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Strategy Analysis and Creation by Simulation in the General Game

Gábor Szűcs, Gábor Neszveda, and Xin Fang

***Abstract*—**In this paper the General Game problem is described. In this problem the competition or cooperation dilemma occurs as the two basic types of strategies. The strategy possibilities have been analyzed for finding winning strategy in uncertain situations (no information about the number of players and their strategy types). The winning strategy is missing, but a good solution can be found by simulation by varying the ratio of the two types of strategies. This new method has been used in a real contest with human players, where the created strategies by simulation have reached very good ranks. This construction can be applied in other real social games as well.

***Keywords*—**competition, cooperation, finding good strategy, General Game,

I. Introduction

THE foundation of Game Theory was laid by John von Neumann in 1928. He proofed the Theorem Minimax, which is now the basic Theorem of the Game Theory. In the Game Theory the games have been divided into two classes: cooperative and non-cooperative games according to the players are communicating each other or not.

There are many cases when we can not give a theoretically good answer like in the case of the General Game [3]. One player's success depends on the number of players, on all the strategies they give one-by-one and we do not have any previous information on their behavior.

We do not know the number of the players either their behavior in the General Game, but if we had a good parameterized model for analyzing the possible behavior of the whole group then we could give good solutions which could significantly outperform the average.

This problem can be observed in many fields in the world. People face these kinds of problems when they try to optimize the marketing budget of a party among the districts [1]. But many enterprises could face with the same situation when there is a market which was ruled by a big monopoly but the monopoly disappears and many competitive enterprises join the market and there is no relevant, available information, but a big concurrence.

G. Szűcs is with the Department of Telecommunications and Media Informatics, Budapest University of Technology and Economics, Budapest, Hungary (corresponding author; e-mail: szucs@tmit.bme.hu).
X. Fang is student on Corvinus Egyetem Budapest, Hungary
G. Neszveda is student on Corvinus Egyetem Budapest, Hungary

There are lot of games (e.g. Othello [8], Random 2x2 constant sum games, Noisy 2x2 games [9], etc.), which can be found in the international literature showing the wide range of interests.

II. The General Game

A. The Rules of the General Game

There are many types of what we call the General Game. The Colonel Blotto's game [2] is the most known from all. But we created a bigger and more complex problem with a bit different rules based on Mérő ideas [3].

There are 12 towns that you can occupy and you have 120 soldiers. In each game there are two players and both of them give a strategy which contains that how many soldiers they send to each town. There is an order in the towns and you can not send more soldiers in a town than the number of soldiers that you sent in the previous town. It means that if you sent 30 soldiers to the first town, you can not send more than 30 soldiers to the second town.

In each game there are two strategies playing against each other and they got points after the towns they occupy. It is worth 5 points of occupying a town, which means that the occupier sent more soldiers than the opposite player. In the case when the opposite player did not send any soldier to that town then that town is worth only 1 point. In the case when both players sent the same number of soldiers to a town they get 2-2 points. If there is a town, where nobody sent any soldiers then nobody gets point for that town.

In the competition everybody has to give one strategy and this strategy plays against every other strategy one by one. The final rank is based on the total points that one player got from all the game that he played.

B. An Example for the General Game

Let us create an example for the General Game as can be seen in the Table I.

TABLE I
EXAMPLE FOR THE GENERAL GAME

Towns	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
A's strategy	*15*	*15*	*15*	*15*	*10*	*10*	*10*	*10*	*8*	*8*	*4*	*0*
B's strategy	*30*	*30*	*15*	*15*	*6*	*6*	*6*	*6*	*6*	*0*	*0*	*0*
Points for A	0	0	2	2	5	5	5	5	5	1	1	0
Points for B	5	5	2	2	0	0	0	0	0	0	0	0

In this example A gets 0+0+2+2+5+5+5+5+5+1+1+0=31 points and B gets 5+5+2+2+0+0+0+0+0+0+0+0 = 14points. If there is "n" number of player then A and B have to play "n-1" games like this and the final rank comes from the total points they got from these "n-1" games.

C. The Aim of the Game

When this game was created by Mérő [3] with very similar rules, he wanted to observe the perfect mixture of competition and cooperation. In this General Game if we send soldiers to a lot of town, it means that we cooperate more, because all together there is more points to collect. If we concentrate our soldiers into the first towns it means that we are more competitive and we want to win against our opponent rather than collect more points. The most cooperative strategy if we send 10 soldiers to all the 12 towns. It can be very efficient for cooperating but it can be abused very easily at the same time.

D. Competition or Cooperation

There are three general strategies of conflict resolution in interpersonal relationships:

- Avoidance behaviors: People employ no or indirect communication with denial, equivocation, changing the subject, noncommittal remarks, unfocused or rephrasing the question, joking.
- Competitive behaviors: Persons involve negative communication with confrontative remarks, personal criticism, rejection, hostile questioning, sarcasm, denial of responsibility.
- Cooperative behaviors: People involve open and positive communication with describing the problem, analytical remarks, open disclosure, soliciting criticism, great empathy, ability to concessions, accepting responsibility.

Avoidance behaviors can not be implemented in the games, so the rest (competition or cooperation) lead to dilemma. This could be the key problem of any strategy maker in the games. There is a problem which is famous for this question, the prisoner's dilemma [4]. This basic prisoner's dilemma can be extended to several (N) people, this can be called N-person prisoner's dilemma [14]. The simplest version of this game, when each person should select between two alternatives: C and D, C represents the intention of the people to cooperation with others and D represents the uncooperative behavior, which leads to defection. Each player who selected C causes each of the other persons to receive $1. Each player who selected D gets $1, but this has no effect on the payoff for others. If everyone selects C, each gets N-1; in case of everyone selects D, each gets 1. Maximal gain is N, when everyone except one player selects C and this player select D.

There is another extension of this problem, the iterated prisoner's dilemma [5]. It was the same question that how much competition and how much cooperation we should use in the well-known prisoner's dilemma if it is iterated. The Tit-for Tat strategy turned out to be the best, which tries to cooperate but if the other is not partner for that he change immediately to compete. According to the experience of the Iterated Prisoner's Dilemma we could think that we will find something similar to this in the General Game. We should cooperate, but compete a little at the same time.

E. Previous Experiments in the General Games

There are several experiments for the Colonel Blotto's game, but we had very important differences in our rules (non-increasing number of soldiers) which makes it incomparable. But László Mérő made some experiments [3] on little groups with a bit different rules. In that game there were only 12 soldiers, not 120. He found that "the winning" strategy was to send 2 soldiers to the first six towns. In three different small groups experiment showed this same result. He interpreted this as the balance of competition and cooperation (because it cooperates, but in half way it competes as well).

F. Missing Winner Strategy

In general, the summation of pay-off functions of two persons is not zero. A finite (nonzero-sum) two-person game is usually referred to as a bimatrix game, since it is completely determined by the pay-off matrices of the players. The game is given by the pair of (A, B) pay-off matrices, and we can define the mixed extension of the game, where the average pay-off E_1 and E_2 (belonging to player 1 and player 2 respectively) are the following:

$$E_1 = p^T \cdot A \cdot q \qquad E_2 = p^T \cdot B \cdot q \tag{1}$$

where the p and q are the distribution vectors of the two players. The pair of (p^0, q^0) strategies is the Nash equilibrium point, if:

$$E_1(p^0,q^0) \geq E_1(p,q^0) \qquad \forall p \tag{2}$$

$$E_2(p^0,q^0) \geq E_2(p^0,q) \qquad \forall q \tag{3}$$

To create a winner strategy in the competition we should know all the strategies which play (we do not know that) and even if knew all the others' strategies, with hundreds of players it would be still a very complex mathematical problem.

First of all there is in no dominant strategy. There is no strategy which could not be defeated by some other strategy. In order to verify this claim let us suppose two types of strategies. One is when the strategy is that we send all the 120 soldiers to the first town, the second type when we do something different.

The first type of strategy can be beaten by the strategy when we send 10 soldiers to each town. The other type of the strategies can be beaten by a strategy which is totally the same but to the last town we send one soldier less and to the first town we send one soldier more.

It means none of the strategy is unbeatable in a game. And there is no Nash equilibrium among the pure strategies if we have two players.

We have already seen that there is no dominant strategy and that even with knowing all the other strategies it is hard to say a winner strategy. But we do not know the strategies even the number of the players. In this case we could analyze the competition with only two players. There is no pure Nash equilibrium (but in our case we deal with not only the Nash equilibrium but the social dilemmas [7]). There must be a mixed Nash equilibrium **Hata! Başvuru kaynağı bulunamadı.**, but every player has more than 76 million possible strategies. In this case even with a computer it would be hard to calculate the mixed Nash equilibrium. Not to mention if we have hundreds of players and not to mention that we do not know the number of players either.

Given these facts we chose to use the method of simulations and we do not want to find optimal solution, but "good". Good solution is a strategy if it gets more points than the average plus variation.

III. The General Game with the Approach of Simulation

A. Simulation Approach

The investigation of highly complex and ill-defined systems is becoming more and more significant in a large number of fields. This applies in particular to problems of economy and related social sciences; there is a tool for problem with heuristic approach: simulation [13].

The work presented here is a part of investigations based on a scientific philosophy of inductive-deductive view. Many problems from various areas (in a wide range of fields, e.g. financial, marketing problems, micro-, and macroeconomy) are collected and it is intended to find solutions which can be applied in a large class of problems. A lot of systems may be investigated by simulation, and it is highly advantageous to use behavior oriented simulation model descriptions. The difficulty of determining the optimal solution is caused not only by the high complexity of the human behavior but also by the facts that such systems are ill-defined.

The Monte Carlo simulation is well-known from the place of Monte Carlo and its gambling history. In the Second World War it became a very popular method to analyze complex problems. This method is efficient to use when the complexity of the problem consumes the capacity of the calculation. This method can be used for approximation in incomplete information in some games [10], for estimation in discrete choice game theoretic models [11].

We used the Excel for building up our simulation which satisfies all the conditions for random numbers that required.

B. The model for the General Game

In our analysis we used 500 strategies sample. And we generated two types of strategy a cooperative one and a competitive one. The sample always contained only these types of strategies. We call the cooperative strategy type "Independent Uniform Distribution" (from now "IUD") and the competitive strategy type "Digressive".

C. IUD and Digressive Strategy

In this case we used IUD to generate 12 numbers then we normalized their sum to 120 and we put them into a non-increasing order to get an appropriate strategy. In this case the number of soldiers to any town is around 10 with a decreasing order.

In the Fig. 1 can be seen 245 strategies which were generated by the method of cooperative strategies. On the X-scale we can see the 245 strategies and on the Y-scale we can see the number of soldiers. The 12 different dots show the number of the soldiers in the 12 different towns. Because of the method we can see that the biggest number represents the number of soldiers in the first town.

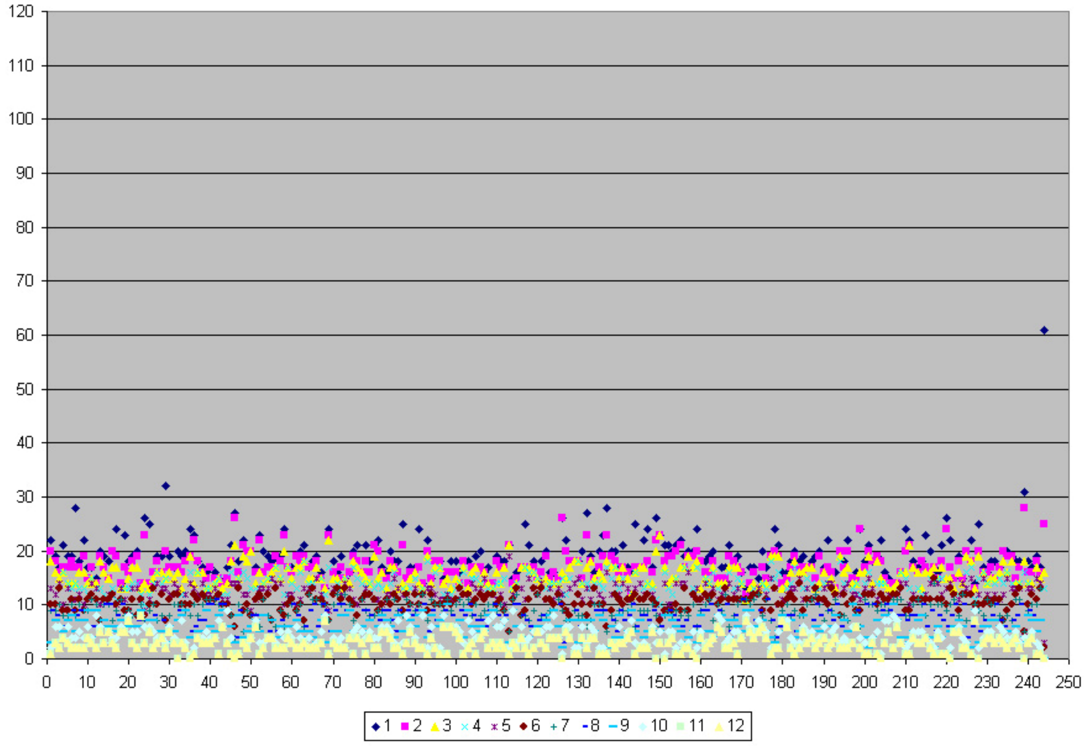

Fig. 1 Generated sample by IUD (245 strategies)

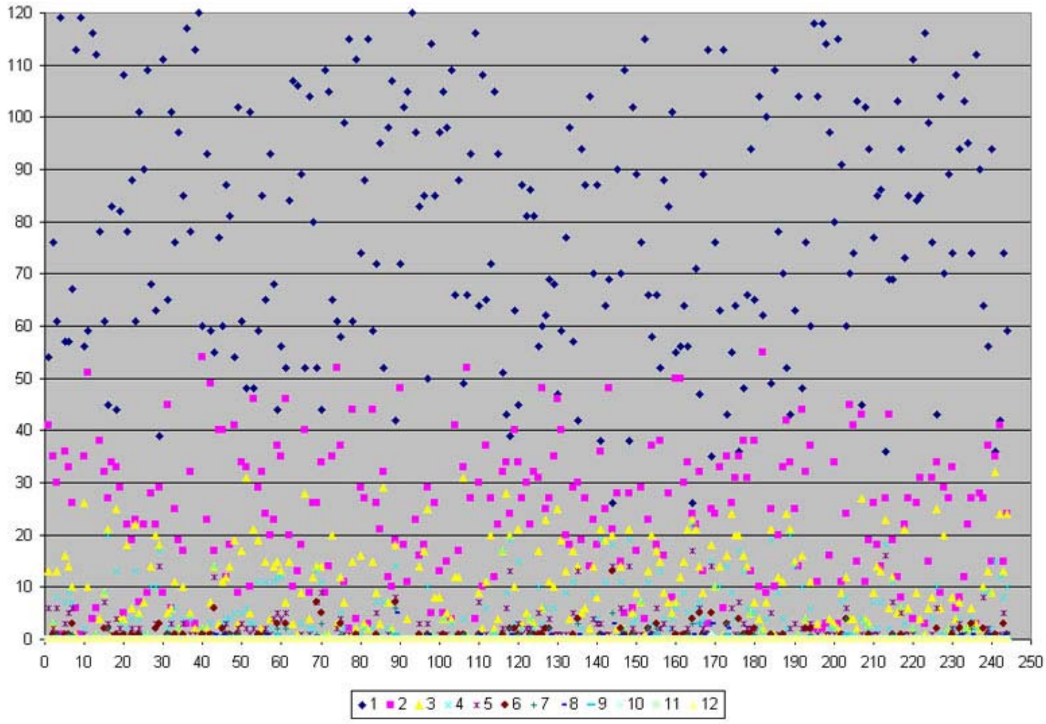

Fig. 2 Generated sample by digressive type

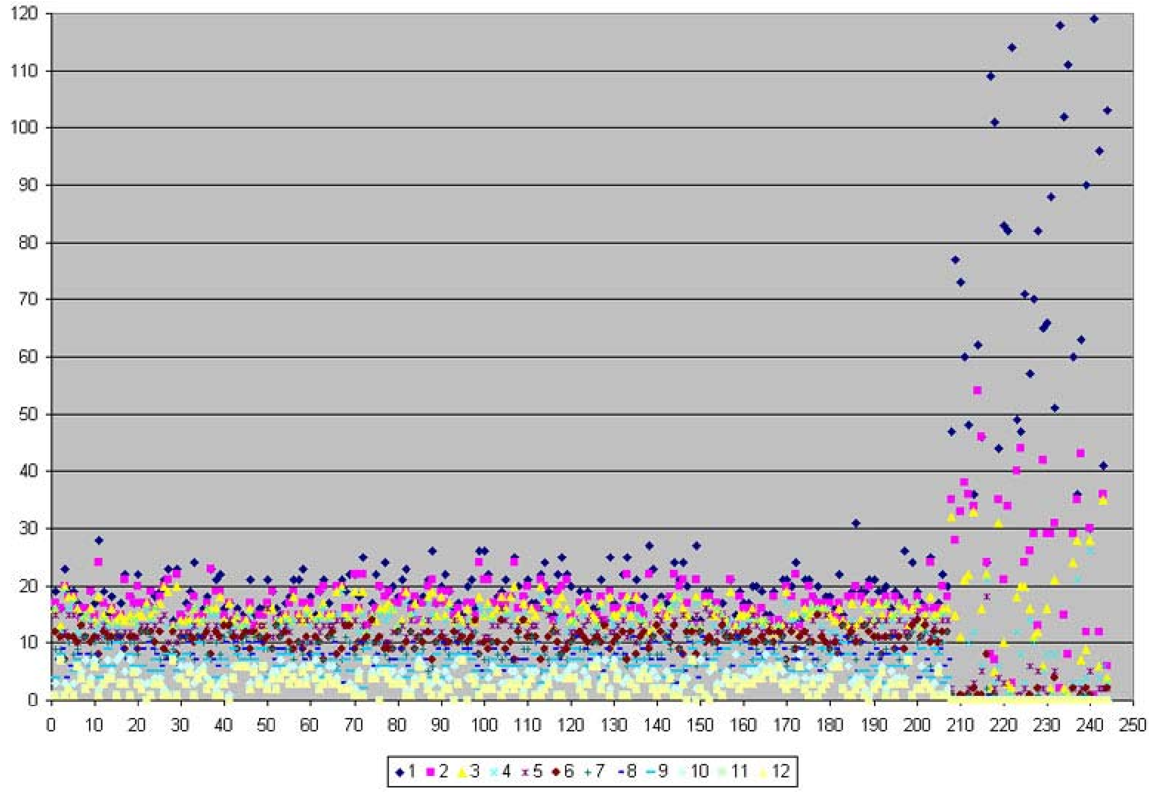

Fig. 3 Generated 85% IUD sample

Digressive strategy: In this case we take an interval of 0-120 and with a uniform distribution we generate a number (called x) out of it. Then from 0-(120-x) we make another random number with the same method. And we generate 12 numbers like this. Usually at the end they are 0. We put then into a non-increasing order to get an appropriate strategy.

In the Fig. 2 can be seen 245 digressive strategies. On the X-scale the strategies on the Y-scale the number of soldiers and the 12 different dots represent the 12 towns. We can see that most of the soldiers are concentrated in the first 3-4 towns. Comparing to the IUD strategies it has a much more variation and the digressive strategy type focus usually on the first 8 towns. It means that the digressive strategy type are more competitive type because it usually does not send any soldier to the last four town, but the IUD almost always sends at least 1 or two soldiers to each town.

D.Set of the strategy samples

It is crucial of reconstructing the real strategy sample and to find good solutions to analyze and create appropriate strategy samples. We observe strategy samples which consist only of the IUD or digressive strategy type. So a strategy cluster can be 100% of IUD or Digressive but it could consist a mixture of the IUD and digressive strategies too.

On the Fig. 3 can be seen a strategy sample which consists 85% IUD and 15% digressive strategy type. From now we will classify the clusters by the percentage of the IUD strategies. Like in the previous example we had an 85% IUD cluster.

E. Finding good strategies

To find equilibria of mixed strategy in noncooperative games of incomplete information is very hard [12]. We can not determine optimal strategy, not even any method to say generally optimal strategies, because it always depends on the specific sample.

We define the good strategy on a given sample with a given mixture of IUD and digressive as a strategy which can win at least five times on different but same mixture of samples.

We find good solution by simulations where we create a sample with a given mixture then we choose the winner strategy. We keep this strategy and create another sample. If this strategy wins then we keep it, if another wins then we keep the new winner. We do it until one strategy wins five times in a row.

TABLE II
OPTIMIZED STRATEGIES ON DIFFERENT SAMPLES

X	FAE %	1	2	3	4	5	6	7	8	9	10	11	12
1	0%	24	22	19	15	13	6	5	5	4	3	2	2
2	5%	25	23	21	17	11	7	6	5	2	1	1	1
3	10%	21	20	20	19	15	7	4	3	3	3	3	2
4	15%	20	19	19	18	15	13	4	3	3	3	2	1
5	20%	20	20	19	14	14	13	5	4	3	3	3	2
6	25%	20	20	18	17	14	13	11	2	2	1	1	1
7	30%	18	18	17	15	14	13	13	3	3	2	2	2
8	35%	19	19	18	14	14	12	10	3	3	3	3	2
9	40%	20	20	18	17	14	13	11	2	2	1	1	1
10	45%	21	19	18	17	6	6	6	6	6	6	5	4
11	50%	18	18	17	15	15	12	11	3	3	3	3	2
12	55%	21	19	18	17	6	6	6	6	6	6	5	4
13	60%	13	13	12	12	12	12	11	10	9	7	5	4
14	65%	13	12	12	12	12	12	12	9	9	7	6	4
15	70%	13	13	13	13	13	12	11	11	8	6	4	3
16	75%	13	13	13	13	13	12	12	9	7	6	6	3
17	80%	13	12	12	12	12	12	12	9	9	7	6	4
18	85%	13	13	13	13	13	12	12	9	7	6	6	3
19	90%	13	13	13	13	13	12	12	9	7	6	6	3
20	95%	14	13	13	13	13	12	11	9	8	6	5	3
21	100%	13	13	13	13	13	13	11	10	7	6	4	4

TABLE III
THE "ALL 10" STRATEGY PERFORMANCE IN DIFFERENT SAMPLES

FAE	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Points	10377.67	10743.33	11096.33	11396.67	11825.33	12195	12680	12982	13406	13944	13922
Str Failure	1.800%	10.000%	19.733%	28.000%	36.067%	42.133%	47.600%	50.600%	51.600%	43.800%	57.400%
FtF Failure	16.9%	19.9%	21.3%	25.3%	28.7%	31.1%	34.2%	37.0%	40.6%	40.2%	50.6%
Opp. AVG	5853	6308	6883	7436	8239	9020	9988	10792	11740	12826	13837
Opp. StDev	2111	2626	3078	3362	3692	3834	3923	3770	3566	2849	1041
Effectiveness	214.2%	168.9%	136.9%	117.8%	97.2%	82.8%	68.6%	58.1%	46.7%	39.2%	8.2%

In the Table II can be found a good strategy for each mixture sample on a scale of 5%, where FAE represents the percentage of the IUD strategy type. It is interesting to see that in each case a good strategy sent soldiers to each town, even in the totally competitive one, in the 0% sample.

F. Sensitivity analysis

Even though finding a good strategy is a very uncertain method itself, it depends really on the rate of the mixture of the sample. In the sensitivity analysis we observe that one strategy is how sensitive to the change of the mixture in a sample. We use the scale of 5% or 10% of rate of mixture in our analysis.

According to our simulations and analysis, being a good strategy is not sensitive to the sample size but the rate of mixture. For analyzing this problem we created a program which makes 11 samples with different rate of mixture and makes a statistic about a strategy in each case.

In the Table III can be seen the strategy all 10's performance in each case. So the strategy which sends 10 soldiers into each town has this result.

In the table *FAE* represents the percentage of the IUD strategy type, *Points* represents the points that the given strategy collected, *Str Failure* represents the percentage of the strategies which had better total points than the given strategy, *FtF Failure* represents the percentage of the strategies which could beat the given strategy in face-to-face, *Opp. AVG* represents the average total points of the opposite players, *Opp StDev* represents the variation of the total points of the opposite strategies, *Effectiveness* gives the percentage that how much the given strategy exceed the average using the variation. According to this analysis we can state that a given strategy is more stable if it has a 0% Str Failure in more cases.

This table has an interesting conclusion about this strategy (all 10) which strategy can be represented as the totally cooperative strategy. This strategy has the highest rank in the totally competitive environment, but being in a more and more cooperative environment he has more and more total points even though he get lower and lower in the rank.

It gives us two conclusions. Firstly, being cooperative can be a very good strategy even in the most competitive environment. Secondly, getting more points does not mean ultimately to be higher in the rank.

IV. Experiment

A. Participants of the contest

We have organized a game with the rules of the General Game and we let 3 weeks for everyone at the University of Corvinus Budapest for participating. There were some precious prizes for the best. Everyone who attended in this game did that voluntary. They could send their strategy by internet or by paper. Everyone could send only one strategy and every strategies had to have a unique ID which was the unique University ID of the student.

At the end of the three weeks we have collected all the strategies and we have added 15 own strategies created by the simulations. All together there were 245 strategies in this experiment.

B. Preliminary sampling

The condition of finding a good solution is to know something about the sample. Without any previous research we can not assume anything about the sample's parameter. But we suppose that there is only cooperative (IUD) and competitive (digressive) strategy type and that there will be more than one hundred strategies which means that the sample size will not significantly matters.

We made a mini sample on university students (given the fact that students will play in the game as well). Eleven students were asked to create a strategy if they should plan in this game. They gave the strategies as can be seen in Table IV.

From the Table IV we have to have prediction about the IUD rate in the experiment. We have generated different samples with a scale of 5% IUD differences and we have calculated the average number of soldiers in each town and standard variation of each town. We have calculated the absolute differences between the generated samples and the preliminary sample and we have created an index number as the sum of the squares of the absolute differences. The best fitting sample has the smallest index.

According the Tables V-VIII and statistics we suppose a strong cooperative environment which can be represented the best in the 90% IUD sample. Technically it means that the typical strategy sends soldiers even to the last town.

TABLE IV
RESULTS OFTHE PRELIMINARY SAMPLE

X	1	2	3	4	5	6	7	8	9	10	11	12	Total points
1	12	12	12	12	12	12	12	12	12	4	4	4	379
2	11	11	11	11	11	11	11	11	11	11	5	5	353
3	18	18	18	18	18	18	2	2	2	2	2	2	338
4	15	15	15	15	10	10	10	10	5	5	5	5	331
5	15	15	15	15	15	15	15	15	0	0	0	0	330
6	10	10	10	10	10	10	10	10	10	10	10	10	283
7	12	12	11	11	10	10	10	10	9	9	8	8	279
8	30	25	20	10	10	10	8	5	1	1	0	0	232
9	35	34	33	2	2	2	2	2	2	2	2	2	228
10	60	20	20	20	0	0	0	0	0	0	0	0	201
11	50	40	10	5	5	4	1	1	1	1	1	1	152

TABLE V
DISTANCE IN AVERAGE BETWEEN PRELIMINARY SAMPLE AND GENERATED SAMPLES

	AVG											
FAE %	1	2	3	4	5	6	7	8	9	10	11	12
0%	74.6	25.5	10.9	4.8	2.4	1.1	0.5	0.2	0.1	0.0	0.0	0.0
5%	73.2	25.5	10.6	5.0	2.5	1.3	0.8	0.5	0.3	0.2	0.1	0.1
10%	70.3	24.0	11.1	5.7	3.2	2.0	1.3	0.9	0.7	0.5	0.3	0.1
15%	66.6	24.7	11.1	6.3	3.6	2.4	1.8	1.3	0.9	0.7	0.4	0.2
20%	62.8	24.7	11.7	6.9	4.2	2.9	2.1	1.6	1.2	0.9	0.6	0.3
25%	61.3	23.8	11.3	7.0	4.7	3.4	2.6	2.1	1.6	1.1	0.7	0.3
30%	59.4	22.3	12.0	7.4	5.2	3.9	3.0	2.3	1.8	1.3	0.9	0.4

35%	56.7	22.4	12.2	8.0	5.7	4.3	3.4	2.7	2.0	1.4	0.9	0.4
40%	52.5	22.6	12.6	8.3	6.1	4.8	4.0	3.1	2.4	1.8	1.2	0.6
45%	50.6	22.1	12.7	8.8	6.6	5.3	4.2	3.4	2.6	1.9	1.3	0.6
50%	47.3	21.5	13.0	9.4	7.2	5.8	4.7	3.8	3.0	2.2	1.5	0.7
55%	44.0	20.5	13.4	10.0	7.9	6.4	5.2	4.2	3.4	2.5	1.6	0.8
60%	41.4	20.6	13.6	10.3	8.3	6.8	5.7	4.5	3.5	2.7	1.8	0.8
65%	38.6	20.6	13.8	10.6	8.7	7.3	6.1	5.0	3.9	2.8	1.8	0.9
70%	36.6	19.2	14.4	11.3	9.2	7.7	6.4	5.3	4.1	3.0	1.9	1.0
75%	33.2	19.7	14.3	11.7	9.8	8.3	6.8	5.6	4.4	3.2	2.1	1.0
80%	30.7	18.9	14.6	12.0	10.1	8.6	7.3	5.9	4.8	3.6	2.3	1.1
85%	27.6	18.2	14.7	12.4	10.7	9.2	7.9	6.5	5.2	3.9	2.6	1.2
90%	**24.6**	**18.1**	**15.1**	**12.9**	**11.2**	**9.7**	**8.2**	**6.8**	**5.4**	**4.0**	**2.6**	**1.3**
95%	22.1	17.6	15.4	13.5	11.8	10.1	8.6	7.1	5.6	4.2	2.8	1.4
100%	18.8	17.1	15.5	13.8	12.2	10.8	9.2	7.7	6.1	4.5	3.0	1.4
AVG	**47.3**	**21.4**	**13.0**	**9.3**	**7.2**	**5.8**	**4.7**	**3.8**	**3.0**	**2.2**	**1.5**	**0.7**

TABLE VI
DISTANCE IN STD BETWEEN PRELIMINARY AND GENERATED SAMPLES

	StDev											
FAE %	1	2	3	4	5	6	7	8	9	10	11	12
0%	22.9	13.2	8.3	4.8	3.0	1.7	1.0	0.7	0.4	0.2	0.0	0.0
5%	24.9	13.5	8.3	5.1	3.3	2.6	2.0	1.6	1.4	1.1	0.7	0.3
10%	28.4	13.2	7.8	5.5	4.2	3.4	2.8	2.4	1.9	1.4	1.0	0.6
15%	28.9	13.3	7.6	5.5	4.3	3.8	3.4	2.8	2.2	1.8	1.2	0.8
20%	29.9	12.6	7.4	5.7	4.8	4.3	3.7	3.1	2.6	2.0	1.4	0.9
25%	31.6	13.0	7.1	5.7	5.1	4.6	4.1	3.4	2.8	2.2	1.5	0.9
30%	32.7	11.4	7.5	5.7	5.3	4.7	4.2	3.5	2.9	2.2	1.7	1.0
35%	33.2	11.7	7.1	6.0	5.5	5.0	4.3	3.6	2.9	2.2	1.6	0.9
40%	32.6	11.7	6.7	5.8	5.4	5.0	4.4	3.7	3.1	2.4	1.8	1.1
45%	32.9	11.1	6.7	6.0	5.5	5.0	4.4	3.8	3.1	2.5	1.8	1.2
50%	32.4	10.9	6.1	5.7	5.4	5.0	4.6	3.8	3.2	2.5	1.9	1.2
55%	32.5	10.4	6.0	5.7	5.4	5.0	4.5	3.9	3.2	2.6	1.9	1.2
60%	31.0	9.7	5.7	5.6	5.4	5.1	4.5	3.9	3.2	2.6	1.9	1.2
65%	30.0	9.9	5.5	5.5	5.3	5.0	4.4	3.8	3.2	2.5	1.8	1.2
70%	29.5	7.8	5.5	5.1	5.1	4.7	4.2	3.7	3.0	2.4	1.9	1.3
75%	27.1	8.7	5.2	5.0	4.8	4.5	4.0	3.5	3.0	2.4	1.8	1.2
80%	25.4	6.9	4.7	4.6	4.5	4.2	3.8	3.3	2.9	2.4	1.9	1.3
85%	22.9	5.9	4.1	3.9	4.0	3.8	3.5	3.1	2.7	2.3	1.8	1.4
90%	**18.4**	**5.4**	**3.5**	**3.4**	**3.5**	**3.3**	**3.0**	**2.7**	**2.4**	**2.1**	**1.7**	**1.3**
95%	14.2	3.7	2.9	2.8	2.7	2.6	2.6	2.3	2.1	1.9	1.7	1.4
100%	3.0	2.5	1.9	1.6	1.6	1.6	1.7	1.8	1.8	1.8	1.7	1.2
AVG	**26.9**	**9.8**	**6.0**	**5.0**	**4.5**	**4.0**	**3.6**	**3.1**	**2.6**	**2.1**	**1.6**	**1.0**

TABLE VII
INDEX FOR THE DISTANCE IN AVERAGE

FAE %	AVG												Sum of squares
0%	47.7	6.3	5.0	6.9	7.0	8.2	6.9	6.9	4.7	4.1	3.4	3.4	2665.2
5%	48.8	6.2	5.3	6.7	6.9	8.0	6.6	6.6	4.5	3.9	3.2	3.3	2749.4
10%	45.9	4.7	4.8	6.0	6.2	7.3	6.1	6.2	4.2	3.6	3.1	3.2	2404.6
15%	42.2	5.5	4.8	5.4	5.8	6.9	5.6	5.8	3.9	3.4	2.9	3.1	2052.8
20%	38.4	5.5	4.2	4.9	5.1	6.4	5.3	5.5	3.6	3.2	2.8	3.1	1709.6

25%	36.9	4.5	4.6	4.7	4.6	5.9	4.7	5.0	3.2	3.0	2.6	3.0	1566.6
30%	35.0	3.1	3.9	4.3	4.2	5.4	4.4	4.7	3.0	2.8	2.5	2.9	1389.5
35%	32.3	3.1	3.7	3.8	3.7	5.0	4.0	4.4	2.8	2.7	2.4	2.9	1187.9
40%	28.1	3.4	3.3	3.5	3.3	4.4	3.4	4.0	2.4	2.3	2.2	2.8	907.3
45%	26.2	2.8	3.2	3.0	2.8	4.0	3.1	3.7	2.2	2.2	2.1	2.7	784.5
50%	23.0	2.2	2.9	2.3	2.2	3.5	2.6	3.3	1.9	1.9	1.9	2.6	598.5
55%	19.6	1.3	2.5	1.7	1.4	2.9	2.2	2.9	1.5	1.6	1.7	2.6	433.6
60%	17.0	1.3	2.3	1.5	1.0	2.4	1.7	2.6	1.3	1.4	1.6	2.5	328.9
65%	14.2	1.3	2.1	1.1	0.7	2.0	1.3	2.1	0.9	1.2	1.6	2.5	231.5
70%	12.2	0.0	1.5	0.5	0.1	1.6	1.0	1.8	0.7	1.1	1.4	2.4	168.0
75%	8.8	0.4	1.6	0.0	0.5	1.0	0.5	1.5	0.4	0.9	1.2	2.4	92.1
80%	6.4	0.3	1.3	0.3	0.7	0.7	0.1	1.2	0.0	0.5	1.0	2.2	51.1
85%	3.3	1.1	1.2	0.7	1.3	0.1	0.5	0.6	0.4	0.2	0.8	2.1	21.6
90%	**0.2**	**1.2**	**0.8**	**1.2**	**1.9**	**0.4**	**0.9**	**0.3**	**0.6**	**0.1**	**0.7**	**2.1**	**13.4**
95%	2.3	1.7	0.5	1.7	2.4	0.9	1.2	0.0	0.8	0.1	0.6	2.0	24.2
100%	5.6	2.1	0.4	2.1	2.9	1.5	1.8	0.6	1.3	0.4	0.4	2.0	60.4
AVG	**23.5**	**2.8**	**2.9**	**3.0**	**3.1**	**3.7**	**3.0**	**3.3**	**2.1**	**1.9**	**1.9**	**2.7**	**641.4**

TABLE VIII
INDEX FOR THE DISTANCE IN STDDEV

FAE %	StdDev												Distance in StdDev
0%	6.4	3.8	1.8	0.2	2.1	3.4	3.9	4.3	4.1	3.7	3.2	3.2	160.5
5%	8.5	4.1	1.8	0.1	1.8	2.6	2.9	3.3	3.2	2.8	2.5	2.9	153.6
10%	11.9	3.8	1.3	0.5	0.8	1.8	2.1	2.6	2.6	2.5	2.2	2.6	198.2
15%	12.4	3.8	1.1	0.4	0.7	1.3	1.5	2.1	2.3	2.2	2.0	2.5	199.2
20%	13.4	3.1	0.9	0.7	0.2	0.9	1.2	1.9	2.0	1.9	1.8	2.3	213.6
25%	15.2	3.6	0.6	0.7	0.1	0.5	0.9	1.5	1.7	1.8	1.7	2.3	261.0
30%	16.3	2.0	1.0	0.7	0.3	0.4	0.7	1.5	1.6	1.7	1.6	2.3	286.4
35%	16.8	2.2	0.6	1.0	0.5	0.1	0.6	1.3	1.7	1.7	1.7	2.3	303.2
40%	16.1	2.3	0.2	0.8	0.4	0.2	0.5	1.2	1.4	1.5	1.5	2.1	277.6
45%	16.5	1.7	0.2	1.0	0.5	0.1	0.5	1.1	1.5	1.5	1.4	2.1	286.9
50%	15.9	1.5	0.4	0.7	0.4	0.1	0.3	1.1	1.4	1.4	1.4	2.0	268.3
55%	16.0	1.0	0.5	0.7	0.3	0.1	0.4	1.1	1.3	1.3	1.3	2.0	269.4
60%	14.6	0.2	0.8	0.6	0.4	0.1	0.4	1.1	1.3	1.4	1.3	2.1	224.4
65%	13.5	0.4	1.0	0.5	0.3	0.1	0.5	1.1	1.3	1.4	1.4	2.0	195.5
70%	13.0	1.6	1.0	0.1	0.0	0.4	0.8	1.3	1.5	1.5	1.4	2.0	185.9
75%	10.6	0.8	1.3	0.0	0.3	0.6	0.9	1.5	1.6	1.5	1.4	2.0	129.2
80%	9.0	2.5	1.8	0.4	0.5	0.9	1.1	1.6	1.7	1.5	1.4	1.9	106.1
85%	6.5	3.5	2.4	1.1	1.0	1.3	1.4	1.9	1.8	1.6	1.4	1.9	80.3
90%	**1.9**	**4.1**	**3.0**	**1.6**	**1.5**	**1.8**	**1.9**	**2.2**	**2.1**	**1.8**	**1.6**	**1.9**	**60.1**
95%	2.3	5.7	3.6	2.2	2.3	2.5	2.4	2.6	2.4	2.0	1.5	1.8	95.1
100%	13.4	7.0	4.6	3.4	3.5	3.5	3.2	3.2	2.7	2.1	1.6	2.0	324.2
AVG	**11.9**	**2.8**	**1.4**	**0.8**	**0.9**	**1.1**	**1.3**	**1.9**	**2.0**	**1.8**	**1.7**	**2.2**	**174.6**

C. Constructing good strategies

From the statistics we got a 90% IUD sample prediction. Before the preliminary sample the participants expected a 80% IUD sample. Finally we created 15 strategies based on this information and we created 12 strategies optimized to a specific sample type and 3 strategies were given by the people who were working on this field and analysis (us) as a comparison of simulated strategies and more intuitive strategies. In both case all the available information was used, but in the first case a computer based algorithm made the strategies and in the second case a human being.

First we created samples with 75% 80% 82,5% 92,5% 97,5% IUD rates and in that samples we were looking for good strategies by simulations.

We created two good strategies optimized in 100% IUD sample because we have found a very strong cooperative environment in our preliminary sample and our analysis showed that cooperative strategies generally have good results.

And to make sure that this method will give good answers we created strategies which were optimized on 20% 25% 40% 45% 50% IUD sample. These strategies were also good to compare their results to the more cooperative ones.

And as we mentioned there were three strategies which were made by human beings with all the available information.

D.Outcome of the experiment

In the experiments everybody participated voluntary from all the departments and faculties of the University. There were 245 strategies in the game from which 15 were ours. As for the outcome, 7 strategies of ours were in the top 10, but not the first.

All the strategies were better than the average and most of them 9 out of 15 were better than the average plus standard deviation. These results show that the optimizing method was successfully.

In the table IX can be seen our strategies, their rank, their total points and their IUD rate sample in which they were optimized. We can state that the human strategies significantly were not different from the strategies which were generated by the simulations. Important to notice that human strategies were also made by knowing all the information from the simulations and analysis.

On the other hand, we can state that the more cooperative strategies were more successful. The strategies optimized less than 50% IUD sample were at the end of the list and the strategies with more than 50% IUD were at the top of the rank. It turned out that our expectation was true, so the environment was more cooperative than competitive and in this case the strategies which were optimized to this environment condition were more successful.

Among the top 20 strategies' total points there were less than 2% differences, which mean that among them the fortune played a big role.

TABLE IX
THE RESULTS OF THE GENERATED STRATEGIES

Rank	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**Total points**	**Opt. IUD %**
2	13	13	13	13	13	12	12	9	7	6	6	3	7495	75%
3	12	12	12	12	12	12	12	12	12	4	4	4	7491	Human
4	13	13	13	13	13	13	11	10	7	6	4	4	7489	100%
5	12	12	12	12	12	12	12	11	9	8	5	3	7477	100%
7	13	13	13	13	13	11	11	10	9	6	5	3	7401	92.50%
8	13	12	12	12	12	12	12	9	9	7	6	4	7384	80%
9	14	13	13	13	13	13	11	10	8	6	4	2	7365	82.50%
11	13	13	13	13	12	12	12	12	9	7	3	1	7350	Human
15	13	13	13	13	13	12	11	9	9	8	4	2	7317	97.50%
34	18	18	17	15	15	12	11	3	3	3	3	2	7104	50%
67	20	20	19	14	14	13	5	4	3	3	3	2	6839	20%
68	22	18	18	16	16	12	4	3	3	3	3	2	6829	Human
81	20	20	18	17	14	13	11	2	2	1	1	1	6730	25%
82	20	20	18	17	14	13	11	2	2	1	1	1	6730	40%
87	21	19	18	17	6	6	6	6	6	6	5	4	6677	45%

V. Discussion and Conclusion

A. Investigation

The general game has the complexity and the uncertainty which makes it very hard to handle and analyze. But in many case we have to face decisions like this.

We supposed that there are only two types of strategies and with them we can generate any sample which is relevant. We gave a method to find good and successful strategies based on the predicted sample type and we tested them in an experiment. We found that with our method we can give outstanding strategies but not surely winner strategy.

In the end we recommend some method to use to improve our results, like using the information about preferred numbers.

There are many problems which are very similar to the General Game, where the problem is complex that we can not give an exact mathematical formula to answer, where there are uncertainty about the number of the players who are involved in the problem and where there is no relevant information about the behavior of the participants.

The General Game is a bit more specific game, because in this situation we have to use a given amount of resources in this uncertain environment. And this game is focusing on the question that in this environment how competitive or cooperative we should be.

A political party has to face with the same problem, when there is an electoral voting system and the party has a given budget to spend on marketing. In this case they have to decide how much they spend on marketing in each district. How much they should focus on only some of the districts or they should spend money in all districts. According to our results they have spend money in each district, but they have to focus on some of them.

The liberalization of energy sector in Hungary raised very similar problems like the General Game. After the disappearance of the monopoly there is a big competition for the market shares but without any information about the concurrence behavior.

In every situation where there is big concurrence because of the low entry cost can be similar to the General Game as well. Typically the digital world is like this. On the internet it is easy to appear and it is also a big question that how much you should cooperate and how much you should compete with other sites and other services in the same sector.

B. Conclusion

What is the advice of the General? The basic principle behind of the construction of this game was to analyze the behavior of competition and cooperation. Mérő created the game for this purpose [3] and he was observing all the results from this aspect. Our results corresponded with the international papers on this topic. You should be cooperative but you have to be a little competitive as well. According to this the general advice is the following. Be cooperative even in the most competitive environment, but never too much. But rather be too cooperative then too competitive.

This result a bit against with the microeconomic theories about the perfect competition. It says that in the perfect competition everybody competes with each other and it is the best and most logical decision for them.

In the world we can see many cases how companies cooperate in a perfect competition. There is indirect way of cooperating like a price cartel when companies do not use lower prices than their costs just because of the competition. The low budget airplane sector went bankruptcy in the United States because of the price competition.

But there are direct ways as well. Nowadays it is typical that there are common innovation projects between companies which are totally concurrence to each other in the same sector. There are such well known innovation projects for IBM, but Arcelor Mittal has the same situation with the Open Innovations project.

In this paper behavior possibilities of the cooperative and non-cooperative persons has been presented, and the constructed method with simulation helps to understand the personal behavior, which can be usable in decision making and in many other fields.

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