larger for viscoelastic fluids than for the equivalent Newtonian fluids.

Viscous potential flow analysis of Kelvin-Helmholtz instability with heat and mass transfer in plane geometry has been carried out by Asthana and Agrawal [17]. They observed that heat and mass transfer has a strong stabilizing effect when the lower fluid is highly viscous and a weak destabilizing effect when the fluid's viscosity is low. Kim et al. [10] investigated the capillary instability problem of vapour liquid system in an annular configuration with heat and mass transfer using viscous potential flow for axisymmetric disturbances. They observed that for irrotational motion of two viscous fluids, heat and mass transfer phenomenon completely stabilizes the interface against capillary effects.

As the electric field plays an important role in many practical problems of chemical engineering and other related fields, there is increasing interests in the study of electrohydrodynamic instability. The capillary instability with heat and mass transfer in an electric field occurs in many practical applications such as ink jet printers, paint spraying, fuel atomization, and etc. An active enhancement of heat transfer by electrohydrodynamics is used in industries including heat exchange manufacturing and power generation. Elhefnawy et al. [3] studied the nonlinear electrohydrodynamic stability of a finitely conducting jet under an axial electric field and observed that the uniform axial electric field has a stabilizing influence. Nonlinear streaming instability of cylindrical structures in finitely conducting fluids under the influence of a radial electric field has been studied by Elhefnawy et al. [4]. Elsayed et al. [11] have studied the effect of general electric field on conducting liquid jets instabilities in the presence of heat and mass transfer. They have observed that the heat and mass transfer has no effect on the stability of the system in the presence of axial electric field as well as radial electric field. Stability of cylindrical conducting fluids with heat and mass transfer in longitudinal periodic electric field has been studied by Elsayed et al. [12]. Moatimid [9] investigated the electrohydrodynamic stability of two inviscid fluids with heat and mass transfer in cylindrical configuration. The two liquid phases were enclosed between two cylindrical surfaces coaxial

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with the interface admitting heat and mass transfer. It was observed that uniform electric field has stabilizing effect. It was also found that the instability criterion is independent of heat and mass transfer coefficient. Elhefnawy and Moatimid [2] have studied the effect of an axial electric field on the stability of cylindrical flows in the presence of mass and heat transfer and absence of gravity. They observed that the electric field has strong stabilizing influence for all short and long wavelengths. Elcoot [1] has studied the nonlinear analysis of capillary instability of viscous fluids in the presence of axial electric field.

Recently, Asthana and Agrawal [18] have studied the viscous potential flow analysis of electrohydrodynamic Kelvin-Helmholtz instability at the plane interface and concluded that the tangential electric field has stabilizing effect on the critical value of relative velocity while relative velocity has destabilizing effect on the critical value of electric field. Awasthi et al. [14] have studied the effect of irrotational shearing stresses on the capillary instability in the presence of heat and mass transfer and found that irrotational shearing stresses stabilize the system. Awasthi and Agrawal [13] has studied the viscous contribution to the pressure for the potential flow analysis of capillary instability with axial electric field and observed that the axial electric field has stabilizing effect on the stability of the system. Awasthi and Asthana [15] have studied the effect of porous medium on the capillary instability when there is heat and mass transfer across the interface and observed that porous medium have stabilizing effect.

In the present article, viscous potential flow analysis of capillary instability with heat and mass transfer in the presence of an axial electric field has been carried out for axisymmetric disturbances. Both the fluids are taken as incompressible, viscous and conducting with different kinematic viscosities, conductivities and permittivities, respectively, which have not been considered earlier. The effect of gravity and free surface charges at the interface is neglected. A dispersion relation is derived and stability is discussed theoretically as well as numerically. A critical value of the electric field as well as the critical wave number is obtained. The effect of the electric field and heat and mass transfer on growth rates is studied. The effect of ratio of electrical conductivities and ratio of permittivity of fluids on stability of the system is also studied and shown graphically. Various neutral curves have been drawn to show the effect of various physical parameters such as electric field, heat transfer capillary number on the stability of the system.

II. PROBLEM FORMULATION

A system of two incompressible and viscous fluids, separated by a cylindrical interface, is considered in an annular configuration as shown in Fig. 1. The undisturbed cylindrical interface is taken at radius R. In the formulation the subscript 1 and 2 denote variables associated with the fluid inside and outside the interface, respectively. In the undisturbed state, viscous fluid of thickness h_1, density ρ_1, viscosity μ_1, electrical conductivity σ_1 and permittivity ε_1 occupies the inner region $R_1 < r < R$ and viscous fluid of thickness h_2, density ρ_2, viscosity μ_2, electrical conductivity σ_2 and permittivity ε_2 occupies the outer region $R < r < R_2$. The bounding surfaces $r = R_1$ and $r = R_2$ are considered to be rigid. The temperatures at $r = R_1, r = R$ and $r = R_2$ are T_1, T_0 and T_2, respectively. Both the fluids are assumed to be incompressible and irrotational. A cylindrical system of coordinates (r, θ, z) is assumed so that in the equilibrium state z-axis is the axis of symmetry of the system. Small axisymmetric disturbances are superimposed on the basic rest state. After disturbance, the interface is given by

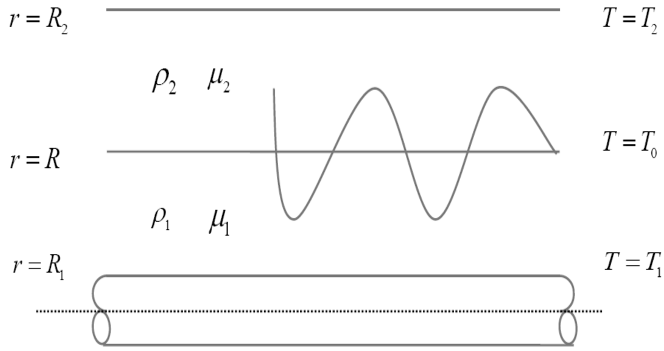

Fig. 1. Equilibrium configuration of the system.

$$F(r, z, t) = r - R - \eta(z, t) = 0 \tag{1}$$

where η is the perturbation in the radius of the interface from the equilibrium value R, and for which the outward unit normal vector is given by

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \left\{ 1 + \left(\frac{\partial \eta}{\partial z} \right)^2 \right\}^{-1/2} \left(\mathbf{e_r} - \frac{\partial \eta}{\partial z} \mathbf{e_z} \right) \tag{2}$$

where $\mathbf{e_r}$ and $\mathbf{e_z}$ are unit vectors along the r and z directions, respectively.

The velocity is expressed as the gradient of the potential function and the potential functions satisfy the Laplace equation as a consequence of the incompressibility constraint. i.e.

$$\nabla^2 \phi_j = 0 \qquad \text{for } (j = 1, 2) \tag{3}$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$.

The two fluids are subjected to an external electric field E_0, acting along z-axis i.e. $\mathbf{E} = E_0 \mathbf{e}_z$.

It is assumed that the quasi-static approximation is valid for the problem, hence the electric field can be derived from electric scalar potential function $\psi(r, z, t)$ such that

$$\mathbf{E}_j = E_0 e_z - \nabla \psi_j, \qquad (j = 1, 2) \tag{4}$$

Gauss's law requires that the electric potentials also satisfy Laplace's equation i.e.

$$\nabla^2 \psi_j = 0, \qquad (j = 1, 2) \tag{5}$$

The boundary conditions at the rigid cylindrical surfaces $r = R_1$ and $r = R_2$ are given by

$$\frac{\partial \phi_j}{\partial r} = 0 \qquad at \qquad r = R_j \qquad (j = 1, 2) \tag{6}$$

$$\frac{\partial \psi_j}{\partial z} = 0 \qquad at \qquad r = R_j \qquad (j = 1, 2) \tag{7}$$

It is assumed that phase-change takes place locally in such a way the net phase-change rate at the interface is equal to zero. The interfacial condition, which expresses the conservation of mass across the interface, is given by the equation (Hsieh [8])

$$\|\rho \left(\frac{\partial F}{\partial t} + \nabla \phi \cdot \nabla F \right) \| = 0 \qquad at \qquad r = R + \eta \tag{8}$$

where $\|X\|$ represents the difference in a quantity across the interface, it is defined as $\|X\| = X_1 - X_2$.
The tangential component of the electric field must be continuous across the interface i.e.

$$\|E_t\| = 0 \tag{9}$$

where $E_t (= |\mathbf{n} \times \mathbf{E}|)$ is the tangential component of the electric field.
There is discontinuity in the normal current across the interface; charge accumulation within a material element is balanced by conduction from bulk fluid on either side of the surface. The boundary condition, corresponding to normal component of the electric field, at the interface is given by

$$\|E_n\| = 0 \tag{10}$$

where $E_n (= \mathbf{n} \cdot \mathbf{E})$ is the normal component of the electric field.
The interfacial condition for energy transfer proposed by Hsieh [8] can be expressed as

$$L\rho_1 \left(\frac{\partial F}{\partial t} + \nabla \phi_1 \cdot \nabla F \right) = S(\eta) \qquad at \qquad r = R + \eta \tag{11}$$

where L is the latent heat released during phase transformation and $S(\eta)$ denotes the net heat flux from the interface. In deriving equation (11), Hsieh [8] assumed that the amount of latent heat released depends mainly on the instantaneous position of the interface.
In the equilibrium state, the heat fluxes in positive radial-direction in the fluid phases 1 and 2 are $-K_1 (T_1 - T_0)/R \ln (R_1/R)$ and $-K_2 (T_0 - T_2)/R \ln (R/R_2)$ respectively where K_1 and K_2 denote the heat conductivities of the two fluids. The net heat flux $S(\eta)$ is expressed as (Nayak and Chakraborty [5])

$$S(\eta) = \frac{K_2 (T_0 - T_2)}{(R + \eta) [\ln R_2 - \ln (R + \eta)]} - \frac{K_1 (T_1 - T_0)}{(R + \eta) [\ln (R + \eta) - \ln R_1]} \tag{12}$$

Expanding $S(\eta)$ about $\eta = 0$ as

$$S(\eta) = S(0) + \eta S'(0) + \frac{1}{2}\eta^2 S''(0) + \frac{1}{6}\eta^3 S'''(0) + \ldots \tag{13}$$

Since $S(0) = 0$, from equation (12) we get

$$\frac{K_2 (T_0 - T_2)}{R \ln (R_2/R)} = \frac{K_1 (T_1 - T_0)}{R \ln (R/R_1)} = G(\text{say}) \tag{14}$$

Hence in the equilibrium state, heat fluxes across the interfaces are equal. The interfacial condition for conservation of momentum is given by;

$$\begin{aligned} &\rho_1 (\nabla \phi_1 \cdot \nabla F) \left(\tfrac{\partial F}{\partial t} + \nabla \phi_1 \cdot \nabla F \right) = \rho_2 (\nabla \phi_2 \cdot \nabla F) \times \\ &\left(\tfrac{\partial F}{\partial t} + \nabla \phi_2 \cdot \nabla F \right) + (p_2 - p_1 + 2 [\![\mu\, \mathbf{n} \cdot \{ (\mathbf{n} \cdot \nabla) \nabla \phi \}]\!] \\ &\quad + \tfrac{1}{2} [\![\varepsilon \left(E_n^2 - E_t^2 \right)]\!] + T\, \nabla \cdot \mathbf{n}) |\nabla F|^2 \end{aligned} \tag{15}$$

where p represents the pressure and T denotes the surface tension. Surface tension has been assumed to be a constant, neglecting its dependence on temperature. Pressure can be obtained using Bernoulli's equation.

III. Linearized equations

It has been observed that the asymmetric disturbances are always stable for capillary instability. A long cylinder of liquid is unstable to the axisymmetric disturbances with wavelengths greater than $2\pi R$, where R is the radius of the cylinder. Hence, we considered only axisymmetric disturbances in this analysis. Now, axisymmetric disturbances are imposed on the equations (8), (9), (10), (11) and (15) and retaining the linear terms we can get the following equations.

$$\|\rho \left(\frac{\partial \phi}{\partial r} - \frac{\partial \eta}{\partial t} \right) \| = 0 \tag{16}$$

$$\| \frac{\partial \psi}{\partial z} \| = 0 \tag{17}$$

$$\|\sigma \left(\frac{\partial \psi}{\partial r} + E_0 \frac{\partial \eta}{\partial z} \right) \| = 0 \tag{18}$$

$$\left[\rho_1 \left(\frac{\partial \phi_1}{\partial r} - \frac{\partial \eta}{\partial t} \right) \right] = \alpha\, \eta \tag{19}$$

$$\|\rho \frac{\partial \phi}{\partial t} + E_0 \varepsilon \frac{\partial \psi}{\partial z} + 2\mu \frac{\partial^2 \psi}{\partial r^2} \| = T \left(\frac{\eta}{R^2} + \frac{\partial^2 \eta}{\partial z^2} \right) \tag{20}$$

where equation (19) is obtained using equations (12)-(14) with equation (11) and

$$\alpha = \frac{G}{LR} \frac{\ln(R_2/R_1)}{\ln(R/R_1) \ln(R_2/R)}$$

Now the normal mode technique is used to find the solution of the governing equations. Let

$$\eta = A\, e^{i\,(k\,z - \omega\,t)} + c.c. \tag{21}$$

where A represents the amplitude of the surface wave, k denotes the real wave number, ω is the growth rate and $c.c.$ refers the complex conjugate of the preceding term.
On solving equations (3) and (5) with the help of boundary conditions we get

$$\phi_j = \left(-\frac{\alpha}{\rho_j} + i\,\omega \right) \left(\frac{I_0(k\,r) K_0'(k\,R_j) - I_0'(k\,R_j) K_0(k\,r)}{D_j(k)} \right) A\, e^{i\,(k\,z - \omega\,t)} + c.c., \quad (j = 1, 2) \tag{22}$$

$$\psi_1 = \frac{i\,k\,(\sigma_2-\sigma_1)\,E_0\,g_2(k)}{\sigma_1 g_2(k)\,G_1(k)-\sigma_2 g_1(k)\,G_2(k)}$$
$$[I_0(k\,r)K_0(k\,R_1)-I_0(k\,R_1)K_0(k\,r)]\;Ae^{i\,(k\,z-\omega\,t)}+c.c. \quad (23)$$

$$\psi_2 = \frac{i\,k\,(\sigma_2-\sigma_1)\,E_0\,g_1(k)}{\sigma_1 g_2(k)\,G_1(k)-\sigma_2 g_1(k)\,G_2(k)}$$
$$[I_0(k\,r)K_0(k\,R_2)-I_0(k\,R_2)K_0(k\,r)]\;Ae^{i\,(k\,z-\omega\,t)}+c.c. \quad (24)$$

where

$$D_j(k)=I_0'(k\,R_j)K_0'(k\,R)\,-I_0'(k\,R)K_0'(k\,R_j),$$
$$g_j(k)=I_0(k\,R_j)K_0(k\,R)-I_0(k\,R)K_0(k\,R_j),$$
$$G_j(k)=I_0'(k\,R)K_0(k\,R_j)-I_0(k\,R_j)K_0'(k\,R),\ (j=1,2)$$

and symbols I_0 and K_0 are modified Bessel functions of first and second kind respectively and prime on modified Bessel functions denotes the differentiation with respect to r when $r=R, R1$ or $R2$.

IV. Dispersion relation

Substituting the values of $\eta, \phi_1,\ \phi_2,\ \psi_1$ and ψ_2 in equation (20) we get the dispersion relation

$$D\,(\omega,k)=a_0\omega^2+i\,a_1\omega-a_2=0 \quad (25)$$

where

$$a_0=\frac{\rho_1\,M_1(k)}{D_1(k)}-\frac{\rho_2\,M_2(k)}{D_2(k)}$$

$$a_1=\alpha\left(\frac{M_1(k)}{D_1(k)}-\frac{M_2(k)}{D_2(k)}\right)+\frac{2\mu_1\,N_1(k)}{D_1(k)}-\frac{2\mu_2\,N_2(k)}{D_2(k)}$$

$$a_2=\frac{2\alpha\mu_1}{\rho_1}\frac{N_1(k)}{D_1(k)}-\frac{2\alpha\mu_2}{\rho_2}\frac{N_2(k)}{D_2(k)}+\frac{T}{R^2}(k^2R^2-1)$$
$$-\frac{k^2E_0^2g_1(k)g_2(k)(\varepsilon_1-\varepsilon_2)(\sigma_1-\sigma_2)}{\sigma_1 g_2(k)\,G_1(k)-\sigma_2 g_1(k)\,G_2(k)},$$
$$M_j=I_0'(k\,R_j)K_0(k\,R)-I_0(k\,R)K_0'(k\,R_j),$$
$$N_j=I_0'(k\,R_j)K_0''(k\,R)-I_0''(k\,R)K_0'(k\,R_j).$$

After using the transformation $\omega\ =\ i\,\omega_0$, the dispersion relation is obtained in growth rate ω_0

$$a_0\omega_0^2+a_1\omega_0+a_2=0 \quad (26)$$

Neutral curves are obtained by putting $\omega_0(k)$. Equation (26) reduces to $a_2=0$, which in turn implies that

$$\frac{2\alpha\mu_2}{\rho_2}\frac{N_2(k_c)}{D_2(k_c)}-\frac{2\alpha\mu_1}{\rho_1}\frac{N_1(k_c)}{D_1(k_c)}-\frac{T}{R^2}(k_c^2R^2-1)$$
$$=\frac{k_c^2(E_0)_c^2g_1(k_c)g_2(k_c)(\varepsilon_1-\varepsilon_2)(\sigma_1-\sigma_2)}{\sigma_2 g_1(k)\,G_2(k)-\sigma_1 g_2(k)\,G_1(k)} \quad (27)$$

where k_c is the critical wave number. For $E_0\ =\ 0$, equation (26) is reduced to dispersion relation as obtained by Kim et al. [10]. In equation (26) putting $\alpha\ =\ 0$ we get the dispersion relation as obtained by Elcoot [1] for his linear theory. Choosing $\mu_1\ =\ 0,\ \mu_2\ =\ 0,\ \alpha\ =\ 0,\ R_1\ \to\ 0,\ R_2\ \to\ \infty$ and $E_0\ =\ 0$, the dispersion relation (26) reduces to form $\omega_0^2\ =\ \frac{T(1-x^2)}{R^3}\left[\frac{xI_1(x)K_1(x)}{\rho_1 I_0(x)K_1(x)+\rho_2 I_1(x)K_0(x)}\right]$, using the results $I_0'(x)\ =\ I_1(x),\ K_0'(x)\ =\ -K_1(x),\ x\ =\ kR,\ \lim_{R_1\to 0}K_0'(kR_1)\ \to\ \infty,\ \lim_{R_2\to\infty}I_0'(kR_2)\ \to\ \infty$. Here the condition for stability is $x>1$ which is well known Rayleigh criteria for a cylindrical jet.

V. Dimensionless form of the dispersion relation

Introducing dimensionless variables

$$\hat r=r/H,\ \ \hat z=z/H,\ \ \hat\eta=\eta/H,\hat t=t/\tau,\ \ \hat\omega_0=\omega_0\tau,$$
$$\hat k=kH,\hat h=h_1/H,\hat R=\hat R_1+\hat h,\ \ \hat E^2=\frac{\varepsilon_2E_0^2H}{T},$$

where the length scale H and time scale τ are defined as $H=R_2-R_1,\ \ \tau=\sqrt{\rho_2H^3/T}$.
Also

$$\rho=\frac{\rho_1}{\rho_2},\mu=\frac{\mu_1}{\mu_2},\kappa=\frac{\upsilon_1}{\upsilon_2},\upsilon_1=\frac{\mu_1}{\rho_1},\ \upsilon_2=\frac{\mu_2}{\rho_2},\sigma=\frac{\sigma_1}{\sigma_2},\varepsilon=\frac{\varepsilon_1}{\varepsilon_2},$$
$$Oh=\frac{\sqrt{\rho_2TH}}{\mu_2},\ \ Ca=\frac{\alpha}{\rho_2/\tau},$$

where Oh is the Ohnesorge number, Ca is the heat transfer capillary number, $\hat h$ is the inner fluid fraction and κ is the kinematic viscosity ratio.
Eliminating the $'\wedge'$ on the dimensionless variables for brevity, the dimensionless form of equation (26) is

$$\left[\left(\rho\frac{M_1(k)}{D_1(k)}-\frac{M_2(k)}{D_2(k)}\right)\right]\omega_0^2+\left[Ca\left(\frac{M_1(k)}{D_1(k)}-\frac{M_2(k)}{D_2(k)}\right)\right.$$
$$\left.+\frac{2}{Oh}\left(\mu\frac{N_1(k)}{D_1(k)}-\frac{N_2(k)}{D_2(k)}\right)\right]\omega_0+\left[\frac{2Ca}{Oh}\left(\kappa\frac{N_1(k)}{D_1(k)}-\frac{N_2(k)}{D_2(k)}\right)\right.$$
$$\left.+\frac{1}{R^2}(k^2R^2-1)+\frac{k^2E^2g_1(k)g_2(k)(\varepsilon-1)(\sigma-1)}{g_1(k)\,G_2(k)-\sigma g_2(k)\,G_1(k)}\right]=0 \quad (28)$$

The expression for neutral curves becomes

$$2Ca\left(\frac{N_2(k_c)}{D_2(k_c)}-\kappa\frac{N_1(k_c)}{D_1(k_c)}\right)\frac{1}{Oh}-\frac{1}{R^2}(k_c^2R^2-1)$$
$$=\frac{k_c^2E_c^2g_1(k_c)g_2(k_c)(\varepsilon-1)(\sigma-1)}{g_1(k_c)\,G_2(k_c)-\sigma g_2(k_c)\,G_1(k_c)} \quad (29)$$

Applying Routh Hurwitz criterion, it is concluded that the system is stable for $E\geq Ec$ and unstable for $E<Ec$.
From equation (28) we can get the value of maximum growth rate $(\omega_0)_m$ and corresponding wave number k_m and the critical wave number k_c can be obtained using equation (29).

VI. Results and discussions

Following parametric values have been considered for the system of interest containing vapour in the inner region and liquid in the outer region.

$$\rho_1=0.0012gm/cm^3,\rho_2=1.0gm/cm^3,\mu_1=0.00018poise,$$
$$\mu_2=0.01poise,T=60.0dyne/cm.$$

The diameters of the inner and outer cylinders are taken as 1 cm and 2 cm, respectively. The conductivity ratio σ and permittivity ratio ε are taken as 0.2 and 0.01, respectively for numerical calculations, otherwise mentioned. At the interface, phase change is taking place. Neutral curves divide the plane into a stable region denoted by S (above the curve) and an unstable region denoted by U (below the curve). In the following the effect of various physical parameters on the onset of instability is interpreted through various Figures and Tables.

In Fig. 2, the neutral curves for the critical wave number k_c versus vapor fraction h have been drawn for various values of heat transfer capillary number Ca when there is

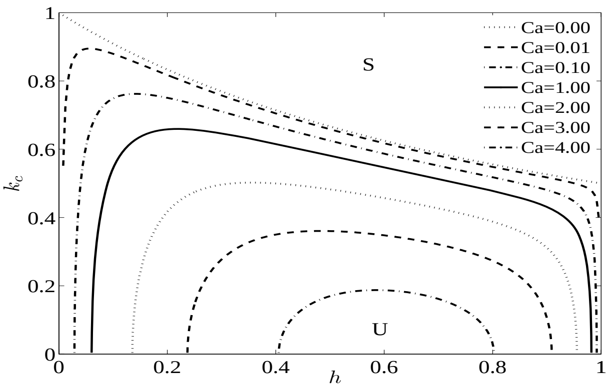

Fig. 2. The neutral curve k_c vs h for different values of Ca when $E = 0$.

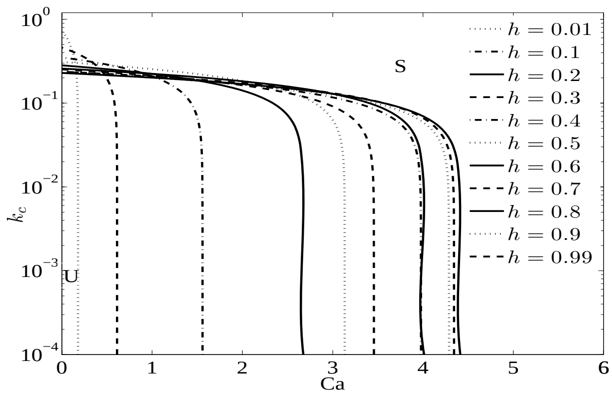

Fig. 5. The neutral curve k_c vs Ca for different values of h when $E = 5$.

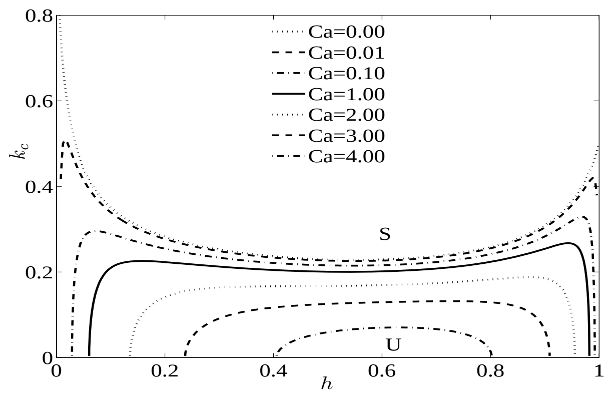

Fig. 3. The neutral curve k_c vs h for different values of Ca when $E = 5$.

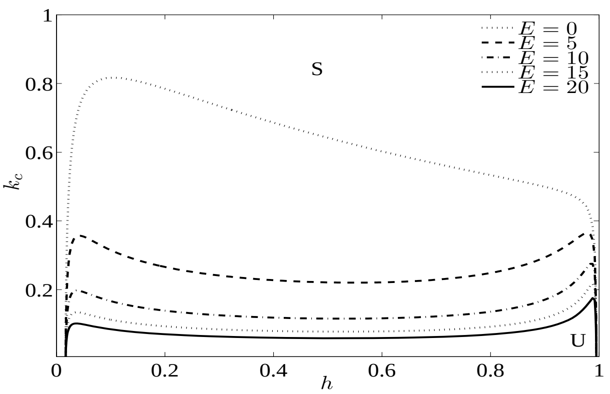

Fig. 4. The neutral curve k_c vs h for different values of E when $Ca = 0.3$.

no electric field i.e. $E = 0$. It has been observed that as Ca increases, the stable region (upper region) grows. Therefore, Ca has a stabilizing effect on the stability of the system. The effect of heat and mass transfer on the stability of the system can be explained in terms of local evaporation and condensation at the interface. At a perturbed interface, crests are warmer because they are closer to the hotter boundary on the vapour side, thus local evaporation takes place, whereas troughs are cooler and thus condensation takes place. The liquid is protruding to a hotter region and the evaporation will diminish the growth of disturbance waves. Fig. 3 shows the neutral curves for the critical wave number k_c versus vapor fraction h for various values of heat transfer capillary number Ca when electric field intensity $E = 5$. The heat and mass transfer phenomenon has stabilizing effect on the stability of the system even in the presence of electric field and this effect is enhanced in the presence of an electric field. For a fixed value of vapour thickness h, on increasing Ca, the critical wave number kc decreases and finally vanishes at threshold Ca.

The neutral curves for critical wave number k_c versus vapor fraction h for various values of electric field intensity E at heat transfer capillary number $Ca = 0.3$ have been shown in Fig. 4. It has been observed that for a fixed value of h and Ca, the critical wave number k_c decreases on increasing electric field intensity E. Therefore, it is concluded that E has stabilizing effect. The variation of the critical wave number for the different values of vapour fraction is illustrated in Fig. 5. As vapour thickness increases, at the crests more evaporation will take place. This additional evaporation will increase the amplitude of the disturbance waves and the system becomes destabilized as observed from Fig. 5.

In Tables I and II, maximum growth rates $(\omega_0)_m$, corresponding wave numbers k_m and critical wave numbers k_c as a function of heat transfer capillary number Ca have been shown for different values of vapour fraction h at various values of electric field intensity E. It is observed that the maximum growth rates decrease on increasing Ca and growth rates vanish at certain Ca, known as threshold Ca. This threshold Ca remains same as the electric field increases at some fixed value of h.

In Fig. 6, the neutral curves for the electric field intensity E versus wave number k have been plotted for various values of heat transfer capillary number Ca when vapor thickness

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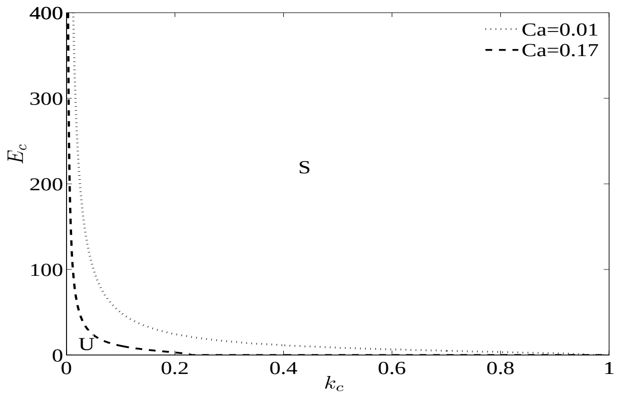

Fig. 6. The neutral curve E_c vs k_c for different values of Ca when $h = 0.01$.

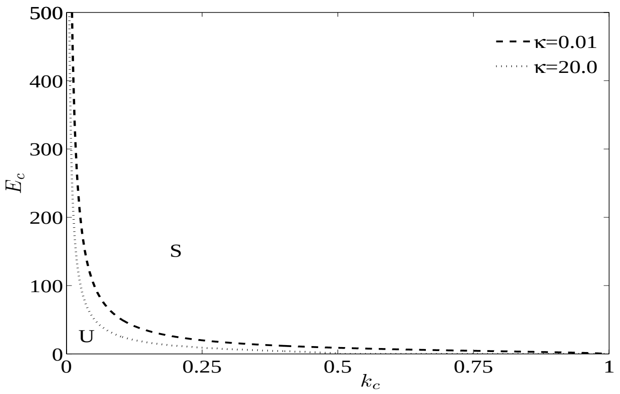

Fig. 7. The neutral curve E_c vs k_c for different values of κ for $Ca = 0.1$ at $h = 0.01$.

TABLE I
MAXIMUM GROWTH RATE $(\omega_0)_m$, CORRESPONDING WAVE NUMBER k_m AND CRITICAL WAVE NUMBER k_c FOR DIFFERENT VALUE OF E AT $h = 0.01$.

	$E = 0$			$E = 5$		
Ca	$(\omega_0)_m$	k_m	k_c	$(\omega_0)_m$	k_m	k_c
0.00	0.5019	0.6707	0.9901	0.3759	0.4940	0.7145
0.02	0.0897	0.6586	0.9333	0.0477	0.4739	0.6734
0.04	0.0355	0.6185	0.8729	0.0185	0.4458	0.6297
0.06	0.0175	0.5703	0.8080	0.0091	0.4137	0.5827
0.08	0.0091	0.5221	0.7374	0.0047	0.3775	0.5317
0.10	0.0047	0.4659	0.6592	0.0024	0.3373	0.4753
0.12	0.0022	0.4016	0.5705	0.0011	0.2892	0.4112
0.14	0.0008	0.3293	0.4651	0.0004	0.2369	0.3352
0.16	0.0002	0.2329	0.3275	0.0001	0.1687	0.2360
0.18	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE II
MAXIMUM GROWTH RATE $(\omega_0)_m$, CORRESPONDING WAVE NUMBER k_m AND CRITICAL WAVE NUMBER k_c FOR DIFFERENT VALUE OF E AT $h = 0.1$.

	$E = 0$			$E = 5$		
Ca	$(\omega_0)_m$	k_m	k_c	$(\omega_0)_m$	k_m	k_c
0.0	0.4306	0.6225	0.9901	0.1756	0.2450	0.3509
0.2	0.0557	0.5984	0.8485	0.0085	0.2329	0.3274
0.4	0.0206	0.5502	0.7834	0.0031	0.2129	0.3021
0.6	0.0095	0.5020	0.7124	0.0014	0.1928	0.2747
0.8	0.0045	0.4458	0.6335	0.0007	0.1727	0.2442
1.0	0.0019	0.3815	0.5435	0.0003	0.1486	0.2094
1.2	0.0007	0.3092	0.4354	0.0001	0.1205	0.1677
1.4	0.0001	0.2048	0.2895	0.00001	0.0803	0.1115
1.6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

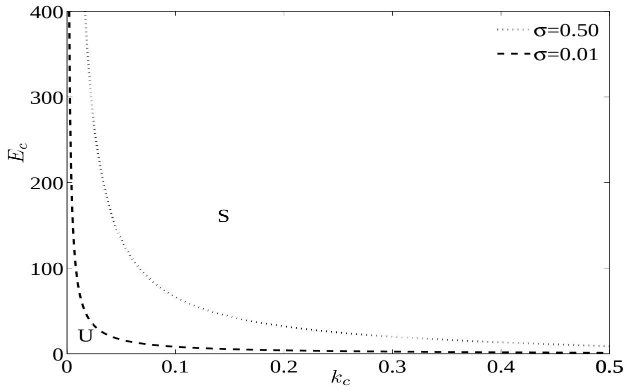

Fig. 8. The neutral curve E_c vs k_c for different values of σ for $Ca = 0.1$ at $h = 0.01$.

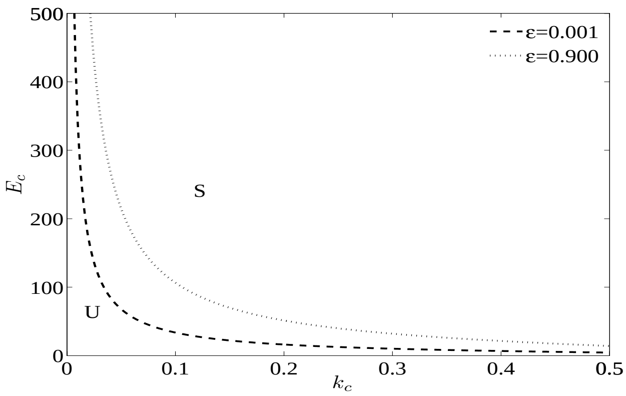

Fig. 9. The neutral curve E_c vs k_c for different values of ε for $Ca = 0.1$ at $h = 0.01$.

$h = 0.01$. A critical value of the electric field has been obtained. If the electric field is less than this critical value, the system is unstable otherwise it is stable. It has been observed that the stable region increases on increasing the heat transfer coefficient Ca and hence it is concluded that Ca has stabilizing effect on the critical electric field. Variation of neutral curves for electric field for different values of kinematic viscosity ratio of two fluids κ have been shown in Fig. 7 for $Ca = 0.1$ and $h = 0.01$. It is concluded that the critical wave number k_c decreases with increasing κ . It is also found that for every E, the threshold κ remains same for some fixed value of Ca and h. As the kinematic

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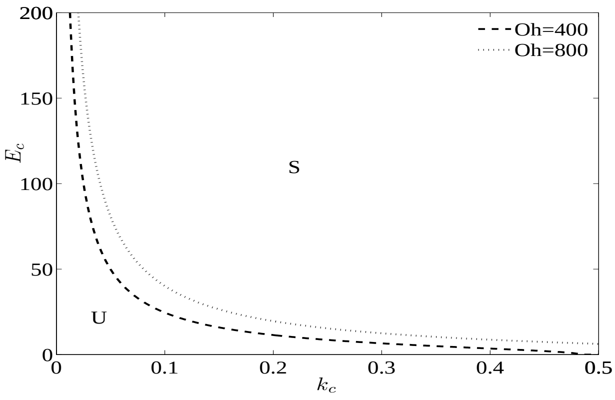

Fig. 10. The neutral curve E_c vs k_c for different values of Oh for $Ca = 0.1$ at $h = 0.01$.

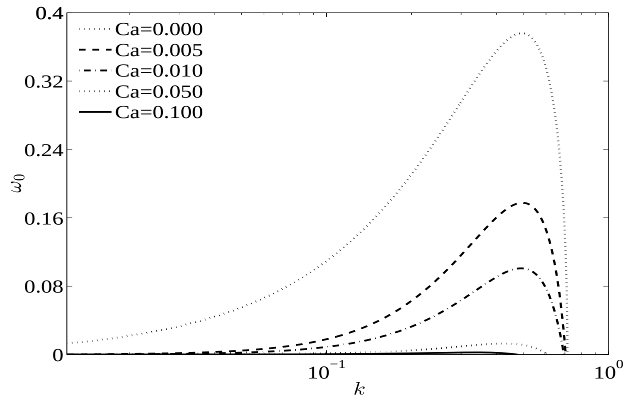

Fig. 12. The growth rate ω_0 vs k for $E = 5$ for different values of Ca at $h = 0.01$.

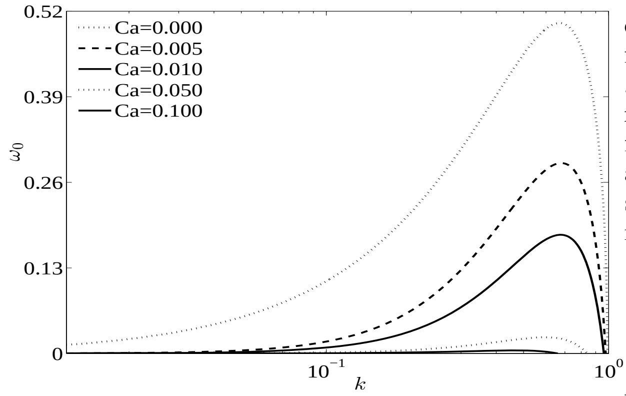

Fig. 11. The growth rate ω_0 vs k for $E = 0$ for different values of Ca at $h = 0.01$.

viscosity ratio κ increases, the viscosity of the inside fluid increases and hence κ has stabilizing effect on the stability of the system.

Variation of neutral curves for electric field for different values of the conductivity ratio σ have been shown in Fig. 8 for $Ca = 0.1$ and $h = 0.1$. The stable region decreases as σ increases and hence it is concluded that σ has destabilizing effect on the critical electric field. The variation of critical electric field for various values of the permittivity ratio ε for $Ca = 0.1$ and $h = 0.1$ have been shown in Fig. 9. It is observed that ε has destabilizing effect on the critical value of electric field.

The evolution of the neutral curves for electric field intensity E versus wave number k for different values of Ohnesorge number has been shown in Fig. 10. It has been observed that Ohnesorge number has destabilizing effect on the stability of the system. Through increasing Ohnesorge number, the viscosity of the outside fluid will decrease and less resistance to the fluid flow will take place. Therefore, the flow will become unstable.

In Figs. 11 and 12, the growth rate values have been compared for the electric field intensity $E = 0$ and 5, for different values of heat transfer capillary number Ca at $h = 0.01$. It is observed that on increasing Ca the growth rates decrease and growth rates in the presence of an electric field decrease faster than the growth rates in the absence of an electric field. It shows that heat and mass transfer has stabilizing effect on the stability of the system and this effect is enhanced in the presence of an electric field.

VII. Conclusion

Viscous potential flow analysis of capillary instability with heat and mass transfer in the presence of an axial electric field has been carried out. The dispersion relation is obtained which is a quadratic equation in growth rate. The stability condition is obtained by applying Routh-Hurwitz criterion for stability. A critical value of electric field as well as critical wave number is obtained. The system is unstable when the electric field is less than the critical value of electric field, otherwise it is stable. It is observed that the heat and mass transfer has stabilizing effect on the stability of the system and this effect is enhanced in the presence of an electric field. The heat and mass transfer completely stabilizes the interface against capillary effects even in the presence of an electric field. It is also observed that the axial electric field increases the stability of the system with or without heat and mass transfer. It is found that the ratio of electric conductivity has destabilizing effect on growth rate. The same nature of result is obtained for the effect of ratio of permittivity on growth rates. The heat and mass transfer, for inviscid fluids, has no effect on the stability of the system, while it has stabilizing effect on the stability for viscous fluids.

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