

Stochastic Model Predictive Control for Linear Discrete-Time Systems with Random Dither Quantization

Tomoaki Hashimoto

Abstract—Recently, feedback control systems using random dither quantizers have been proposed for linear discrete-time systems. However, the constraints imposed on state and control variables have not yet been taken into account for the design of feedback control systems with random dither quantization. Model predictive control is a kind of optimal feedback control in which control performance over a finite future is optimized with a performance index that has a moving initial and terminal time. An important advantage of model predictive control is its ability to handle constraints imposed on state and control variables. Based on the model predictive control approach, the objective of this paper is to present a control method that satisfies probabilistic state constraints for linear discrete-time feedback control systems with random dither quantization. In other words, this paper provides a method for solving the optimal control problems subject to probabilistic state constraints for linear discrete-time feedback control systems with random dither quantization.

Keywords—Optimal control, stochastic systems, discrete-time systems, probabilistic constraints, random dither quantization.

I. INTRODUCTION

THE quantized control of systems is one of the most important research topics in recent years. This is partly because various quantizing devices, such as communication networks and discrete-level actuators/sensors, are useful to reduce installation and maintenance costs. On the other hand, in considering the control systems with quantized signals, it is important to avoid the performance degradation caused by quantization errors, which is a challenging control problem.

So far, several kinds of quantizers have been proposed for linear discrete-time feedback control systems. In particular, this study focuses on the so-called random dither quantizer [1] that transforms a given continuous-valued signal to a discrete-valued signal by using an artificially added random signal. It has been shown in [1] that the random dither quantizers exhibit much better performance than the simple uniform quantizers. However, the constraints imposed on state variables and control inputs have not yet been taken into consideration for the design of feedback control systems with random dither quantization.

T. Hashimoto is with the Department of Mechanical Engineering, Osaka Institute of Technology, Asahi-ku 535-8585, Japan (e-mail: info@thashi.net).

This work was supported in part by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C), Grant Number JP15K06139. This work was also supported in part by Japan Science Technology Agency, CREST.

Model predictive control (MPC), also known as receding horizon control [2]-[4], is a well-established control method in which the current control input is obtained by solving a finite-horizon open-loop optimal control problem using the current state of the system as the initial state, and this procedure is repeated at each sampling instant. An important advantage of MPC is its ability to deal with constraints on state and control variables, which makes it one of the most successful control methodologies because it enables control performance to be optimized while considering any constraints on state and control variables [5]-[7].

Although a certain class of MPC methods [8]-[11] does not provide a systematic method to handle uncertain disturbances, another class of MPC methods [12]-[14] guarantee constraint fulfillment under uncertain disturbances. In this study, we focus on the class of MPC problems in which a performance index is minimized subject to state constraints under uncertain disturbances. In fact, the methods of MPC against uncertain disturbances can be classified into deterministic and stochastic approaches.

In the deterministic approach, most works are based on the min-max approach, where a performance index is minimized over the worst possible disturbance [12], [13]. In this approach, however, the control performance often results in too conservative because no statistical properties of uncertain disturbances are taken into consideration.

The other approach is addressed by stochastic MPC (SMPC) where the expected values of the performance indices and probabilistic constraints are considered by exploiting the statistical information of uncertain disturbances. It is known that a small relaxation of the probability requirement sometimes can lead to a significant improvement in the achievable control performance. In general, however, probabilistic constraints are generally intractable in an optimization problem. In recent decades, much attention has been paid to this difficulty of the stochastic MPC problem. For example, the methods proposed in [15]-[19] enable us to address unknown arbitrary probability distributions of stochastic disturbances, including non-Gaussian, infinitely supported, and time-variant distributions, only under the assumption of known expectation and variance in the disturbance. These studies aim to provide a SMPC method to successfully deal with probabilistic constraints with a lower computational load. For this purpose, concentration inequalities [20] were applied to transform probabilistic constraints on state variables into deterministic constraints on

control inputs.

The design method of feedback control systems for quantized control systems subject to probabilistic state constraints has not yet been established. Thus, the objective of this paper is to propose a SMPC method that fulfills probabilistic state constraints for linear discrete-time feedback control systems with random dither quantization. In other words, this paper enables us to solve the optimal control problems subject to probabilistic state constraints for linear discrete-time feedback control systems with random dither quantization.

This paper is organized as follows: In Section II, we introduce some notations. In Section III, the system model and random dither quantizers are formulated. In Section IV, we formulate the SMPC problem for linear discrete-time systems with stochastic disturbances. The main results are provided in Section V. Finally, some concluding remarks are given in Section VI.

II. NOTATION

In this section, we introduce some notations that are adopted throughout this paper. Let the sets of real and natural numbers be denoted by \mathbb{R} and \mathbb{N} , respectively. Let the set of non-negative real numbers be denoted by \mathbb{R}_+ .

For matrix A , let A' and $\text{tr}A$ denote the transpose and trace of A , respectively. For matrices $A = \{a_{i,j}\}$ and $B = \{b_{i,j}\}$, let the inequalities between A and B , such as $A > B$ and $A \geq B$, indicate that they are component-wise satisfied, i.e., $a_{i,j} > b_{i,j}$ and $a_{i,j} \geq b_{i,j}$ hold true for all i and j , respectively.

Similarly, let multiplication $A \circ B$ indicate that it is applied component-wise, i.e., $A \circ B = \{a_{i,j} \times b_{i,j}\}$ for all i and j . Let $\mathbf{1}$ indicate the column vector whose every element is equal to 1.

Let a probability space be denoted by $(\Omega, \mathcal{F}, \mathcal{P})$, where $\Omega \subseteq \mathbb{R}$ is the sampling space, \mathcal{F} is the σ -algebra, and \mathcal{P} is the probability measure [21]. Here, Ω is non-empty and is not necessarily finite.

Let $\mathcal{P}(E)$ denote the probability that event E occurs. If $\mathcal{P}(E) = 1$ holds true, E almost surely occurs. For a random variable $z : \Omega \rightarrow \mathbb{R}$ defined by $(\Omega, \mathcal{F}, \mathcal{P})$, let the expected value and variance of z be denoted by $\mathcal{E}(z)$ and $\mathcal{V}(z)$, respectively. For a random vector $z = [z_1, \dots, z_n]'$, whose components are random variables $z_i : \Omega \rightarrow \mathbb{R}$ ($i = 1, \dots, n$) defined on the same probability space $(\Omega, \mathcal{F}, \mathcal{P})$, let the same notations $\mathcal{E}(z)$ and $\mathcal{V}(z)$ be adopted to denote $\mathcal{E}(z) = [\mathcal{E}(z_1), \dots, \mathcal{E}(z_n)]'$ and $\mathcal{V}(z) = [\mathcal{V}(z_1), \dots, \mathcal{V}(z_n)]'$ for notational simplicity. Furthermore, let the covariance matrix $\mathcal{C}_v(z)$ be defined by $\mathcal{C}_v(z) := \mathcal{E}[\{z - \mathcal{E}(z)\}\{z - \mathcal{E}(z)\}']$.

Let q denote the static nearest-neighbor quantizer toward $-\infty$ with the quantization interval d as shown in Fig. 1.

III. SYSTEM MODEL

First, we consider the following linear discrete-time system Σ_P :

$$x(t+1) = Ax(t) + Bv(t), \quad (1)$$

$$v(t) = q(u(t)), \quad (2)$$

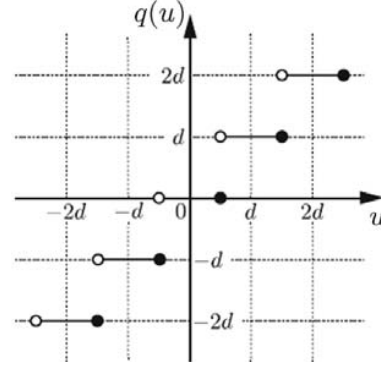


Fig. 1 Quantizer q with quantization interval d

where $t \in \mathbb{N}$ is the time step, $x(t) : \mathbb{N} \rightarrow \mathbb{R}^n$ is the state, $u(t) : \mathbb{N} \rightarrow \mathbb{R}^m$ is the control input. Note that the simple uniform quantizer $v(t) = q(u(t))$ is introduced into Σ_P , where q is defined in Section II and Fig. 1.

Suppose that system coefficients A and B are constant known matrices. The pair (A, B) is assumed to be controllable. All components of state $x(t)$ are deterministically observable, that is, the state $x(t)$ is known at present time t . Thus, we assume that $\mathcal{E}(x(t)) = x(t)$ and $\mathcal{V}(x(t)) = 0$.

Next, we consider the following linear discrete-time system Σ_Q :

$$x(t+1) = Ax(t) + Bv(t), \quad (3)$$

$$v(t) = q(u(t) + \eta(t)), \quad (4)$$

where $\eta(t) : \mathbb{N} \rightarrow \mathbb{R}^m$ is an independent and identically distributed random variable with the uniform probability distribution on $[-d/2, d/2]$. The signal η is called the random dither signal and the quantizer is called the random dither quantizer. Note that the random dither quantizer $v(t) = q(u(t) + \eta(t))$ is introduced into Σ_Q .

Here, note that the system Σ_Q is equivalently transformed into the following system:

$$x(t+1) = Ax(t) + B(u(t) + w(t)), \quad (5)$$

$$w(t) = v(t) - u(t), \quad (6)$$

where w denotes the quantization error. From the definition of the random dither quantizer, we note that the quantization error is also a random variable.

A schematic view of system Σ_Q described by (3) and (4) is shown in Fig. 2. In contrast, a schematic view of system Σ_Q described by (5) and (6) is shown in Figs. 3.

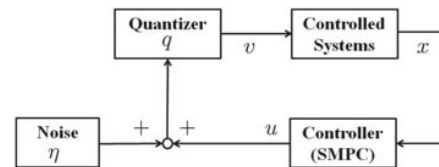
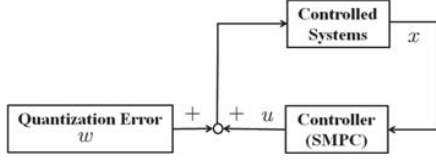


Fig. 2 A schematic view of system Σ_Q described by (3) and (4)

The following lemma proved in [1] shows the properties of the expectation and variance of the quantization error w .

Fig. 3 A schematic view of system Σ_Q described by (5) and (6)

Lemma 1 ([1]): For the random dither quantizer defined in section II, suppose that the quantization interval d is given. Then,

$$\mathcal{E}(w(t)) = 0, \quad (7)$$

$$\mathcal{V}(w(t)) \leq \frac{d^2}{4}. \quad (8)$$

This lemma plays an important role to solve the SMPC problem introduced in the next section.

IV. SMPC PROBLEMS

In this section, we formulate the SMPC problem of system (5). The control input at each time t is determined to minimize the performance index given by

$$J := \phi[x(t+N)] + \sum_{k=t}^{t+N-1} L[x(k), u(k)], \quad (9a)$$

where $N \in \mathbb{N}$ denotes the length of the evaluation interval. Moreover, let ϕ and L be defined by

$$\phi := \mathcal{E}[x(t+N)'Px(t+N)], \quad (9b)$$

$$L := \mathcal{E}[x(k)'Qx(k)] + u(k)'Ru(k), \quad (9c)$$

where let P , Q , and R be weighting coefficients that are positive definite constant matrices. Note that $\phi \in \mathbb{R}_+$ is the terminal cost function and $L \in \mathbb{R}_+$ is the stage cost function over the evaluation interval.

Let the probability in vector form be denoted by

$$p(t) = \begin{bmatrix} p_1(t) \\ \vdots \\ p_2(t) \end{bmatrix} : \mathbb{N} \rightarrow [0 \ 1]^n,$$

which means that each component $p_i(t)$ belongs to $[0 \ 1]$ for each time t .

For notational convenience, we introduce the so-called expanded vectors as follows: Let $\mathbf{X} \in \mathbb{R}^{nN}$, $\mathbf{U} \in \mathbb{R}^{mN}$ and $\mathbf{W} \in \mathbb{R}^{\ell N}$ be defined by

$$\mathbf{X}(t) := \begin{bmatrix} x(t+1) \\ \vdots \\ x(t+N) \end{bmatrix},$$

$$\mathbf{U}(t) := \begin{bmatrix} u(t) \\ \vdots \\ u(t+N-1) \end{bmatrix},$$

$$\mathbf{W}(t) := \begin{bmatrix} w(t) \\ \vdots \\ w(t+N-1) \end{bmatrix}.$$

Note that \mathbf{X} , \mathbf{U} and \mathbf{W} consist of the system state, control input and uncertain disturbance, respectively, over the evaluation interval.

Similarly, we introduce the so-called expanded matrices as follows: Let $\mathbf{A} \in \mathbb{R}^{nN \times n}$, $\mathbf{B} \in \mathbb{R}^{nN \times mN}$, $\mathbf{Q} \in \mathbb{R}^{nN \times nN}$, $\mathbf{R} \in \mathbb{R}^{mN \times mN}$, and $\mathbf{p} \in \mathbb{R}^{nN}$, be defined by

$$\mathbf{A} := \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}.$$

$$\mathbf{B} := \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix},$$

$$\mathbf{Q} := \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & Q & 0 \\ 0 & \cdots & 0 & P \end{bmatrix},$$

$$\mathbf{R} := \begin{bmatrix} R & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R \end{bmatrix},$$

$$\mathbf{p}(t) = \begin{bmatrix} p(t+1) \\ \vdots \\ p(t+N) \end{bmatrix}.$$

Using the expanded vectors and matrices denoted by the aforementioned notation, the performance index in (9) can be rewritten as follows:

$$J[x(t), \mathbf{X}(t), \mathbf{U}(t)] = \mathcal{E}[x(t)'Qx(t)] + \mathcal{E}[\mathbf{X}(t)'\mathbf{Q}\mathbf{X}(t)] + \mathbf{U}(t)'\mathbf{R}\mathbf{U}(t), \quad (10)$$

In addition, (5) over the evaluation interval can be rewritten as:

$$\mathbf{X}(t) = \mathbf{A}x(t) + \mathbf{B}(\mathbf{U}(t) + \mathbf{W}(t)). \quad (11)$$

Then, $\mathcal{E}(\mathbf{X}(t))$ and $\mathcal{V}(\mathbf{X}(t))$ are given by

$$\mathcal{E}(\mathbf{X}(t)) = \mathbf{A}x(t) + \mathbf{B}\mathbf{U}(t) + \mathbf{B}\mathcal{E}(\mathbf{W}(t)), \quad (12a)$$

$$\mathcal{V}(\mathbf{X}(t)) = (\mathbf{B} \circ \mathbf{B})\mathcal{V}(\mathbf{W}(t)). \quad (12b)$$

In (12a), we apply $\mathcal{E}(x(t)) = x(t)$ because the present state $x(t)$ is a deterministic vector. Note that the performance index (9a) can be transformed into the following:

$$J = x(t)'Qx(t) + \mathbf{U}(t)'\mathbf{R}\mathbf{U}(t) + \text{tr}[\mathbf{Q}\mathcal{C}_v(\mathbf{X}(t))] + \mathcal{E}(\mathbf{X}(t))'\mathbf{Q}\mathcal{E}(\mathbf{X}(t)). \quad (13)$$

Here, we introduce the following assumption.

Assumption 1: Each element of $x(t)$, $\mathbf{U}(t)$ and $\mathbf{W}(t)$ are assumed to be independent for each time t .

Noting that covariance matrix $\mathcal{C}_v(\mathbf{X}(t))$ is independent of $\mathbf{U}(t)$, we have the following:

$$\begin{aligned}\mathcal{C}_v(\mathbf{X}(t)) &= \mathcal{E} \{ [\mathbf{X}(t) - \mathcal{E}(\mathbf{X}(t))] [\mathbf{X}(t) - \mathcal{E}(\mathbf{X}(t))]^T \} \\ &= \mathcal{E} \{ [\mathbf{B}\mathbf{W}(t) - \mathbf{B}\mathcal{E}(\mathbf{W}(t))] [\mathbf{B}\mathbf{W}(t) - \mathbf{B}\mathcal{E}(\mathbf{W}(t))]^T \}.\end{aligned}$$

Substituting (12a) into (13) and neglecting the terms that do not contain $\mathbf{U}(t)$, we obtain

$$\begin{aligned}\min_{\mathbf{U}(t)} J[x(t), \mathbf{X}(t), \mathbf{U}(t)] &= \\ \min_{\mathbf{U}(t)} \left\{ \begin{aligned} &\mathbf{U}'(t) (\mathbf{B}'\mathbf{Q}\mathbf{B} + \mathbf{R}) \mathbf{U}(t) \\ &+ 2(\mathbf{A}x(t) + \mathbf{B}\mathcal{E}(\mathbf{W}(t)))' \mathbf{Q}\mathbf{B}\mathbf{U}(t) \end{aligned} \right\}. \end{aligned} \quad (14)$$

Note that the minimization problem of J in (9) subject to (11) has been reduced to a quadratic programming problem with respect to \mathbf{U} .

Here, we impose the following probabilistic constraint on the optimization problem:

$$\mathcal{P}(\mathbf{D}\mathbf{X}(t) < \mathbf{h}) \geq \mathbf{p}. \quad (15)$$

where $\mathbf{D} \in \mathbb{R}^{s \times nN}$, $0 < \mathbf{h} \in \mathbb{R}_+^s$, $\mathbf{p} \in [0, 1]^s$, and $s \in \mathbb{N}$ are given constant parameters.

In general, to solve the quadratic programming problem with probabilistic constraints is not straightforward. In [18]-[19], it has been shown that the probabilistic constraints can be converted into deterministic constraints using the concentration inequalities.

V. MAIN RESULTS

In this section, the main results are shown below. The following proposition proved in [18] plays an important role to derive the main results.

Proposition 1 ([18]): Here, we consider the following system subject to probabilistic constraint (15):

$$\mathbf{X}(t) = \mathbf{A}x(t) + \mathbf{B}\mathbf{U}(t) + \mathbf{C}\mathbf{W}(t). \quad (16)$$

Suppose that the following condition holds true for system (16):

$$\mathbf{D}\mathbf{B}\mathbf{U}(t) \leq \mathbf{h} - \mathbf{D}(\mathbf{A}x(t) + \mathbf{C}\mathcal{E}(\mathbf{W}(t))) - \mathbf{V}(t), \quad (17)$$

where the i th element of $\mathbf{V} \in \mathbb{R}^s$ is given by:

$$\mathbf{V}_i = \sqrt{\frac{\mathbf{p}_i}{1 - \mathbf{p}_i}} ((\mathbf{D}\mathbf{C} \circ \mathbf{D}\mathbf{C})\mathcal{V}(\mathbf{W}(t)))_i. \quad (18)$$

Then, the probabilistic condition (15) is satisfied.

Using Proposition 1, we can state the following proposition.

Proposition 2: Suppose that the following condition holds true for system (11):

$$\mathbf{D}\mathbf{B}\mathbf{U}(t) \leq \mathbf{h} - \mathbf{D}(\mathbf{A}x(t) + \mathbf{B}\mathcal{E}(\mathbf{W}(t))) - \mathbf{V}(t), \quad (19)$$

where the i th element of $\mathbf{V} \in \mathbb{R}^s$ is given by:

$$\mathbf{V}_i = \sqrt{\frac{\mathbf{p}_i}{1 - \mathbf{p}_i}} ((\mathbf{D}\mathbf{B} \circ \mathbf{D}\mathbf{B})\mathcal{V}(\mathbf{W}(t)))_i. \quad (20)$$

Then, the probabilistic condition (15) is satisfied.

Proof: The proof can be completed by substituting $\mathbf{C} = \mathbf{B}$ into (17) and (18). ■

Using Proposition 2, we can state the following theorem.

Theorem 1: For the system Σ_Q with the random dither quantizer, suppose that the following condition holds:

$$\mathbf{D}\mathbf{B}\mathbf{U}(t) \leq \mathbf{h} - \mathbf{D}\mathbf{A}x(t) - \mathbf{V}(t), \quad (21)$$

where the i th element of $\mathbf{V} \in \mathbb{R}^s$ is given by:

$$\mathbf{V}_i = \sqrt{\frac{\mathbf{p}_i}{1 - \mathbf{p}_i}} \left((\mathbf{D}\mathbf{B} \circ \mathbf{D}\mathbf{B}) \mathbf{1} \frac{d^2}{4} \right)_i. \quad (22)$$

Then, the probabilistic condition (15) is fulfilled.

Proof: Substituting (7) into (19), we obtain (21). From (8), we consider the worst case as follows:

$$\mathcal{V}(w(t)) = \frac{d^2}{4}. \quad (23)$$

Substituting (23) into (20), we obtain (22). Therefore, we can see that if deterministic constraint (21) on $\mathbf{U}(t)$ is fulfilled, then the probabilistic constraint (15) on $\mathbf{X}(t)$ is also fulfilled. This completes the proof. ■

Remark 1: From Theorem 1, the minimization problem of (9) subject to probabilistic constraint (15) is reduced to a quadratic programming problem (14) subject to deterministic constraint (21), which can be solved using a conventional algorithm [22].

Remark 2: Suppose that not only probabilistic state constraint (15) but also control input constraint are imposed on the optimization problem as follows:

$$\mathbf{F}\mathbf{U}(t) < \bar{\mathbf{U}}, \quad (24)$$

where $\mathbf{F} \in \mathbb{R}^{f \times mN}$, $\bar{\mathbf{U}} \in \mathbb{R}^f$ and $f \in \mathbb{N}$ are given constant parameters. Then, the optimization problem can be reduced to a quadratic programming problem (14) subject to the following constraint:

$$\begin{bmatrix} \mathbf{D}\mathbf{B} \\ \mathbf{F} \end{bmatrix} \mathbf{U}(t) \leq \begin{bmatrix} \mathbf{h} - \mathbf{D}\mathbf{A}x(t) - \mathbf{V}(t) \\ \bar{\mathbf{U}} \end{bmatrix}. \quad (25)$$

To solve a quadratic programming problem (14) subject to constraint (25) is also straightforward using a conventional algorithm [22].

VI. CONCLUSION

In this study, we have examined a design method of SMPC for linear discrete-time systems with the random dither quantizer. The solution method to the optimal control problems subject to probabilistic state constraints was proposed for quantized control systems under stochastic quantization errors. The optimal control problems subject to probabilistic constraints were reduced to quadratic programming problems with deterministic constraints that can be solved using a conventional algorithm. To verify the effectiveness of the proposed method using numerical simulations is a possible future work.

It is known that not only uncertain disturbances but also time delays may cause instabilities and lead to more complex analysis [23]-[28]. The stabilization problem of random dither systems with time delays is also a possible future work.

REFERENCES

- [1] R. Morita, S. Azuma, T. Sugie, Performance Analysis of Random Dither Quantizers in Feedback Control Systems, *Preprints of the 18th IFAC World Congress*, pp. 11296-11301, 2011.
- [2] T. Hashimoto, Y. Yoshioka, T. Ohtsuka, Receding Horizon Control with Numerical Solution for Thermal Fluid Systems, *Proceedings of SICE Annual Conference*, pp. 1298-1303, 2012.
- [3] T. Hashimoto, Y. Yoshioka, T. Ohtsuka, Receding Horizon Control with Numerical Solution for Spatiotemporal Dynamic Systems, *Proceedings of IEEE Conference on Decision and Control*, pp. 2920-2925, 2012.
- [4] T. Hashimoto, Y. Takiguchi and T. Ohtsuka, Output Feedback Receding Horizon Control for Spatiotemporal Dynamic Systems, *Proceedings of Asian Control Conference*, 2013.
- [5] T. Hashimoto, Y. Yoshioka and T. Ohtsuka, Receding Horizon Control for Hot Strip Mill Cooling Systems, *IEEE/ASME Transactions on Mechatronics*, Vol. 18, No. 3, pp. 998-1005, 2013.
- [6] T. Hashimoto, Y. Yoshioka and T. Ohtsuka, Receding Horizon Control With Numerical Solution for Nonlinear Parabolic Partial Differential Equations, *IEEE Transactions on Automatic Control*, Vol. 58, No. 3, pp. 725-730, 2013.
- [7] T. Hashimoto, Y. Takiguchi and T. Ohtsuka, Receding Horizon Control for High-Dimensional Burgers' Equations with Boundary Control Inputs, *Transactions of the Japan Society for Aeronautical and Space Sciences*, Vol. 56, No.3, pp. 137-144, 2013.
- [8] R. Satoh, T. Hashimoto and T. Ohtsuka, Receding Horizon Control for Mass Transport Phenomena in Thermal Fluid Systems, *Proceedings of Australian Control Conference*, pp. 273-278, 2014.
- [9] T. Hashimoto, Receding Horizon Control for a Class of Discrete-time Nonlinear Implicit Systems, *Proceedings of IEEE Conference on Decision and Control*, pp. 5089-5094, 2014.
- [10] T. Hashimoto, Optimal Feedback Control Method Using Magnetic Force for Crystal Growth Dynamics, *International Journal of Science and Engineering Investigations*, Vol. 4, Issue 45, pp. 1-6, 2015.
- [11] T. Hashimoto, R. Satoh and T. Ohtsuka, Receding Horizon Control for Spatiotemporal Dynamic Systems, *Mechanical Engineering Journal*, Vol. 3, No. 2, 15-00345, 2016.
- [12] M. V. Kothare, V. Balakrishnan and M. Morari, Robust Constrained Model Predictive Control Using Linear Matrix Inequalities, *Automatica*, Vol. 32, pp. 1361-1379, 1996.
- [13] P. Scokaert and D. Mayne, Min-max Feedback Model Predictive Control for Constrained Linear Systems, *IEEE Trans. Automat. Contr.*, Vol. 43, pp. 1136-1142, 1998.
- [14] T. Hashimoto, I. Yoshimoto, T. Ohtsuka, Probabilistic Constrained Model Predictive Control for Schrödinger Equation with Finite Approximation, *Proceedings of SICE Annual Conference*, pp. 1613-1618, 2012.
- [15] T. Hashimoto, Probabilistic Constrained Model Predictive Control for Linear Discrete-time Systems with Additive Stochastic Disturbances, *Proceedings of IEEE Conference on Decision and Control*, pp. 6434-6439, 2013.
- [16] T. Hashimoto, Computational Simulations on Stability of Model Predictive Control for Linear Discrete-time Stochastic Systems, *International Journal of Computer, Electrical, Automation, Control and Information Engineering*, Vol. 9, No. 8, pp. 1385-1390, 2015.
- [17] T. Hashimoto, Conservativeness of Probabilistic Constrained Optimal Control Method for Unknown Probability Distribution, *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol. 9, No. 9, pp. 11-15, 2015.
- [18] T. Hashimoto, A Method for Solving Optimal Control Problems subject to Probabilistic Affine State Constraints for Linear Discrete-time Uncertain Systems, *International Journal of Mechanical and Production Engineering*, Vol. 3, Issue 12, pp. 6-10, 2015.
- [19] T. Hashimoto, Solutions to Probabilistic Constrained Optimal Control Problems Using Concentration Inequalities, *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering*, Vol. 10, No. 10, pp. 441-446, 2016.
- [20] S. Boucheron, G. Lugosi and P. Massart *Concentration Inequalities: A Nonasymptotic Theory of Independence*, Oxford University Press, 2013.
- [21] B. Øksendal, *Stochastic Differential Equations: An Introduction with Applications*, Springer, 6th edition, 2010.
- [22] J. Nocedal and S. J. Wright, Numerical Optimization, *Springer Series in Operation Research and Financial Engineering*, Springer, 2006.
- [23] T. Hashimoto, T. Amemiya and H. A. Fujii, Stabilization of Linear Uncertain Delay Systems with Antisymmetric Stepwise Configurations, *Journal of Dynamical and Control Systems*, Vol. 14, No. 1, pp. 1-31, 2008.
- [24] T. Hashimoto, T. Amemiya and H. A. Fujii, Output Feedback Stabilization of Linear Time-varying Uncertain Delay Systems, *Mathematical Problems in Engineering*, Vol. 2009, Article ID. 457468, 2009.
- [25] T. Hashimoto and T. Amemiya, Stabilization of Linear Time-varying Uncertain Delay Systems with Double Triangular Configuration, *WSEAS Transactions on Systems and Control*, Vol. 4, No.9, pp.465-475, 2009.
- [26] T. Hashimoto, Stabilization of Abstract Delay Systems on Banach Lattices using Nonnegative Semigroups, *Proceedings of the 50th IEEE Conference on Decision and Control*, pp. 1872-1877, 2011.
- [27] T. Hashimoto, A Variable Transformation Method for Stabilizing Abstract Delay Systems on Banach Lattices, *Journal of Mathematics Research*, Vol. 4, No. 2, pp.2-9, 2012.
- [28] T. Hashimoto, An Optimization Algorithm for Designing a Stabilizing Controller for Linear Time-varying Uncertain Systems with State Delays, *Computational Mathematics and Modeling*, Vol.24, No.1, pp.90-102, 2013.