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Structural Damage Detection via Incomplete Modal Data Using Output Data Only

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Abstract**—**Structural failure is caused mainly by damage that often occurs on structures. Many researchers focus on to obtain very efficient tools to detect the damage in structures in the early state. In the past decades, a subject that has received considerable attention in literature is the damage detection as determined by variations in the dynamic characteristics or response of structures. The study presents a new damage identification technique. The technique detects the damage location for the incomplete structure system using output data only. The method indicates the damage based on the free vibration test data by using 'Two Points Condensation (TPC) technique'. This method creates a set of matrices by reducing the structural system to two degrees of freedom systems. The current stiffness matrices obtain from optimization the equation of motion using the measured test data. The current stiffness matrices compare with original (undamaged) stiffness matrices. The large percentage changes in matrices' coefficients lead to the location of the damage.

TPC technique is applied to the experimental data of a simply supported steel beam model structure after inducing thickness change in one element, where two cases consider. The method detects the damage and determines its location accurately in both cases. In addition, the results illustrate these changes in stiffness matrix can be a useful tool for continuous monitoring of structural safety using ambient vibration data. Furthermore, its efficiency proves that this technique can be used also for big structures.

Keywords**—**Damage detection, two points–condensation, structural health monitoring, signals processing, optimization.

I. Introduction

THIS study indicates the damage that is based on the vibration by using "Two Points –Condensation (TPC) technique", this technique is to create a set of matrices by reducing the whole structure stiffness matrix to two degree of freedoms matrices and compare the condensed stiffness matrices' coefficients that calculated by using measured test data with theoretical condensed stiffness matrices' coefficients, according to the large percentage changes in matrices' coefficients, location of the damage can be easily estimated.

TPC technique is an analytical method that reduces the theoretical system rigidity matrix into a set of two degree of freedom stiffness matrices using the Guyan condensation which is the simplest and most popular condensation approach and has been widely used for large number of engineering problems and implemented into many commercial finite element analysis codes [1]. By comparing the theoretical structure rigidity (the rigidity of structure model according to the drawings) with the calculated rigidity (that calculated using signal data), the large changes in stiffness matrices' coefficients imply the locations of damage.

In TPC technique, calculate the real stiffness matrices by finding optimal solution of Equation of Motion (1) that satisfy the system real properties.

$$[M]_{2x2}\{\ddot{u}\}_{2x1} + [C]_{2x2}\{\dot{u}\}_{2x1} + [K]_{2x2}\{u\}_{2x1} = 0 \quad (1)$$

where K: Stiffness Matrix of system, M: Mass Matrix of system, C: Damping Matrix of system, $\ddot{u}$: The acceleration vector, $\dot{u}$: The velocity vector, u: The displacement vector.

The optimization uses the recorded acceleration signals $\ddot{u}$ and velocity $\dot{u}$ calculated by using measured acceleration, and displacement u calculated from velocity while using the theoretical mass matrix M of real structure because it can be calculated easily and it does not change mostly [2].

Damping matrix C is calculated by considering the damping proportional to the mass and the stiffness [3]:

$$C = a_0 M + a_1 K \quad (2)$$

The damping ratio for the n^{th} mode of such a system is:

$$\xi_n = \frac{a_0}{2}\frac{1}{\omega_n} + \frac{a_1}{2}\,\omega_n \quad (3)$$

The coefficients a_0 and a_1 can be determined from the i^{th} and j^{th} modes' damping ratios ξ_i and ξ_j, respectively. Damping ratios are calculated using Half-Power Bandwidth Method [4].

TPC technique uses the multi objective optimization. When there is more than one design objective for the problems, the solution defines as Multi-objective optimization. The TPC technique finds the optimal stiffness value to satisfy the equation of motion, where the input data included the mass matrix and initial stiffness matrix (calculated undamaged stiffness matrix) in addition, the corresponding vectors of acceleration, velocity, and displacement. The damping matrix optimized due to the stiffness updating. The solution is controlled by frequencies' satisfaction.

The damage location is obtained by observation on the value of changes in the stiffness coefficients of the two degrees of freedom systems.

II. System Condensation

A useful method of accomplishing the reduction of the stiffness matrix is to identify those degrees of freedom to be condensed or reduced as secondary degrees of freedom, and

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express them in the term of the remaining primary degrees of freedom. The Guyan condensation method divides the degrees of freedom into secondary and primary DOF's, where those degrees of freedom arranged to put the (secondary) degrees of freedom as the first (s) coordinates.

The stiffness equation for the structure may be rewritten using partition matrices as:

$$\begin{bmatrix} [K_{ss}] & [K_{sp}] \\ [K_{ps}] & [K_{pp}] \end{bmatrix} \begin{Bmatrix} \{u_s\} \\ \{u_p\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{F_p\} \end{Bmatrix} \tag{4}$$

where $\{u_s\}$ is the displacement vector corresponding to the s degrees of freedom to be reduced and $\{u_p\}$ is the vector corresponding to the reigning p independent degrees of freedom [5].

The transform matrix [T] obtained using (5) and (6). The reduced stiffness matrix $[\bar{K}]$ can be expressed as a transformation of the system stiffness matrix [K] as shown in (7). The reduced mass matrix is calculated by (8).

$$\begin{Bmatrix} \{u_s\} \\ \{u_p\} \end{Bmatrix} = \begin{bmatrix} [\bar{T}] \\ [I] \end{bmatrix} \{u_p\} = [T]\{u_p\} \tag{5}$$

$$[\bar{T}] = -[K_{ss}]^{-1}[K_{sp}] \tag{6}$$

$$[\bar{K}] = [T]^T[K][T] \tag{7}$$

$$[\bar{M}] = [T]^T[M][T] \tag{8}$$

III. Experimental Study

To apply the developed damage detection algorithm given in this study, the beam's cross section is a rectangular cross section 10cm by 2.19cm and the material properties of the beam are taken as density $7.85\text{x}10^{-6}$ ton/cm^3 and Young's modulus E 2100 ton/cm^2 for modeling in the MATLAB software. In damage model, the element 2 has extra thickness 0.38cm represents the damage (the changing in thickness leads to change in stiffness) the total thickness for element 2 is 2.57cm as shown in Fig. 1.

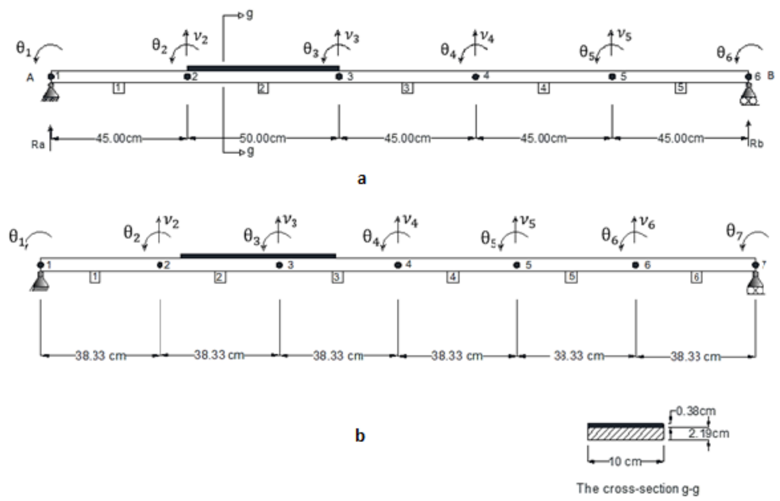

Fig. 1 Damage model steel beam A-B shown the DOF's, (a) 5-element test beam, (b) 6-element test

The considered degree of freedoms (DOFs) are vertical displacements and rotations as shown in Fig. 1, where the DOFs (θ_1, θ_2, θ_3, θ_4, θ_5 and θ_6) refer to the rotations and the DOFs (ν_2, ν_3, $\nu_4$and ν_5) refer to the vertical displacements.

The mass matrix that is used in this study is Consistent Mass Matrix while the stiffness matrix that is used is the Euler–Bernoulli beam Element Stiffness Matrix [6].

At first case (Fig. 1 (a)) the global stiffness matrix of the five element system is condensed to DOFs: $\nu_2 - \nu_4$, $\nu_2 - \nu_5$, $\nu_3 - \nu_4$, $\nu_3 - \nu_5$ and $\nu_4 - \nu_5$. The resultant matrices and condensed structure's systems are given in Fig. 2.

The beam is modeled as five element system that make the damage location is all through the element length (see Fig. 3) (between nodes 2-3) while in the second case six equal length elements is used which means the damage location is between the nodes 2 and 4 (see Fig. 1 (b)).

In the first case, the damaged beam prototype is used with four accelerometers placed on the nodes 2, 3, 4 and 5 as shown in Fig. 3.

The mass of the accelerometer is 2.85 Kg (including the apparatus for connections) see Fig. 5; these masses are added to the global mass matrix of the system as lumped mass at the nodes that accelerometers located. In this technique the time domain and incomplete system is considered. The system vibrated by impact excitation. The recorded acceleration signals for five element beam test and six element beam test shown in Fig. 6.

To obtain velocity and displacement that is necessary to use in the equation of motion, the trapezoidal rule for numerical integration is applied to acceleration data [7]. The modes' frequencies are obtained by using fast Fourier transforming (FFT). The frequencies used for control the solution and obtaining the damping coefficients [8].

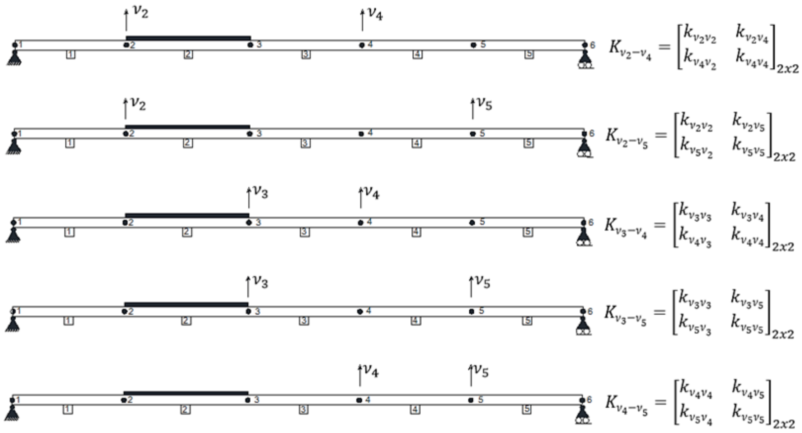

Fig. 2 Reduced system of the beam to 2 DOF's system to different DOF's

Fig. 3 The test of 5 element steel simply supported beam with accelerometers

Fig. 4 Damage element in beam

Fig. 5 Data-acquisition and Accelerometer

Equation (9) is representing the changes in the stiffness matrices using (TPC).

$$\Delta K_{x-y} = \begin{bmatrix} \Delta k_{xx} & \Delta k_{xy} \\ \Delta k_{yx} & \Delta k_{yy} \end{bmatrix} \quad (9)$$

Comparing the Δk_{xx} with the Δk_{yy}, it is seen that percentage change of k_{xx} is greater than the percentage change of k_{yy}that means the damage lies on the left side of the element. The result of the test of five element steel beam using TPC technique is shown in the Table I, while The result of the test of six element steel beam (Fig. 1 (b)) using TPC technique are shown in Table II.

TABLE I
THE CHANGES IN THE STIFFNESS COEFFICIENTS USING THE TPC TECHNIQUE FOR THE 5 ELEMENT BEAM

DOFs for condensed matrix	Exact Changes in the stiffness matrix (ΔK %)	Changes in the stiffness matrix using TPC (ΔK %)
$[v_2 - v_4]$	$\begin{bmatrix} 22.74 & 15.08 \\ 15.08 & 7.20 \end{bmatrix}$	$\begin{bmatrix} \mathbf{21.61} & \mathbf{14.18} \\ \mathbf{14.18} & \mathbf{6.60} \end{bmatrix}$
$[v_2 - v_5]$	$\begin{bmatrix} 22.80 & 14.18 \\ 14.18 & 4.70 \end{bmatrix}$	$\begin{bmatrix} \mathbf{21.75} & \mathbf{15.12} \\ \mathbf{15.12} & \mathbf{4.81} \end{bmatrix}$
$[v_3 - v_4]$	$\begin{bmatrix} 21.52 & 16.93 \\ 16.93 & 11.68 \end{bmatrix}$	$\begin{bmatrix} \mathbf{20.13} & \mathbf{18.02} \\ \mathbf{18.02} & \mathbf{11.28} \end{bmatrix}$
$[v_3 - v_5]$	$\begin{bmatrix} 17.86 & 12.26 \\ 12.26 & 5.74 \end{bmatrix}$	$\begin{bmatrix} \mathbf{17.20} & \mathbf{13.31} \\ \mathbf{13.31} & \mathbf{5.03} \end{bmatrix}$
$[v_4 - v_5]$	$\begin{bmatrix} 17.86 & 12.26 \\ 12.26 & 5.74 \end{bmatrix}$	$\begin{bmatrix} \mathbf{17.32} & \mathbf{10.84} \\ \mathbf{10.84} & \mathbf{5.55} \end{bmatrix}$

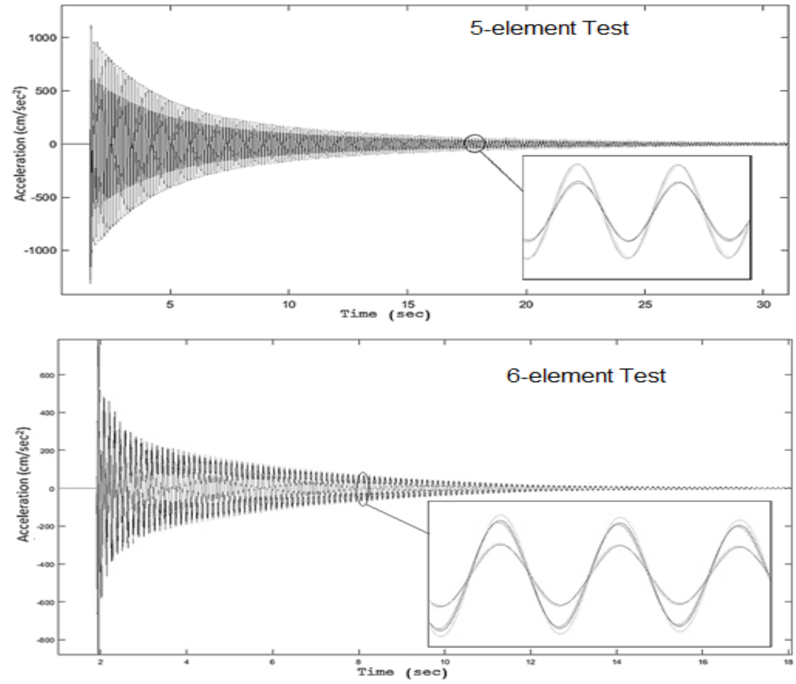

Fig. 6 Recorded acceleration signals for 5 elements steel beam (case-1) and 6 element steel beam, (case-2) where the proportional damping is clear in both cases

TABLE II
THE CHANGES IN THE STIFFNESS COEFFICIENTS USING THE TPC TECHNIQUE FOR THE 6 ELEMENT BEAM

DOFs for condensed matrix	Exact Changes in the stiffness matrix (ΔK %)	Changes in the stiffness matrix using TPC (ΔK %)
$[v_2 - v_3]$	$\begin{bmatrix} 6.03 & 14.83 \\ 14.83 & 16.87 \end{bmatrix}$	$\begin{bmatrix} 6.61 & 15.53 \\ 15.53 & 17.01 \end{bmatrix}$
$[v_2 - v_6]$	$\begin{bmatrix} 7.10 & 14.36 \\ 14.36 & 5.04 \end{bmatrix}$	$\begin{bmatrix} 7.75 & 14.26 \\ 14.26 & 5.31 \end{bmatrix}$
$[v_3 - v_5]$	$\begin{bmatrix} 17.02 & 14.32 \\ 14.32 & 6.67 \end{bmatrix}$	$\begin{bmatrix} 17.35 & 14.52 \\ 14.52 & 6.99 \end{bmatrix}$
$[v_4 - v_5]$	$\begin{bmatrix} 6.40 & 9.83 \\ 9.83 & 4.07 \end{bmatrix}$	$\begin{bmatrix} 6.94 & 9.71 \\ 9.71 & 4.64 \end{bmatrix}$

IV. THE DISCUSSION AND CONCLUSION

Considering Table II: For the first result of DOF's $[v_2 - v_3]$, the node 2 is outside the damage while the node 3 contain the damage, therefore the large change concentrated at v_2.

For the result of DOF's $[v_2 - v_6]$ the both degrees of freedom belong to nodes that located outside the damage, but node 2 is closer to damage than node 6 that explain large change in coefficient $k_{v_2 v_2}$compare with coefficient $k_{v_6 v_6}$. The DOF's $[v_3 - v_5]$ choose because the position of node 3 at the damage and the position of the node 5 away from the damage, that produce large change in coefficient $k_{v_3 v_3}$.

The result of ΔK for $[v_4 - v_5]$ shows the damage to the left side of element although the both nodes far away from damage.

The method shows large change in stiffness at the node where it near to the damage location, while there small change at the recorded nodes that far from the damage.

Comparison of results shows that the Guyan condensation is suitable for use in this technique for beam type structure, the method provide a good result for beam type of structure.

According to Table I the TPC technique shows the location of damage regardless the place of the acceleration sensors; the technique demonstrates the location of damage when the acceleration sensor nears the damage as well as it distant from the damage.

The method can show the damage using output data only for incomplete system so it can be used for health monitoring of structures using ambient vibration data via less number of sensors.

For large structures, this method may have some limitations due to the static condensation errors, where control of the frequencies is not enough to obtain stable result.

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