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Study of Unsteady Behaviour of Dynamic Shock Systems in Supersonic Engine Intakes

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***Abstract*—**An analytical investigation is performed to study the unsteady response of a one-dimensional, non-linear dynamic shock system to external downstream pressure perturbations in a supersonic flow in a varying area duct. For a given pressure ratio across a wind tunnel, the normal shock's location can be computed as per one-dimensional steady gas dynamics. Similarly, for some other pressure ratio, the location of the normal shock will change accordingly, again computed using one-dimensional gas dynamics. This investigation focuses on the small-time interval between the first steady shock location and the new steady shock location (corresponding to different pressure ratios). In essence, this study aims to shed light on the motion of the shock from one steady location to another steady location. Further, this study aims to create the foundation of the Unsteady Gas Dynamics field enabling further insight in future research work. According to the new pressure ratio, a pressure pulse, generated at the exit of the tunnel which travels and perturbs the shock from its original position, setting it into motion. During such activity, other numerous physical phenomena also happen at the same time. However, three broad phenomena have been focused on, in this study - Traversal of a Wave, Fluid Element Interactions and Wave Interactions. The above mentioned three phenomena create, alter and kill numerous waves for different conditions. The waves which are created by the above-mentioned phenomena eventually interact with the shock and set it into motion. Numerous such interactions with the shock will slowly make it settle into its final position owing to the new pressure ratio across the duct, as estimated by one-dimensional gas dynamics. This analysis will be extremely helpful in the prediction of inlet 'unstart' of the flow in a supersonic engine intake and its prominence with the incoming flow Mach number, incoming flow pressure and the external perturbation pressure is also studied to help design more efficient supersonic intakes for engines like ramjets and scramjets.

***Keywords*—**Analytical investigation, compression and expansion waves, fluid element interactions, shock trajectory, supersonic flow, unsteady gas dynamics, varying area duct, wave interactions.

NOMENCLATURE

X	Location at any point in the inlet duct
A	Area of the inlet duct
M_s	Mach Number of the shock
M_1	Mach Number of the flow upstream of the shock
Z	Pressure ratio across the wave/shock
Γ	Ratio of specific heat capacities of air
$P_{0,\, inf}$	Stagnation pressure at the entry of the inlet
$T_{0,\, inf}$	Stagnation Temperature at the inlet entry
M_{inf}	Mach Number at the inlet entry
P_1	Static Pressure of the flow upstream of the shock
T_1	Static Temperature of the flow upstream of the shock
u_1	Speed of the flow upstream of the shock
P_3	Static Pressure of the flow downstream of the perturbation wave
T_3	Static Temperature of the flow downstream of the perturbation wave
u_3	Speed of the flow downstream of the perturbation wave
P_{2i1}	Static Pressure of the flow downstream of the shock and upstream of the contact surface
T_{2i1}	Static Temperature of the flow downstream of the shock and upstream of the contact surface
u_{2i1}	Speed of the flow downstream of the shock and upstream of the contact surface
P_{2i2}	Static Pressure of the flow downstream of the contact surface and upstream of the perturbation wave
T_{2i2}	Static Temperature of the flow downstream of the contact surface and upstream of the perturbation wave
u_{2i2}	Speed of the flow downstream of the contact surface and upstream of the perturbation wave
P_{exit}	Static Pressure of the flow at the exit of the inlet
P_{inf}	Static Pressure of the flow at the entry of the inlet
A_i	Area at the entry of the inlet duct
A_e	Area at the exit of the inlet duct

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I. INTRODUCTION

TODAY, supersonic intakes for engines are designed for steady flow conditions. During certain paths of flight like take-off or landing, the altitude of the flight changes. With altitude, there is a pressure change as well. Because of the change in external pressure conditions, the flow inside the inlet duct is perturbed. The current designs only account for the steady states, the initial the final, of the shock within. If we understand the physics of the shock movement between these two steady states, we can design more efficient inlets and eliminate the possibility of having an inlet 'unstart'.

This investigation aims to enable the study of 'unsteady gas dynamics' resulting from downstream pressure perturbations and as a result, movement of shock in an upstream supersonic diverging nozzle. This study has been restricted to downstream pressure perturbations only. The perturbations can be in the form of a compression wave or an expansion wave.

Any compression or expansion wave travelling in an area varying duct undergoes changes in its Mach number and strength (pressure ratio across the wave). All such waves communicate the external perturbations from the exit to the shock and passes on this information via interactions. Every interaction with the shock imparts it a certain velocity which slowly changes as it travels in an area varying duct, slowing down, it eventually settles into a final position owing to the new pressure ratio across the duct.

II. Assumptions

1. The flow is quasi-one-dimensional.
2. The inlet free-stream velocity is perpendicular to the plane of inlet entry.
3. The gas is ideal and the flow is inviscid and isentropic except the non-isentropic waves in the flow.
4. No high temperature effects in gas dynamics are considered i.e., the composition and specific heat values remain fixed.
5. Flow conditions upstream of the shock are constant.
6. No addition or removal of heat from the flow is present.

III. Traversal of a Wave

Any wave, compression or expansion, when travels through an area variation duct goes through changes in its Mach number and strength (i.e., pressure ratio across it). Such a relationship can be mathematically formulated using the Method of Characteristics and Whitham's Rule [3], [4], which has been discussed at length in "Shock Propagation in a Varying Area Duct" Raja Keshav [1].

$$G(M_s, M_1)dM_s + \frac{dA}{A} = 0 \tag{1}$$

where,

$$G(M_s, M_1) = \frac{(M_s - M_1)(1 + 2\mu + (M_s - M_1)^{-2})B_1}{(M_s - M_1)^2 - 1 + \frac{\gamma+1}{2}M_1(M_s - M_1)}$$

$$H(M_s, M_1)dz + \frac{dA}{A} = 0 \tag{2}$$

where,

$$H(M_s, M_1) = \frac{\left(\frac{\gamma+1}{4\gamma}\right)(1 + 2\mu + (M_s - M_1)^{-2})B_1}{(M_s - M_1)^2 - 1 + \frac{\gamma+1}{2}M_1(M_s - M_1)}$$

Here:

$$B_1 = 1 + \frac{2}{\gamma+1}\frac{1-\mu^2}{\mu} + \frac{(\gamma+1)M_1(M_s - M_1)}{\mu\left(2\gamma(M_1 - M_s)^2 - (\gamma - 1)\right)}$$

and,

$$\mu = \left(\frac{2 + (\gamma-1)(M_1 - M_s)^2}{2\gamma(M_1 - M_s)^2 - (\gamma - 1)}\right)^{1/2}$$

The relationship between area change, value of G, change in Mach number and value of H, change in pressure ratio across the wave for a compression and expansion wave is summarized in Tables I and II. It may be noted that the equations hold true for compression and expansion wave in the same way, however, its effect may be different depending upon the type of the wave.

Tables I and II consolidate the effect of converging and diverging area duct on the speed and strength of both compression and expansion waves.

TABLE I
Effect of Wave Speeds Due to Area Change

Type of Wave	Area Variation	dA	G	dM_s	Wave Speed
Compression Wave	Diverging	>0	>0	<0	Slows down
			<0	>0	Speeds up
	Converging	<0	>0	>0	Speeds up
			<0	<0	Slows down
Expansion Wave	Diverging	>0	>0	<0	Slows down
			<0	>0	Speeds up
	Converging	<0	>0	>0	Speeds up
			<0	<0	Slows down

TABLE II
Effect of Wave Strengths Due to Area Change

Type of Wave	Area Variation	dA	H	dz	Wave Strength
Compression Wave	Diverging	>0	>0	<0	Decreases
			<0	>0	Increases
	Converging	<0	>0	>0	Increases
			<0	<0	Decreases
Expansion Wave	Diverging	>0	>0	<0	Increases
			<0	>0	Decreases
	Converging	<0	>0	>0	Decreases
			<0	<0	Increases

IV. Fluid Elements Interactions

As mentioned in the previous section, any wave weakens or strengthens as it travels in a varying area duct. So, the pressures in the adjacent fluid elements are not pertaining to the isentropic conditions.

When a wave travels through a varying area duct, the processed fluid elements towards the downstream of the wave are not in isentropic harmony with each other. In other words, the pressure, temperature, Mach number and other properties are different from what it would have been if it were only adjusted for the area change of the duct. Extrapolating this, it can also be said that the stagnation pressure values towards the downstream of the wave are different for different fluid elements. For the sake of simplicity, we shall talk in terms of stagnation pressure which is easier to deal with, as it is constant in an isentropic flow and doesn't change for different areas.

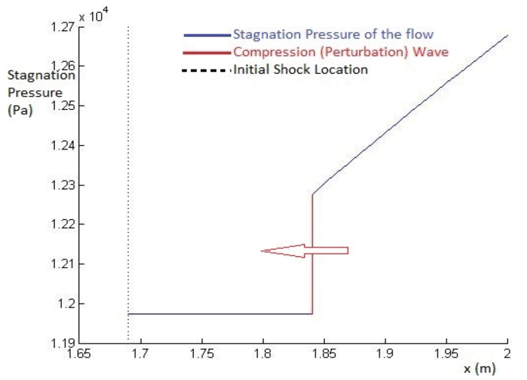


Fig. 1 Flow schematic for a compression (perturbation) wave travelling upstream

Fig. 1 has plotted for stagnation pressure values against location in the duct. As the compression wave is moving upstream, it can be seen that the stagnation pressure values are inconsistent downstream. Basically, this indicates that the adjacent elements downstream of the moving compression wave have pressure and velocity mismatches.

This can also be thought of like a shock tube problem with non-zero flow velocities on either side of the ruptured diaphragm initially itself. The rupture of the diaphragm brings in contact two adjacent fluid elements with intense pressure mismatches. Adding on that, there can be velocity mismatches too for the case mentioned above. However, the underlying principle is similar. The fluid will try to do something to relieve itself of such mismatches.

This will result in emanation of two waves in opposite directions and a contact surface at the interface of the fluid elements. This is referred to as *Fluid Elements Interactions* phenomenon. This results in imparting new velocities to both these new waves with an 'intermediate region' between the two. Further, a contact surface is formed, which separates the temperature discontinuity in the intermediate region, however, the pressure and velocity of the flow remain continuous. The nature, speed and strength of the waves emanating will certainly depend on the conditions and the kind of mismatch that is causing such emanations. The analysis of this has been dealt in the subsequent sections.

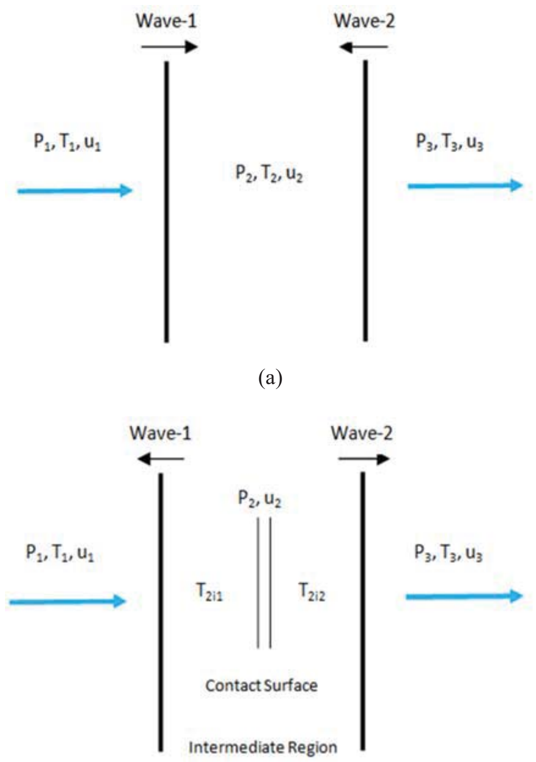
For simplifying the analysis, a certain stagnation pressure threshold is taken into account, greater than which, the Fluid Element Interactions are to be considered. Once the stagnation pressure difference in the adjacent fluid elements is higher than a pre-defined stagnation pressure threshold limit, all the fluid elements in between and including them are combined into two big fluid elements - one upstream and another downstream. The properties of these are computed by taking the average of all the fluid elements that were combined to make this new fluid element.

This analysis is, again, similar to a shock-tube analysis with non-zero flow velocities on either side of the ruptured diaphragm initially itself, as stated earlier as well.

V. Wave Interactions

In most unsteady gas-dynamics problems, multiple compression and expansion waves are formed. All of them have their own speed, strength, and direction of propagation and in a given domain of interest, many waves can collide among themselves and give rise to new waves with new characteristics. This phenomenon is known as Wave Interactions.

When two different waves interact each other, or process the same fluid domain, a contact surface is created at the interface, as is observed in shock tube analysis, with non-zero flow velocities on either side of the ruptured diaphragm. A similar situation exists when any two waves interact. Further, it results in imparting new velocities and strengths to both the waves with an 'intermediate region' between the two. The contact surface separates the temperature discontinuity in the intermediate region, however, the pressure and velocity of the flow remain continuous. This entire interaction is also discussed in Chakravarthy's [2] thesis work and at length by Hamilton and Blackstock [5].



(b)

Fig. 2 (a) Flow schematic just before the instant of wave interaction (b) Flow schematic just after the instant of wave interaction

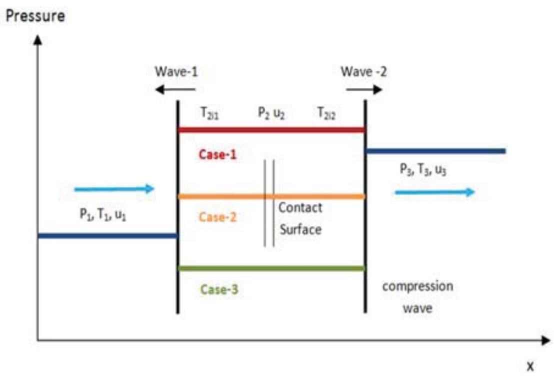
Figs. 2 (a) and (b) show schematics of the problem just before and just after the event of interaction, respectively. As it can be seen in Fig. 2 (b), the contact surface separates a temperature discontinuity but allows pressure and velocity to be continuous across it in the intermediate region.

VI. Computation of Properties of the Emanating Waves

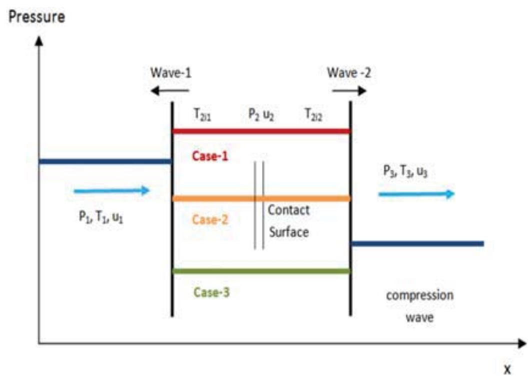
An iterative procedure is used to compute the properties in intermediate region with the constraints of having continuous velocity and pressure because simple normal shock relations cannot be used to solve for such a system. The conditions upstream of the shock i.e. P_1, u_1, T_1 and the conditions downstream of the perturbation wave i.e. P_3, u_3, T_3 are known. A guess is made for the shock velocity. Using Rankine - Hugoniot moving normal shock relations, P_{2i1}, u_{2i1}, T_{2i1} are computed. Since the pressure and flow velocity in the intermediate region are continuous, they are used as boundary conditions for the problem. In this case, P_{2i2} is taken equal to P_{2i1} and u_{2i1} is iterated with u_{2i2}. Having known P_{2i2} and P_3, all properties across the residual wave can be computed. Now, the shock velocity is iterated to get continuous flow velocity i.e., u_{2i1} equal to u_{2i2}, in the intermediate region.

So, the wave traversing towards upstream will have its strength as downstream by upstream pressure (i.e., P_2/P_1) and similarly, for the downstream traversing wave, the strength is P_2/P_3. The speeds (or Mach numbers) of the emanating waves are computed using moving shock relations across the wave as we know the flow conditions both on upstream and downstream sides.

As stated before, any such interactions are happening because of mismatch of either the pressure, or the velocity or both these properties of the adjacent fluid elements. The nature of the waves depends on these mismatches as they are the cause of the waves generating in the first place. All cases shall be evaluated and analyzed later in this section.



(a)



(b)

Fig. 3 (a) New Waves Properties for 3 cases for $P_3 < P_1$(b) New Waves Properties for 3 cases for $P_3 > P_1$

In case of a pressure mismatch, if the upstream fluid element is at a lower pressure than the downstream fluid element, then a compression wave is sent upstream and an expansion wave is sent downstream and vice-versa. In case of a velocity mismatch, i.e., if the velocity of the upstream fluid element is higher than the velocity of the downstream fluid element, then a compression wave is sent out in both directions which reduces the flow upstream and enhances the flow downstream separating the fluid elements out and vice-versa. If both the mismatches are present, then the dominant ones decide the nature of the waves emanated which are purely governed by the moving shock relations.

In general, it can be said that if the velocity mismatch is dominant, same nature of waves will emanate and if pressure mismatch is dominant, opposite nature of waves will emanate. So, in Figs. 3 (a) and (b), cases 1 and 3 depict velocity mismatch dominance as the waves emanated are of the same nature while case 2 depicts pressure mismatch domination as both waves are opposite in nature.

Figs. 3 (a) and (b) depict the three cases possible for a both the sets of initial conditions. The speeds are determined again by using moving shock relations and making use of the upstream and downstream pressures and temperatures. In Figs. 3 (a) and (b), case-1 corresponds to the formation of two compression waves which indicates a dominant velocity mismatch; case-2 corresponds to the formation one compression and one expansion wave which indicates a dominant pressure mismatch and case-3 corresponds to the formation of two expansion waves which again indicates dominant velocity mismatch.

VII. Computation of Shock Trajectory

In this study, only the diverging portion of the inlet is considered with an area ratio (A_e: A_i) of 3.0(A_i = 0.1 sq. m and with A_e = 0.3 sq. m) and a total length of the inlet duct as 0.2 meters.

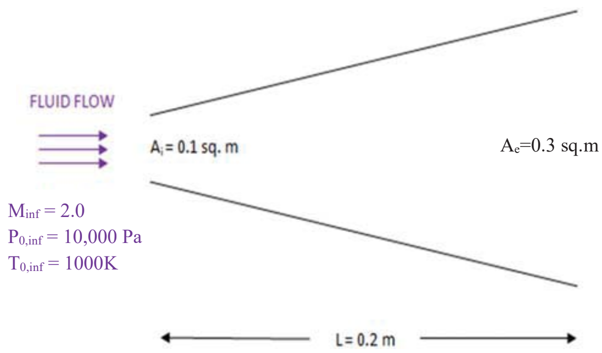
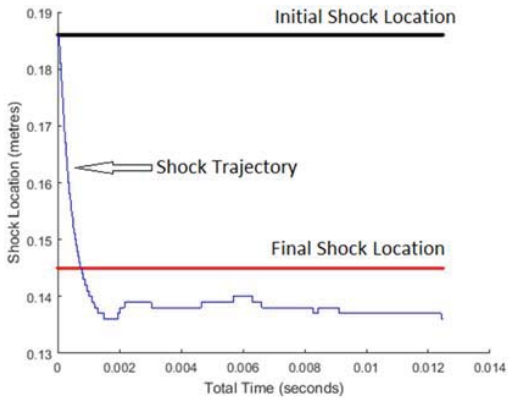


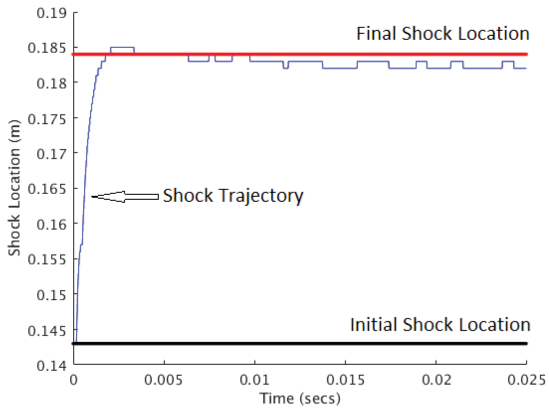
Fig. 4 Schematics of the duct

The conditions at inlet are M_{inf} = 2.0, $P_{0,inf}$ = 10,000 Pa and $T_{0,inf}$ = 1000K. With these conditions and a given operating pressure ratio (P_{exit} /P_{inf}), the initial shock location is computed from the quasi-steady one-dimensional gas dynamics concepts. This system is now disturbed by increasing the pressure at the exit, i.e., P_{exit} by a known value. This pressure will be referred to as 'new pressure' for the further discussions. This disturbance will be in the form of either a compression wave (in case of higher P_{exit} than before) or in the form of an expansion wave (in case of lower P_{exit} than before). Owing to the new pressure ratio, the shock will sit at a new location, which can be computed using quasi one-dimensional gas dynamics concepts. Figs. 5 (a) and (b) show the unsteady location of shock trajectory with time as it moves from its initial location (as per the initial pressure ratio) to its final location (as per the new or final pressure ratio).

In Fig. 5 (a), the operating pressure ratio is set at 2.0. It is to be noted that the initial shock location and final shock location is computed using one-dimensional gas dynamics concepts for the original and new pressure ratio. The initial shock location is at 0.186m and the final shock location is at 0.145m as is indicated in the figure too.



(a)



(b)

Fig. 5 (a) Shock Location versus Time for an external compression wave perturbation (b) Shock Location versus Time for an external expansion wave perturbation

An external pressure perturbation in the form of a compression wave is formed by increasing the P_{exit} by 500 Pa. This wave travels upstream towards the shock location to set into motion. On its way, more waves can be generated owing to fluid element interactions for the given stagnation pressure threshold of 0.5%. This means that if the difference in the stagnation pressure between any two elements (provided there is no wave in between) is greater than 0.5% of the average value, two waves will be created as discussed in the earlier section. All such waves interacting within themselves and with the shock will take the shock to its new location which is fairly close to the final shock location of 0.145m.

Similarly, in Fig. 4 (b), the operating pressure ratio is set at 2.4 and the initial shock location is at 0.143m and the final shock location is at 0.184m as is indicated in the figure too. Again, an external perturbation in the form of an expansion wave is formed by decreasing the P_{exit} by 500 Pa. As in the previous case, same phenomena occur which sets the shock in its final location.

VIII. Discussions on Shock Trajectories

In Figs. 4 (a) and (b), it is important to note that in both cases, the unsteady shock trajectory starts at the respective initial shock location and finally sets fairly close to the final shock location. Further, it can be observed that the error in the final shock location and unsteady shock trajectory settling location is smaller for the second case (expansion wave perturbation) than the first case (compression wave perturbation). This is because the unsteady shock will always keep receiving compression and expansion waves until it settles at its correct final location because the exit pressure is always fixed at its new pressure value. However, the communication between exit and the unsteady shock is much faster for the second case as compared to the first case, owing to the distance between them, so, it looks like its settling faster. However, if infinite time is allowed, the unsteady shock will properly coincide with the final shock location in both the cases, but is computationally very expensive and takes a very long time for the simulation to run.

As far as the shock trajectory is concerned, there are two reasons that account for the errors in it. Firstly, the stagnation pressure threshold for fluid elements interactions wave emanations, is ideally zero but it is set to be 0.5% here in both cases for computational simplicity. Secondly, a contact surface is formed at the interactions of any two waves and fluid elements interactions, which will then propagate upstream or downstream of the duct with a common velocity. This contact surface propagation will either stretch or compress the contact surface as the temperature and Mach numbers on either side of contact surface are not the same. So, when it propagates to a new location with a different area, it will compress or expand differently on both sides. Hence, it will send out compression or expansion waves accordingly to neutralize this effect as contact surface is just a surface and cannot be stretched or compressed. However, these waves are found to much weaker than those emanating due to fluid element interactions or wave interactions and thus, are neglected for this analysis.

IX. Conclusion

This study is intended to establish the foundation of physics of the unsteady phenomena in fluid flows. Covering the basic aspects of wave generation and traversal, the unsteady trajectory of the shock has been sketched and the model can be improved upon by incorporating many more features like high temperatures activating its internal energy modes. Further, it can be extrapolated to study different phenomena such as inlet 'unstart' or regular and Mach reflections transition for an unsteady flow. Further studies currently are underway to make the current model better by including the features mentioned in the previous section.

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