Steering Velocity Bounded Mobile Robots in Environments with Partially Known Obstacles

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Abstract—This paper presents a method for steering velocity bounded mobile robots in environments with partially known stationary obstacles. The exact location of obstacles is unknown and only a probability distribution associated with the location of the obstacles is known. Kinematic model of a 2-wheeled differential drive robot is used as the model of mobile robot. The presented control strategy uses the Artificial Potential Field (APF) method for devising a desired direction of movement for the robot at each instant of time while the Constrained Directions Control (CDC) uses the generated direction to produce the control signals required for steering the robot. The location of each obstacle is considered to be the mean value of the 2D probability distribution and similarly, the magnitude of the electric charge in the APF is set as the trace of covariance matrix of the location probability distribution. The method not only captures the challenges of planning the path (i.e. probabilistic nature of the location of unknown obstacles), but it also addresses the output saturation which is considered to be an important issue from the control perspective. Moreover, velocity of the robot can be controlled during the steering. For example, the velocity of robot can be reduced in close vicinity of obstacles and target to ensure safety. Finally, the control strategy is simulated for different scenarios to show how the method can be put into practice.

Keywords—Steering, obstacle avoidance, mobile robots, constrained directions control, artificial potential field.

I. INTRODUCTION

MOBILE robots can be widely used for various applications such as search and rescue, part delivery in industrial environments, tour guide in museums, etc. Mobile robots are classified into several categories. Differential drive robots, car like systems (skid steered systems) and omnidirectional robots are the three important categories of mobile robots [1]. Differential drive robots have three wheels, two of which are active wheels and are on the sides of robot. The third wheel which is on the other end of robot is passive and is called caster wheel. Depending on the modeling approach, the control inputs for differential drive robot models are either the angular velocities of the wheels or forward/angular velocities of robot. Car like robot models have four wheels on the sides of the robot just like automobiles. The forward velocity and the steering radius of the system are the control inputs for these robots. Omnidirectional robots are the last category of mobile robots that exist in various wheel number configurations [1].

Navigation of mobile robots has two main steps: Path planning and control. Path planning of the robot is defined as the process of constructing a path for the states of the robot where certain constraints might be imposed on the states of the system such as obstacle avoidance [1], [2]. On the other hand, control of robot is the process of synthesizing time and/or state dependent signals applying which to the inputs of the robot results in the output of the system to follow the prescribed path. In other words, the output of the path planning algorithm is fed as the desired input for the control algorithm. And the goal of control unit is ensuring that the robot follows the desired path. Path planning and control of robots are usually achieved in two different steps; however, there are methods that can simultaneously address both problems within the same framework.

Many researchers have studied the problem of steering and path planning of the mobile robots in environments with obstacles. This is due to the fact that in real applications there always exists obstacles within the environment and the robot should avoid them. APF [2]-[5] and its combination with intelligent methods such as evolutionary algorithms [6], fuzzy logic or particle swarm optimization [7] are widely used for obstacle avoidance of mobile robots. APF provides a rigorous approach for path planning of mobile robots among obstacles and it enables incorporating the properties of obstacles such as size, danger or certainty of the knowledge on location of obstacles within the path planning problem formulation. Another benefit of this method is that it can easily be augmented on top of other techniques to result in approaches which can satisfy multiple needs in the area of navigation of mobile robots. For instance, [8] has combined the method of APF with the project management techniques to navigate a rescue robot among obstacles while completing the rescue mission in the field. APF Method can also be enhanced by incorporating the observations from natural phenomenon such as evolution [9], animal behavior [10] or water fall [11].

Alike path planning, the problem of controlling mobile robots has been widely studied. Reference [12] uses a backstepping-like feedback linearization for tracking control of differential-drive wheeled mobile robots with nonholonomic constraints. Approximate feedback linearization is used in [13] for control of differential-drive wheeled robots. Adaptive control is another popular approach for control of differential drive robots which can take into account the modeling uncertainties [14]. Additionally, using adaptive control strategies enables using more complex models of the robots. Similarly, researchers have used robust control strategies to deal with uncertainty problems [15], [16]. The non-holonomic property of differential drive robots makes

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design of controller very hard for these robots, especially if certain transient properties such as fast response are required [17], [18], or if certain constraints are imposed on the inputs [19], [20], [22] or states [21] of the systems. Recently, many researchers have focused on developing methods that address these control challenges in the stabilization or tracking control of mobile robots [17]-[23].

This paper presents an approach for steering input constrained mobile robots in the environments with partially known obstacles. In other words, the exact location of obstacles is unknown, but a probability distribution associated with the position of the obstacles is known. The method is based on APF [24], [25] and CDC [19], [23] and executes the path planning, obstacle avoidance and control simultaneously. The statistical parameters of the probability distribution, i.e. the average and trace of covariance of the probability distribution function, define the location of the electric charges modeling the obstacles and the magnitude of the electric charge respectively. With this approach, the uncertainty in the location of a certain obstacle results in a larger electric charge which eventually results in a path that is further away from that specific obstacle. Calculating the vector sum of the force vectors, the direction of the prescribed path is determined at each point. The desired direction of motion is fed to the CDC as the input. The desired velocity for the robot is determined based on the distance between the current location of the robot and final location of the robot to ensure zero final velocity for the robot. In other words, while the generated force vector is used to direct the robot toward the assigned target point, it is used as a reference direction for the constrained directions method to produce the necessary control satisfying the input bounds to guide the robot.

This paper is organized as follows: Section II explains the APF method, Section III briefly explains the constrained directions methods, Section IV describes our proposed methodology, Section V provides simulation results and analysis, and Section VI concludes the paper.

II. APF METHOD

The potential field method has been widely used for path planning of mobile robots when obstacles exist in the environment [1], [2], [4], [8], [9]. In this method, an APF is constructed for robot's workspace and target(s) are assumed to produce attractive force while obstacles produce repulsive forces. First, a mathematical formulation for the attractive and repulsive potential fields needs to be constructed. The most commonly used attractive potential function and the corresponding attractive force is as [8], [25], [26]:

$$U_{att}(q) = \frac{1}{2}k_a\|q_{goal} - q\|$$ (1)

$$F_{att} = -\nabla U_{att} = k_a(q_{goal} - q)$$ (2)

where $q = [x \ y]^T$ denotes the position of the robot in the workspace, k_a is a positive scaling factor, q_{goal} is the position of the target point, $\|.\|$ is the 2-norm operator which

calculates the distance between the current position of robot (q) and the goal position of robot (q_{goal}). The attractive potential is conic in shape and the resulting attractive force has constant amplitude except at the position of the target where U_{att} is singular. Also, the attractive force converges linearly toward zero as the robot approaches the goal.

One commonly used repulsive potential function and the resulting repulsive force is as [25]:

$$U_{rep} = \begin{cases} \frac{1}{2}k_r\left(\frac{1}{\rho(q,q_{obs})} - \frac{1}{\rho_0}\right)^2 & if \ \rho(q,q_{obs}) \le \rho_0 \\ 0 & otherwise \end{cases}$$ (3)

$$F_{rep} = -\nabla U_{rep} =$$
$$\begin{cases} k_r\left(\frac{1}{\rho(q,q_{obs})} - \frac{1}{\rho_0}\right)\frac{1}{\rho^2(q,q_{obs})}\nabla\rho(q,q_{obs}), & if \ \rho(q,q_{obs}) \le \rho_0 \\ 0, & if \ \rho(q,q_{obs}) > \rho_0 \end{cases}$$ (4)

where k_r is a positive scaling factor, $\rho(q,q_{obs})$ denotes the minimal distance from the robot q to the obstacle, assuming obstacles as points, q_{obs} denotes the position of the obstacle, and ρ_0 is a positive constant denoting the distance of influence of the obstacle. The direction of motion of the robot is determined based on the vector sum of the two force vectors applied to the robot [25], [26]:

$$F_{total} = F_{att} + F_{rep}$$ (5)

III. CONSTRAINED DIRECTIONS CONTROL

The directions of motion a mobile robot can achieve are determined by the mechanical constraints governing its motion and limitations on the magnitude of control inputs [19]. By identifying the achievable directions for a robot, CDC defines a set named E_{reach} containing all the directions that can be achieved. Finding a member in E_{reach} that is closest to the desired direction of motion of the robot at each instant of time, steers the robot toward the defined target while moving on the desired path or on the closest path possible. The desired direction of movement for the robot can be formulated as:

$$e_{des} = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix}$$ (6)

where $\dot{x}_d$ and $\dot{y}_d$ denote the desired velocities in x and y directions respectively. The desired direction of motion can be alternatively defined as:

$$e_{des} = v_d\begin{bmatrix} cos\theta_d \\ sin\theta_d \end{bmatrix}$$ (7)

where θ_d denotes the desired direction of motion for robot and v_d is the desired velocity which can be assumed as one if the velocity of motion is not a factor to be controlled. Therefore control problem can be mathematically formulated as finding a member in E_{reach} that better matches the control objective function. In other words, the control design can be rephrased as: Find $e_i \in E_{reach}$ which solves

$$Min \ \Psi(e_i, e_{des})$$ (8)

where Ψ denotes an appropriately defined metric within E_{reach} which evaluates the desirability of a given direction of motion [20].

IV. PROPOSED METHODOLOGY

Consider a two wheeled differential drive mobile robot. The position of the center of mass of the robot in a fixed frame of reference is denoted by (X, Y). A finite number of obstacles are assumed to be located in the plane of movement of robot and we assume the obstacles to be fixed with unknown locations. The only information on the obstacles is their number and a probability distribution associated with the location of each single obstacle. Let the probability distribution associated with position of the obstacles be denoted by $f_{obs_i}(m_{obs_i}, \mu_{obs_i})$ where m_{obs_i} and μ_{obs_i} are the mean and trace of the covariance matrix of the probability distribution function respectively. Let the position of the target point be denoted by $q_{goal} = (q_{goal,x}, q_{goal,y})$. It is assumed that in this problem achieving the goal configuration in minimum time is the objective, so when allowed and safe, the robot can move at its maximum allowed speed. APF method is used to determine the desired direction of movement of the robot. In fact, m_{obs_i} determines the position of the repulsive electric charges and μ_{obs_i} determines the magnitude of the electric charge and the resultant force vector. Every obstacle repels the robot when it is located within the distance of influence as formulated in (3), (4). Similarly, an attractive force is exerted to the robot at each instant of time which is directed towards the goal. The sum of attractive and repulsive forces applied to the robot, determine the direction of movement of the robot. In our proposed method, we assume this direction as the desired direction of movement for the robot and it is fed into the CDC to find the desired control signals which can simultaneously satisfy the constraints on the inputs as well as steer the robot toward the desired target. If the total force applied to the robot is represented by F_{total}, according to (5) we can assume the components of this total force are:

$$F_{total} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} \tag{9}$$

where F_x and F_y are the components of the force in the x and y directions, respectively. The direction of this force can be found by using the four quadrant tangent inverse function represented by:

$$\theta_F = atan2(F_x, F_y) \tag{10}$$

In order to be consistent with the problem formulation of CDC in (7), we assume:

$$e_{des} = v_d \begin{bmatrix} cos\ \theta_F \\ sin\ \theta_F \end{bmatrix} \tag{11}$$

Let's define v_d as the desired velocity to safely steer the robot towards the desired target. Mathematically v_d can be defined as:

$$v_d = \alpha V_{maximum} \tag{12}$$

where α is a scaling factor for adjusting and controlling the speed of robot and is defined as:

$$\alpha = \frac{R}{R_{maximum}} \tag{13}$$

where R is the distance between the current position of the robot and the target point and $R_{maximum}$ is the distance between the initial starting point of the robot and the target point.

To steer the robot toward the goal, CDC is used [20]. The search procedure can be formulated as minimizing the angle between the state vector χ and the desired direction of movement. We can assume the state vector χ of the differential drive robot to be as:

$$\chi = \begin{bmatrix} X \\ Y \\ \beta \end{bmatrix} \tag{14}$$

where X and Y show the position of the robot in the Cartesian space and β shows the heading of the robot. Next step is defining the state equations of the system. The state vector of the system is defined just as the state vector χ in (14) which includes the position of the center of mass of the robot and the heading of the robot. The state equation of the system is found via modeling the kinematics of the system. Fig. 1 shows the schematics of the mobile robot used within this paper.
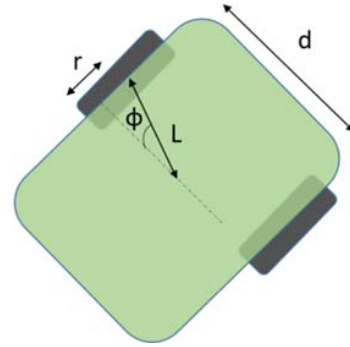

Fig. 1 Schematics of the differential drive robot

Due to the linearity of the equations of motion with respect to inputs for differential drive robots, state equation of the system can be written as:

$$\dot{\chi} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\beta} \end{bmatrix} = Bu \tag{15}$$

Assuming a kinematic model for the differential drive robot with two symmetric wheels on the front, the state equations representing this system can be derived as follows as suggested in [20], [23]:

$$B = \frac{r}{d}\begin{bmatrix} Lcos(\beta + \phi) & dcos(\beta) - Lcos(\beta + \phi) \\ dsin(\beta) - Lsin(\beta - \phi) & Lsin(\beta - \phi) \\ -1 & 1 \end{bmatrix} \quad (16)$$

$$u = \begin{bmatrix} \omega_l \\ \omega_r \end{bmatrix} \quad (17)$$

where r is the radius of the wheels, d the width of the robot, L the distance between center of gravity of the robot to the center of the wheel of the robot, β the heading of the robot, ϕ is a constant angle between vector connecting the center of wheel to the center of gravity of the robot, and the line connecting the two wheels together and crossing the center of the robot, and u is the input vector of the system which consists of angular velocity of the left and right wheels as shown by ω_l and ω_r.

As suggested by the CDC, the control input is chosen such that the reachable directions of movement of robot is closest to the desired direction of movement, which is in our method dictated by the APF method. Since the velocity vector can show the direction of movement of the robot, the goal is to minimize the angle between the velocity vector and the direction produced by the APF. This is equivalent to maximizing the cosine function of the angle between these two vectors.

Assuming the velocity vector as $V = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}$, this maximization problem can be formulated in mathematical representation as:

$$\max_u Cos\,\theta_{V,e_{des}} \quad (18)$$

where $\theta_{V,e_{des}}$ is the angle between velocity vector and the direction vector of artificial potential field. Equation (18) can be further developed as:

$$\max_u \frac{<V,e_{des}>}{\|V\|\|e_{des}\|} = \frac{<V,e_{des}>}{\|V\|\|V_d\|} = \frac{\dot{X}Cos\theta_F + \dot{Y}Sin\theta_F}{\|V\|\|V_d\|} \quad (19)$$

Since velocity vector is the first two elements of the state equations' matrix, we can write:

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\dot{\chi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} \quad (20)$$

or

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}Bu = \frac{r}{d}*\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}*$$
$$\begin{bmatrix} Lcos(\beta + \phi) & dcos(\beta) - Lcos(\beta + \phi) \\ dsin(\beta) - Lsin(\beta - \phi) & Lsin(\beta - \phi) \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \omega_l \\ \omega_r \end{bmatrix} =$$
$$\frac{r}{d}\begin{bmatrix} Lcos(\beta + \phi) & dcos(\beta) - Lcos(\beta + \phi) \\ dsin(\beta) - Lsin(\beta - \phi) & Lsin(\beta - \phi) \end{bmatrix}\begin{bmatrix} \omega_l \\ \omega_r \end{bmatrix} \quad (21)$$

Consider the system $\dot{\chi} = Bu$ subject to constrained inputs $\mathbf{u} \in U_{adm}$ which describes the kinematics of the 2WDMR, where $\dot{\chi} \in R^3$, $\mathbf{B} \in R^{3\times 2}$ and $\mathbf{u} \in R^2$. Let $\mathbf{e}_{des} \in R^2$ be the desired direction vector for the robot which is dictated by the direction of total force resulted from applying the method of APF. The set of control inputs that minimize the angle

between the desired and actual directions of motion can be found by:

$$\mathbf{u}^* = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{e}_{des} \quad (22)$$

If the resulted control input is out of bound, a scaling factor $0 < \gamma \leq 1$ can be used to scale down the control input inside the admissible set of inputs. The proof for derivation of this control law is as: Optimal control values can be found by solving $\mathbf{Bu} = \mathbf{e}_{des}$. If $e_{des} \notin E_{reach}$, based on the projection theorem, among all vectors in E_{reach}, the angle between the projection of e_{des} on E_{reach} and e_{des} is minimum. Therefore, the control input $\mathbf{u}^* = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{e}_{des}$ minimizes $\theta_{V,e_{des}}$. If the resultant control input u^* is not inside the control bounds, the synthesized control u^* is scaled down such that the out of bound control signal sits on the boundaries of U_{adm}. In other words, u^* should be scaled down by $0 < \gamma < 1$ such that the new input belongs to U_{adm} [20].

V. SIMULATION RESULTS AND ANALYSIS

The method proposed in the paper is simulated in this section. The control objective is steering the robot from an initial position of (0,0) to the final position of (3,2) while avoiding stationary obstacles that are randomly distributed on the field. It is assumed that the probability distribution associated with the position of each obstacle is a normal distribution. Following the proposed procedure for constructing the APF, the location of each obstacle is considered to be the mean value of the 2D probability distribution. Similarly, the magnitude of the electric charges on the obstacles is set as the trace of the covariance matrix of the location probability distribution. The bound on the velocity of the wheels is assumed to be 50 rad/sec and the parameters of the robot are assumed to be:

$$\phi = 30°, r = 0.1\,m, l = 0.3\,m, d = 0.1299\,m \quad (23)$$

TABLE I
PROBABILITY DISTRIBUTION FOR OBSTACLE LOCATIONS

Obstacles Number	Mean	Trace of Covariance Matrix
1	(0.5,-1)	4.2
2	(2,0)	3.5
3	(1.5,1.5)	2
4	(2.7,0.1)	0.9
5	(3.2,1)	2.1
6	(0.5,2)	1.6
7	(1.2,0.5)	1.55
8	(2,0.9)	1.1
9	(2.4,1.5)	1.6
10	(0.5,1)	3.1

Ten obstacles are assumed to be in the environment between the robot and its final goal. The exact location of these obstacles is unknown, however, a probability distribution function is known. For simulation, it has been assumed that the probability distribution function is Gaussian with known mean and covariance matrix. Table I shows the

probability distribution functions associated with the obstacles.
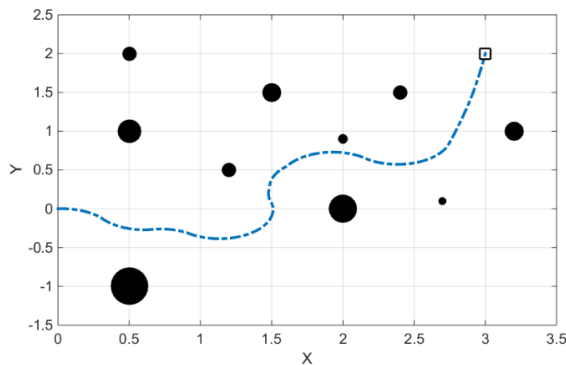

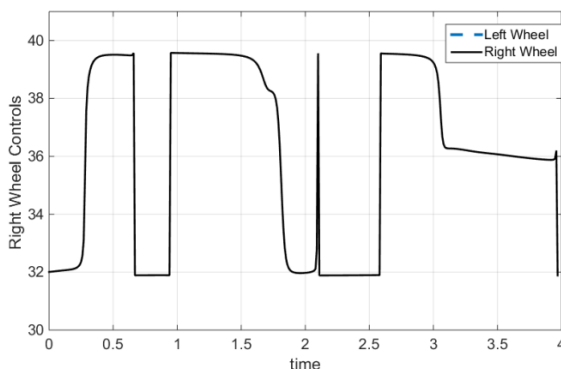Fig. 2 Robot path among obstacles
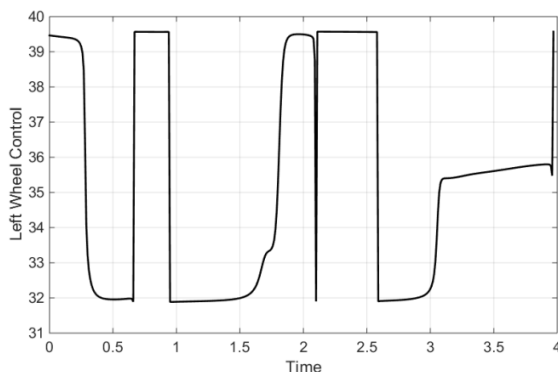

Fig. 3 Right wheel control signal

Fig. 4 Left wheel control signal

For the mentioned steering scenario, the proposed path planning and control strategy is applied and the results of simulations are provided in this section. Fig. 2 shows the path of the robot during the steering process where the rectangle shows the position of the final point and the radius of each obstacle (illustrated as black dot) is proportional to the charge associated with each obstacle. Figs. 3 and 4 show the synthesized control signals for the right wheel and the left wheel of the robot respectively and Fig. 5 shows the time evolution of the states of the system (state trajectories). As the

results show, the robot is steered from the initial position to the final position and the bound on the controls are well-kept.
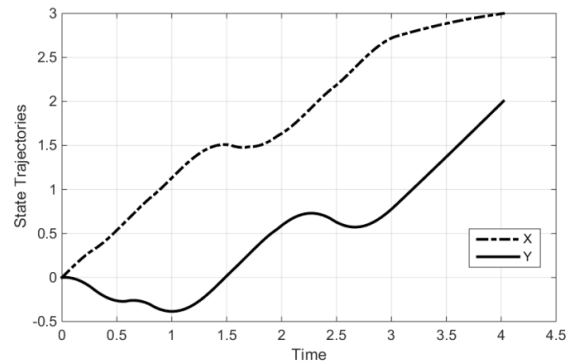

Fig. 5 State trajectories

VI. CONCLUSION

A method has been presented for steering of mobile robots with velocity constraints in environments with partially known obstacles. The exact location of the obstacles is unknown and only a probability distribution for the location of the obstacles is known. This method solves the path planning and control problem simultaneously. APF method was used to find a desired direction of movement at each point while the CDC uses the generated directions to synthesize the control signals required for steering the robot. Velocity of the robot is controlled during the steering and guaranteed not to exceed the imposed bounds, and robot can have a reduced speed in close vicinity of obstacles and target to ensure safety. The presented method is simulated for a steering situation with obstacles and the results demonstrate the success of the method.

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