Streamwise Conduction of Nanofluidic Flow in Microchannels

Yew Mun Hung, Ching Sze Lim, Tiew Wei Ting, Ningqun Guo

Abstract—The effect of streamwise conduction on the thermal characteristics of forced convection for nanofluidic flow in rectangular microchannel heat sinks under isothermal wall has been investigated. By applying the fin approach, models with and without streamwise conduction term in the energy equation were developed for hydrodynamically and thermally fully-developed flow. These two models were solved to obtain closed form analytical solutions for the nanofluid and solid wall temperature distributions and the analysis emphasized details of the variations induced by the streamwise conduction on the nanofluid heat transport characteristics. The effects of the Peclet number, nanoparticle volume fraction, thermal conductivity ratio on the thermal characteristics of forced convection in microchannel heat sinks are analyzed. Due to the anomalous increase in the effective thermal conductivity of nanofluid compared to its base fluid, the effect of streamwise conduction is expected to be more significant. This study reveals the significance of the effect of streamwise conduction under certain conditions of which the streamwise conduction should not be neglected in the forced convective heat transfer analysis of microchannel heat sinks.

Keywords—fin approach, microchannel heat sink, nanofluid, streamwise conduction

I. INTRODUCTION

THE choice of working fluid in cooling device has been a key issue for the sake of heat transfer enhancement. Addition of solid nanoparticle-suspensions to the base fluid to improve the heat transfer characteristics of conventional fluids is one of the innovative methods adopted. This type of fluid is coined as "nanofluid", which exhibits anomalously increased effective thermal conductivity with a small volume fraction of the nanoparticles [1]. Nanofluid is a novel type of engineered colloids consisting of suspended solid nanoparticles whose size ranges from 1 nm to 100 nm. For the last two decades, research on nanofluids started to attract attention due to their notable enhanced heat transfer characteristics compared to those of the conventional fluids. Over the last five years, a number of review articles on nanofluids have been well-documented [2]–[4], and there have been numerous experimental studies emphasizing the enhancement effect of the effective thermal conductivity of nanofluids. Nanofluid possesses high thermal conductivity even with the presence of only a small fraction of nanoparticles [5]–[10].

Yew Mun Hung is with School of Engineering, Monash University, 46100 Bandar Sunway, Malaysia (phone: +603-55146251; fax: +603-55146207; e-mail: hung.yew.mun@monash.edu).

Ching Sze Lim is with Faculty of Engineering and Technology, Multimedia University, 75450 Melaka, Malaysia (e-mail: cslim1987@hotmail.com).

Tiew Wei Ting is with School of Engineering, Monash University, 46100 Bandar Sunway, Malaysia (e-mail: ting.tiew.wei@monash.edu).

Ninqun Guo is with School of Engineering, Monash University, 46100 Bandar Sunway, Malaysia (e-mail: anthony.guo@monash.edu).

These studies reported that the increase of thermal conductivity in the nanofluids would be in the range of 30% to 150% compared to the base fluids. Unlike the milli-sized or micro-sized particle which would lead to the clogging problem in tubes during operation, nanoparticles are free from this problem. In addition, nanofluids induce little or no penalty in pressure drop due to the fact that nanoparticles are ultrafine and nanofluid behaves more like a single-phase fluid than a solid-liquid mixture [7].

Most existing analytical studies on microchannel heat sink neglected the effect of streamwise conduction despite the fact that this effect has been justified to significantly affect the heat transport rate in proximity to the entrance region of the fluid flow for conventional size channels [11]–[13].

Due to the fact that the characteristic time of convection and conduction are comparable for flows with small Peclet number, the streamwise conduction becomes indispensable. Since the fluid flow in micro-scale devices is typically characterized by finite Peclet number, the incorporation of the effect of streamwise conduction is a necessity in the thermal analysis of microchannel heat sinks [14]–[17].

Furthermore, the streamwise conduction is expected to play an important role in contributing heat transfer enhancement of nanofluids due to the fact that the effective thermal conductivity of nanofluids is anamolously increased compared to the convectional fluids.

The present study emphasizes on the effect induced by the streamwise conduction on the thermal performance of nanofluid flow in a microchannel heat sink subjected to a constant wall temperature boundary condition.

An analytical model is proposed based on the fin approach. By comparing the models with and without the incorporation of streamwise conduction, the analysis emphasizes details of the variations incurred by the streamwise conduction on the convective transport of nanofluids. The effects of the pertinent physical parameters, such as the Peclet number, thermal conductivity ratio and nanoparticle volume fraction, are investigated and discussed.

II. PROBLEM STATEMENT

Three main transport properties involved in nanofluids which induce significantly different heat transfer performance from conventional fluids are thermal conductivity k, viscosity μ and heat capacity c_p. Thermal conductivity is a key parameter in enhancing the heat transfer performance.

The suspended nanoparticles with higher thermal conductivity than the base fluids are responsible in enhancing the thermal conductivity of nanofluids. The effective thermal conductivity of nanofluid depends on shape of particles and volume fraction of nanoparticles suspended in base fluid.

A model which developed by Hamilton and Crosser [18] is employed:

$$k_{nf} = \zeta k_f \qquad (1)$$

where ζ is expressed as:

$$\zeta = \left(\frac{\kappa + (n-1) - (n-1)\phi(1-\kappa)}{\kappa + (n-1) + \phi(1-\kappa)} \right) \qquad (2)$$

where n is an empirical shape factor which is given by

$$n = \frac{3}{\chi} \qquad (3)$$

The parameter ϕ in (2) is denoted as the nanoparticle volume fraction while the parameter κ is the thermal conductivity ratio, given by

$$\kappa = \frac{k_p}{k_f} \qquad (4)$$

It should be noted that (1) is valid when the thermal conductivity ratio ζ is more than 100. The sphericity χ is defined as the ratio of the surface area of a sphere with a volume equal to that of the particle to the surface area of the particle. For particles of other shapes, the shape factor n can be allowed to vary from 0.5 to 6.0. The effective viscosity of nanofluid is modelled using the Brinkman's equation which is extended from the Einstein model [19]

$$\mu_{nf} = \eta \mu_f \qquad (5)$$

where μ_f is the dynamic viscosity of base fluid and the ratio η is defined as

$$\eta = \frac{1}{(1-\phi)^{5/2}} \qquad (6)$$

The heat capacity of nanofluid employed is given by [20]

$$(\rho c_p)_{nf} = (1-\phi) + \phi \psi \qquad (7)$$

where ρ is density and ψ is the specific heat capacity ratio given by

$$\psi = \frac{(\rho c_p)_p}{(\rho c_p)_f} \qquad (8)$$

The problem considered in this study is the forced convection of nanofluidic flow through a microchannel heat sink taking into account the streamwise conduction. The microchannel heat sink consists of a number of channels, where each channel has a length L, height H, and width a. The thickness of the plate is t. The bottom surface of the microchannel heat sink ($y=0$) is kept at constant temperature (T_w) and the top surface ($y=H$) is insulated.
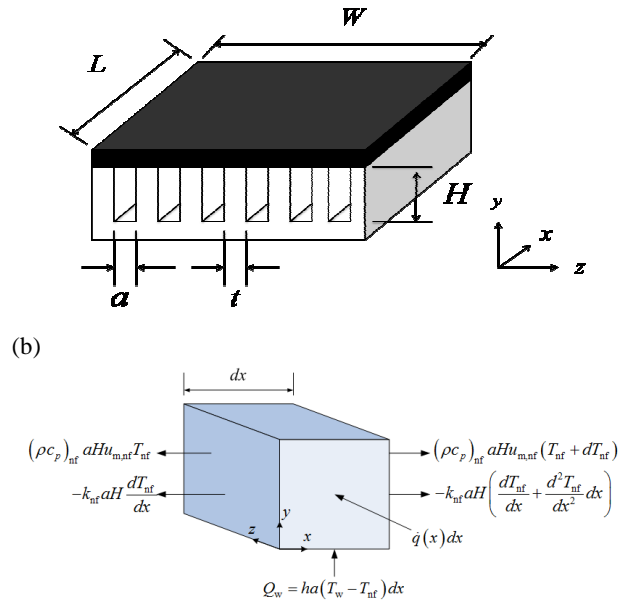
(a)

(b)

Fig. 1 (a) Schematic diagram of the microchannel heat sink. (b) Differential control volume for the derivation of energy equation in the liquid domain

III. MATHEMATICAL FORMULATION

Fin approach is one of the analytical approaches adopted due to its effectiveness and relative simplicity in the analysis of the transport phenomena, based on the assumption of one dimensional heat conduction along the fin or the channel plate height. The heat transfer analysis is simplified by considering constant heat transfer coefficient and uniform fluid temperature in the transverse direction of the flow [17]. The temperatures of the fluid and the solid plate are assumed to have been properly integrated over the width of the channel and the thickness of the plate, respectively, to yield two-dimensional averaged fluid temperature $T_f(x,y)$ and solid plate temperature $T_s(x,y)$. Fin approach is based on one-dimensional heat conduction along the height of the solid plate and the fin equation for the solid plate [21] is given by.

$$\frac{d^2 T_s}{dy^2} - \frac{2h}{k_s t}[T_s - T_{nf}(x)] = 0 \qquad (9)$$

The nanofluid temperature is further averaged over the height of the channel to yield the bulk mean temperature

$T_{nf}(x)$ over the rectangular cross section. In (9), k_s is the thermal conductivity of the solid plate whereas h denotes the interfacial heat transfer coefficient. The appropriate thermal boundary conditions are expressed as

$$T_s(x,0) = T_w, \quad \frac{\partial T_s}{\partial y}\bigg|_{y=H} = 0 \tag{10}$$

where T_w denotes the constant temperature at the solid wall. The application of these boundary conditions on (9) yields the solid plate temperature distribution

$$T_s(x,y) = T_{nf}(x) + \frac{\cosh\left[\sqrt{2Bi}(H-y)/t\right]}{\cosh\left(\sqrt{2Bi}H/t\right)}[T_w - T_{nf}(x)] \tag{11}$$

where Bi is the Biot number defined as

$$Bi = \frac{ht}{k_s} \tag{12}$$

The heat transfer rate per unit length of the channel from the solid plate to the working fluid can be determined from Fourier's law of heat conduction as

$$\dot{q}(x) = -k_s t \frac{dT_s}{dy}\bigg|_{y=0}$$
$$= k_s \sqrt{2Bi} \tanh\left(\frac{\sqrt{2Bi}H}{t}\right)[T_w - T_{nf}(x)] \tag{13}$$

To derive the energy equation for the working fluid inside the microchannel heat sink, the conservation of energy to the infinitesimal control volume depicted in Fig. 1(b) is applied and the ordinary differential equation obtained is expressed as

$$-k_{nf}aH\frac{d^2 T_{nf}(x)}{dx^2} + (\rho c_p)_{nf} u_{m,nf} aH \frac{dT_{nf}(x)}{dx}$$
$$= \left[k_s \sqrt{2Bi}\tanh\left(\sqrt{2Bi}\frac{H}{t}\right) + ha\right][T_w - T_{nf}(x)] \tag{14}$$

by assuming the fluid flow to be unidirectional and hydrodyanmically and thermally fully developed with a mean velocity $u_{m,nf}$. The thermophysical properties of the nanofluid are assumed to be independent of temperature. With T_0 the nanofluid temperature at the entrance and assuming that the exit of the microchannel is connected to an adiabatic section, the corresponding thermal boundary conditions for the fluid are given by

$$T_{nf}(0) = T_0, \quad \frac{dT_{nf}}{dx}\bigg|_{x=L} = 0 \tag{15}$$

by employing the following dimensionless variables

$$X = \frac{x}{L}, \quad \theta_{nf}(X) = \frac{T_w - T_{nf}(x)}{T_w - T_o} \tag{16}$$

the energy equation of (14) and the thermal boundary conditions of (15) are nondimensionalized, respectively, as

$$C_1 \frac{d^2 \theta_{nf}}{dX^2} - C_2 \frac{d\theta_{nf}}{dX} = C_3 \theta_{nf} \tag{17}$$

$$\theta_{nf}(0) = 1, \quad \frac{d\theta_{nf}}{dX}\bigg|_{X=1} = 0 \tag{18}$$

$$C_1 = \frac{\zeta \gamma^2}{\alpha} \tag{19}$$

$$C_2 = \frac{\left[(1-\phi) + \phi\psi\right]Pe\gamma}{\eta}\left(\frac{\alpha+1}{2\alpha}\right) \tag{20}$$

$$C_3 = \left[k_r \sqrt{2Bi}\tanh\left(\frac{\varepsilon\alpha\sqrt{2Bi}}{1-\varepsilon}\right) + \frac{k_r \varepsilon Bi}{(1-\varepsilon)}\right] \tag{21}$$

where the Peclet number is

$$Pe = \frac{\rho c_p u_m D_h}{k_f} \tag{22}$$

and

$$\alpha = \frac{H}{a}, \quad \gamma = \frac{H}{L}, \quad \varepsilon = \frac{a}{a+t}, \quad D_h = \frac{2aH}{(a+H)}, \quad k_r = \frac{k_s}{k_{nf}} \tag{23}$$

By solving (17) with the boundary conditions of (18), the closed form analytical solution for the dimensionless nanofluid temperature profile can be obtained as

$$\theta_{nf}(X) = 2C_1\{(\omega+\lambda)\exp[\omega+\lambda+(\omega-\lambda)X]$$
$$-(\omega-\lambda)\exp[\omega-\lambda+(\omega+\lambda)X]\}/\{2C_1\lambda[\exp(\omega+\lambda) \tag{24}$$
$$+\exp(\omega-\lambda)] + C_2[\exp(\omega+\lambda) - \exp(\omega-\lambda)]\}$$

where

$$\omega = \frac{C_2}{2C_1} \tag{25}$$

$$\lambda = \frac{\sqrt{C_2{}^2 + 4C_1 C_3}}{2C_1} \qquad (26)$$

Hereafter, for the purpose of comparison of results, the model with streamwise conduction term incorporated in the energy equation is denoted as Model 1, and that without streamwise conduction term as Model 2 when $C_1 = 0$.

IV. RESULTS AND DISCUSSION

Fig. 2 depicts the effect of Peclet number variation on the dimensionless fluid temperature for fixed values of aspect ratios ($\alpha = 1$ and $\gamma = 0.2$), thermal conductivity ratio ($k_r = 200$) and porosity ($\varepsilon = 0.5$), nanoparticle volume fraction ($\phi = 0.1$) and heat capacity ratio ($\psi = 0.5$) when $Bi = 0.1$. In Fig. 2, it can be observed that the temperature gradient becomes steeper as Pe reduces, indicating that the nanofluid temperature $T_{nf}(x)$ increases more rapidly along the channel as Pe decreases. The dimensionless nanofluid temperature reaches zero at about $X = 0.2$ when $Pe = 1$, showing that the fluid flows isothermally at $T_{nf}(x) = T_w$ without absorbing heat for the remaining of the channel as T_w is the maximum temperature that the fluid can reach. For larger Peclet numbers, the dimensionless fluid temperature distribution remains above zero throughout the entire channel length, therefore the nanofluid absorbs heat for the entire channel length. Therefore, reduction in the Peclet number decreases the distance in which the heat transfer to the nanofluid takes place. To characterize the role of streamwise conduction, comparison between the models with and without considering streamwise conduction is performed. The discrepancy between the two models is larger for small Pe and decreases with Pe. The dimensionless nanofluid temperature obtained from Model 1 deviates noticeably from that obtained from Model 2 at $Pe = 1$, indicating that the existence of heat transfer due to streamwise conduction leads to increasing difference between wall-fluid temperature within the entrance section. However, for larger Peclet numbers, the dimensionless temperatures from the Model 1 overlap with those from the Model 2, implying that the streamwise conduction induces pronounced effect only when Pe is sufficiently small, and the streamwise conduction can be neglected in the analysis of heat transfer for large Pe. The nanofluid achieves thermal equilibrium with the base wall at the axial location which is denoted as the equilibrium point where the dimensionless nanofluid temperature is zero.

For $Pe = 1$, the distance from the entrance of the channel to the equilibrium point of Model 1 appears to be longer than that of Model 2. Therefore, the streamwise conduction causes the nanofluid to reach thermal equilibrium with the base wall at the axial location farther from the channel inlet, and the presence of the streamwise conduction increases the distance in which the heat transfer to the nanofluid takes place.

To quantify the total amount of heat transported by the nanofluid, the differential equation, (17), governing the dimensionless fluid temperature distribution is integrated over the entire length of the microchannel. By utilizing the boundary conditions in (18), it can be shown that

$$-C_1 \left.\frac{d\theta_{nf}}{dX}\right|_{X=0} + C_2 \left[1 - \theta_{nf}(1)\right] = C_3 \int_0^1 \theta_{nf}\, dX \qquad (27)$$

In (27), the first term on the left-hand side concerns the dimensionless form of the heat transport due to the streamwise conduction, Ω_{cond}, while the second term is considered as the dimensionless form of convective heat transfer, Ω_{conv}. The integral on the right-hand side is proportional to the total heat transfer from the solid plate and the base wall, which is absorbed by the nanofluid and transported through streamwise conduction and convection.

Therefore, it is indicated that the area under the curve of the dimensionless nanofluid temperature distribution represents the total amount of heat absorption of the nanofluid throughout the entire length of the microchannel.
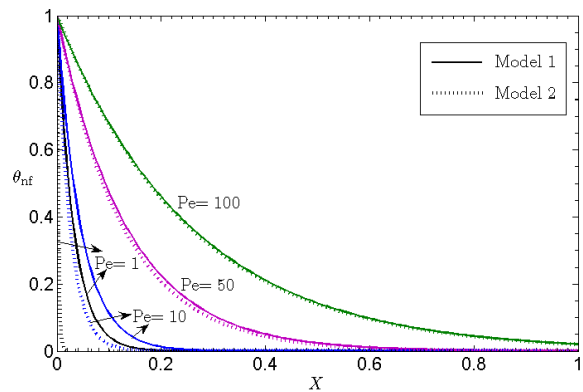

Fig. 2 Comparison of dimensionless nanofluid temperature distributions between Model 1 and Model 2 along the axial direction of the channel with Pe as a parameter when $Bi = 0.1$

As observed from Fig. 2, when the Biot number is increased, the equilibrium points for both models shift toward the entrance of the microchannel and the total heat transfer by the nanofluid decreases, as the area under the curve of the dimensionless nanofluid temperature profile is susceptible to shrinkage. An increase in Bi shortens the distance in which heat transfer to the nanofluid takes place for both models, attributed to the fact that the interfacial heat transfer coefficient between solid and nanofluid is related to the Biot number. As the Biot number increases, the convective heat transfer rate increases and the effect of streamwise conduction becomes less pronounced.
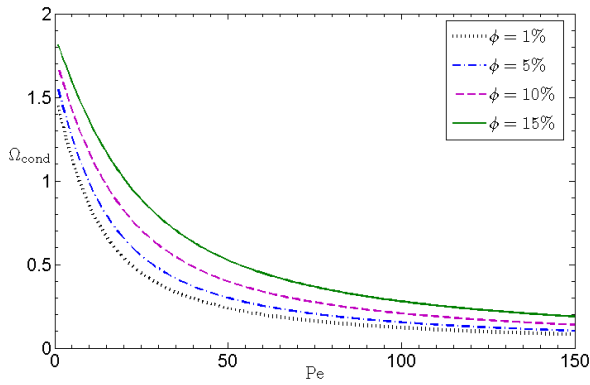
Fig. 3   Dimensionless heat transfer contributed by streamwise conduction as a function of Peclet number, with volume fraction of nanoparticle being a parameter
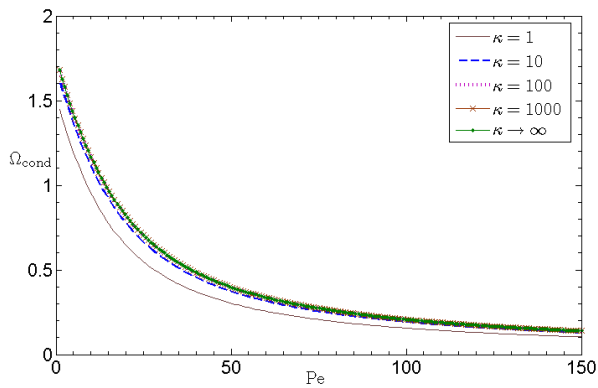

Fig. 4   Dimensionless heat transfer contributed by streamwise conduction as a function of Peclet number, with thermal conductivity ratio being a parameter

Fig. 3 and 4 show Ω_{cond} of Model 1 as a function of Pe, with ϕ and κ being the parameter, respectively, when $\text{Bi} = 0.1$. It is observed from Fig. 3 that Ω_{cond} decreases with Peclet number and increases with nanoparticle volume fraction ϕ. For a given Peclet number, nanofluid with largest nanoparticle volume fraction, $\phi = 15\%$, provides the largest Ω_{cond}. Therefore, it can be deduced that addition of nanoparticles in the base fluid further enhances the contribution of the streamwise conduction to the heat transfer process. In Fig. 4, we can observe that Ω_{cond} increases with the thermal conductivity ratio of the nanoparticle to the base fluid, κ, when κ is sufficiently small (from $\kappa = 1$ to $\kappa = 10$). However, Ω_{cond} is insensitive to the variation of κ when this ratio is larger than 100. Therefore, practically it can be said that the streamwise conduction is not affected by κ as this ratio is typically sufficiently large for most of the combinations of nanoparticle material and base fluid.

Fig. 5 depicts Ω_{conv} of Model 1 as a function of Pe, with ϕ being the parameter when $\text{Bi} = 0.1$. Contrary to Fig. 3, it can be observed that Ω_{conv} increases with Peclet number and decreases with nanoparticle volume fraction ϕ. For a given Peclet number, nanofluid with smallest nanoparticle volume

fraction, $\phi = 1\%$, provides the largest Ω_{conv}. Hence, it can be concluded that the addition of nanoparticles in the base fluid enhances the streamwise conduction while on the other hand reduces the contribution of the convective heat transfer.
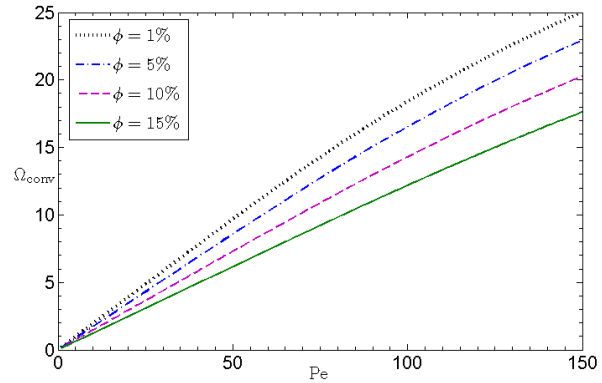

Fig. 5   Dimensionless heat transfer contributed by convection as a function of Peclet number, with volume fraction of nanoparticle being a parameter

To compare the significance of the streamwise conduction with respect to the convection, a parameter

$$M = \frac{|\Omega_{\text{cond}}|}{|\Omega_{\text{conv}}|} \tag{28}$$

which is the ratio of streamwise conduction to heat convection of the nanofluid is defined. Fig. 6 illustrates the variation of M as a function of Pe when $\text{Bi} = 0.1$ and $\phi = 10\%$. For small Peclet number, M is significantly large. The order of magnitude of M is 10, indicating that the streamwise conduction dominates over the convective heat transport. However, when Pe increases, M decreases drastically, showing that the convection heat transfer overcomes and dominates over the conduction heat transfer. Beyond $\text{Pe} = 10$, M is less than unity and the heat transported by the fluid is mainly due to convection. On the other hand, M increases with the nanoparticle volume fraction for all Peclet numbers. Therefore, the degree of significance of the effect of streamwise conduction decreases with Pe and increase with ϕ.
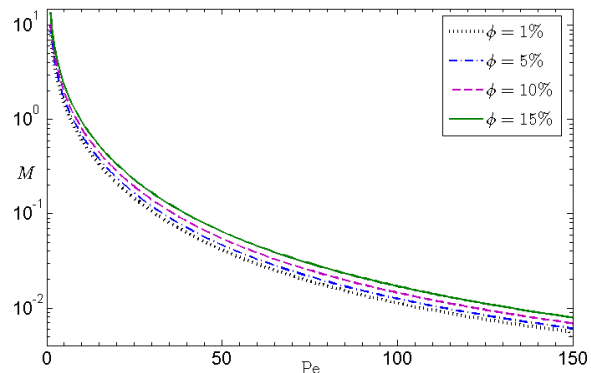

Fig. 6 Variation of the ratio of streamwise conduction to heat convection as a function of Peclet number, with volume fraction of nanoparticle being a parameter

V. CONCLUSION

It is observed that the nanofluid temperature distribution is a strong function of the Peclet number when the streamwise conduction is incorporated in the energy equation. The deviation of the nanofluid temperature distribution between the models with and without considering streamwise conduction effect increases as Peclet number decreases. A decrease in the Peclet number shortens the distance where the heat transfer to the nanofluid takes place along the channel length. The nanofluid temperature profiles of both models depict significant discrepancy when the Peclet number is sufficiently small. The nanoparticle volume fraction induces pronounced effect on the heat transfer contributed by the streamwise conduction. However, it is insensitive to the variation of thermal conductivity ratio. The heat transport by streamwise conduction decreases with the Peclet number while the reverse is true for the heat transfer by convection. For small Peclet number, the streamwise conduction dominates over the heat transport due to convection. Beyond $Pe = 10$, the heat transported by the fluid is mainly due to convection and the degree of significance of the effect of streamwise conduction decreases dramatically with Peclet number. It can be concluded that the effect of streamwise conduction on the nanofluidic flow in microchannel heat sink is significant albeit not dominant particularly for small Peclet number and high nanoparticle volume fraction of the nanofluid.

ACKNOWLEDGMENT

REFERENCES

[1] S.U.S. Choi, "Enhancing thermal conductivity of fluids with nanoparticles," *Developments and Applications of Non-newtonian Flows*, vol. ASME FED 231, pp. 99-105, 1995.

[2] W. Daungthongsuk, and S. Wongwises, "A critical review of convective heat transfer of nanofluids," *Renewable and Sustainable Energy Reviews*, vol. 11, pp.797-817, 2007.

[3] W. Yu, D.M. France, J.L. Routbort, and S.U.S. Choi, "Review and comparison of nanofluid thermal conductivity and heat transfer enhancements," *Heat Transfer Engineering*, vol. 29, pp. 432-460, 2008.

[4] S. Kakaç, and A. Pramuanjaroenkij, "Review of convective heat transfer enhancement with nanofluids," *International Journal of Heat and Mass Transfer*, vol.52, pp. 3187–3196, 2009.

[5] J.A. Eastman, S.U.S. Choi, S. Li, W. Yu, and L.J. Thompson, "Anomalously increased effective thermal conductivities of ethylene glycol-based nano-fluids containing copper nano-particles," *Applied Physics Letters*, vol. 78, pp. 718-720, 2001.

[6] S.U.S. Choi, Z.G. Zhang, W. Yu, F.E. Lockwood, and E.A. Grulke, "Anomalous thermal conductivity enhancement in nano-tube suspensions," *Applied Physics Letters*, vol. 79, pp. 2252-2254, 2001.

[7] Y. Xuan, and Q. Li, "Investigation on convective heat transfer and flow features of nanofluids," *Journal of Heat Transfer*, vol. 125, pp. 151-155, 2003.

[8] J.A. Eastman, S.U.S. Choi, S. Li, and L.J. Thompson, "Enhanced thermal conductivity through the development of nanofluids," *in Proceedings of the Symposium on Nanophase and Nanocomposite Materials II*, USA, 1997, pp. 3-11.

[9] S. Lee, S.U.S. Choi, S. Li, and J. A. Eastman, "Measuring thermal conductivity of fluids containing oxide nanoparticles," *Journal of Heat Transfer*, vol. 121, pp. 280-289, 1999.

[10] B.W. Wang, L.P. Zhou, and X.F. Peng, "A fractal model for predicting the effective thermal conductivity of liquid with suspension of nanoparticles," *International Journal of Heat and Mass Transfer*, vol. 46, pp. 2665-2672, 2003.

[11] T.V. Nguyen, "Laminar heat transfer for thermal developing flow in ducts," *International Journal of Heat and Mass Transfer*, vol. 35, pp. 1733-1741, 1992.

[12] J. Lahjomri, and A. Oubarra, "Analytical solution of the Graetz problem with axial conduction," *ASME Journal of Heat Transfer*, vol. 121, pp. 1078-1083, 1999.

[13] B. Weigand, and D. Lauffer, "The extended Graetz problem with piecewise constant wall temperature for pipe and channel flows," *International Journal of Heat and Mass Transfer*, vol. 47, pp. 5303-5312, 2004.

[14] C.Y. Zhao, and T.J. Lu, "Analysis of microchannel heat sink for electronics cooling," *International Journal of Heat and Mass Transfer*, vol. 45, pp. 4857-4869, 2002.

[15] G. Hetsroni, A. Mosyak, E. Pogrebnyak, and L.P. Yarin, "Heat transfer in micro-channels: Comparison of experiments with theory and numerical results," *International Journal of Heat and Mass Transfer*, vol. 48, pp. 5580-5601, 2005.

[16] A. Husain, and K.Y. Kim, "Optimization of a microchannel heat sink with temperature dependent fluid properties," *Applied Thermal Engineering*, vol. 28, pp. 1101-1107, 2008.

[17] S. Muniandy, and Y.M. Hung, "Analysis of streamwise conduction in forced convection of microchannels using fin approach," *Journal of Zhejiang University – Science A*, vol. 12, pp. 655-664, 2011.

[18] R.L. Hamilton, and O.K. Crosser, "Thermal conductivity of heterogeneous two components systems," *Industrial and Engineering Chemistry Fundamentals*, vol. 1, pp. 187-191, 1962.

[19] H.C. Brinkman, "The viscosity of concentrated suspension and solutions," *Journal of Chemistry Physics*, vol. 20, pp. 571-581, 1952.

[20] Y. Xuan, and W. Roetzel, "Conceptions for heat transfer correlation of nanofluids," *International Journal Heat Mass Transfer*, vol. 43, pp. 3701-3707, 2000.

[21] F.P. Incropera, D.P. Dewitt, T.L. Bergman, and A.S. Lavine, "Introduction to heat transfer," 5th ed., New York: Wiley, 2007, pp. 137-145.