Statistical Modeling of Local Area Fading Channels Based on Triply Stochastic Filtered Marked Poisson Point Processes

Jihad S. Daba, J. P. Dubois

***Abstract*—**Fading noise degrades the performance of cellular communication, most notably in femto- and pico-cells in 3G and 4G systems. When the wireless channel consists of a small number of scattering paths, the statistics of fading noise is not analytically tractable and poses a serious challenge to developing closed canonical forms that can be analysed and used in the design of efficient and optimal receivers. In this context, noise is multiplicative and is referred to as stochastically local fading. In many analytical investigation of multiplicative noise, the exponential or Gamma statistics are invoked. More recent advances by the author of this paper utilized a Poisson modulated-weighted generalized Laguerre polynomials with controlling parameters and uncorrelated noise assumptions. In this paper, we investigate the statistics of multi-diversity stochastically local area fading channel when the channel consists of randomly distributed Rayleigh and Rician scattering centers with a coherent Nakagami-distributed line of sight component and an underlying doubly stochastic Poisson process driven by a lognormal intensity. These combined statistics form a unifying triply stochastic filtered marked Poisson point process model.

***Keywords*—**Cellular communication, femto- and pico-cells, stochastically local area fading channel, triply stochastic filtered marked Poisson point process.

I. Introduction

THE rapid growth of cellular communication has caused an exponential growth in research to optimally design transceivers that account for the fading and reduced signal-to-noise-ratio that notoriously deteriorate the performance of these systems.

Key to the design of efficient demodulation schemes are accurate modeling and in-depth stochastic analysis of wireless channels. The network structure to support cellular communications consists of various components at different scales ranging from mega- (global) and macro- to pico- and as little as femto-cellular sizes. Classically, these channels have been assumed to be fully developed, that is, comprising an infinite number of scatterers, leading to well-known multipath fading models such as Rayleigh, Rician, lognormal, and Nakagami-*m*. While these models work well for large cellular structures, mainly, mega-, macro-, and micro-cells which dominate 2G, 2.5G and some 3G cellular systems, they are known not to accurately characterize smaller cellular structures. With the emerging and planning of new wireless technologies, cell sizes have been reduced to pico- and femto-levels especially for the next generation cellular systems, as well as a multitude of other WPAN and ad-hoc network technologies in the planning [1]. The channel fading models in these small cells, which we term stochastically local area fading channel (SLAC), would no longer follow classical fully-developed noise models, thus triggering the need to develop newer more accurate stochastic models for fading.

There are also many other propagation scenarios where the received signal comprises a small random number of multipath waves. While this typically occurs for narrowband receiver operation, directional antennas and wideband signals also increase the likelihood of partially developed fading. In fact, directive antennas or arrays tend to amplify several of the strongest multipath waves arriving in a particular direction while attenuating the remaining waves [2], [3]. Also, wideband receivers have the ability to reject multipath components that arrive with largely different time delays, effectively retaining only a small number of multipath waves [4], [5], which we consider random for more accurate analysis. In such fading environment, directive antennas and wideband receivers have a tendency to increase the coherent specular power-to-diffuse power ratio, also known as the Rician K-factor [6]. Therefore, the likelihood of partially developed Rician statistics in a SLAC is high.

In this paper, we develop statistical models for noise in stochastically local area fading channels which prove to be more accurate for the pico- and femto-cells than the classical fading noise models. We extend the results developed in our previous work [7] to consider the case when the specular line of sight power component is random, and when the fading channel consists of a doubly stochastic Poissonian number of Rayleigh and Rician scattering centers.

Since the statistics of speckle noise in ultrasound imaging systems are based on the same physical assumptions as those of fading noise in wireless systems, and since both type of noise are multiplicative, the stochastic models derived in this paper will also prove useful for the development of optimal detection, estimation, segmentation, and filtering schemes in coherent imaging systems [8]-[14].

II. Triply Stochastic Filtered Poisson Point Process

One common approach to modeling random scattering is to assume that the backscattered return within a resolution element arises from the collection of elemental point

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scatterers. The total backscattered field is then formed by the sum of the scattered fields from all of the elemental scatterers. The amplitude of the scattered field from each of these elemental scatterers will in general be a function of their size and physical properties. Thus, in general, the amplitude of the field scattered by each elemental scatterer can be a random variable.

For most wireless multipath channels, the locations of the elemental scatterers can be viewed as random, and furthermore, the number of elemental scatters within a channel will be a random variable. One of the most effective methods for modeling "elements" that occur randomly in space is to use a point process [15]. If, in addition, there is attached to each point (random location) a random quantity that can be represented by one or more random variables (in this case the amplitude A_k and phase ϕ_k of the backscattered field), the natural stochastic model to use is that of a marked point process [15]. Furthermore, for many multipath channels, the number of elemental scatterers in disjoint regions will be statistically independent integer-valued random variables and the point process is termed a compound point process [15]. If in addition, the point process satisfies a technical condition called Khinchine orderliness [15] then the point process will be a Poisson point process. The number "N" of elemental scatterers within a channel will be a Poisson random variable with intensity or rate λ, which in turn is random. For this reason, a *triply stochastic compound Poisson point process* is a useful model of random scattering in wireless channels.

The triply stochastic characteristic arises from associating random scattering amplitudes, a random specular line of sight power (caused by mobility), and a random process intensity to the underlying doubly stochastic point process analyzed in our previous work [7].

When the number of scattering points within a channel is sufficiently large that the central limit theorem holds and the scattered field is approximately a circular complex Gaussian random variable, we say that the fading noise (or speckle noise) is fully developed. But if the number of points is relatively small typically less than 10 to 20, the fading noise is not fully developed. In this case, the triply stochastic compound point process is useful for characterizing the partially developed fading, and estimates of the intensity λ can be used to characterize the average received diffuse power [16]-[20]. We now investigate the statistics of triply stochastic compound point processes for the purpose of characterizing the stochastic nature of the received fading power.

A. Conditional Probability Density Function of SLAC

The conditional fading power $\nu \mid N, A_k, V_0$ for the generalized random walk in the complex plane model

$$\nu_N = \left| \sum_{k=1}^{N} A_k e^{j\varphi_k} + V_0 \right|^2 \tag{1}$$

can be generated from the definite integral

$$p_{\nu_N}(\nu_N) = \frac{1}{2}\int_0^\infty u J_0(u\sqrt{\nu_N}) e^{-\frac{u^2 P_c}{4}} \left[\prod_{k=1}^{N} J_0(a_k u) \right] du,$$
$$P_c = |V_0|^2, \tag{2}$$

where $J_0(.)$ is the zero-order Bessel function of the first kind and P_c is the power of the specular coherent scatterer (direct line of sight or coherent uniform background). By setting $g(u) = e^{-\frac{u^2 P_c}{4}} \prod_{k=1}^{N} J_0(a_k u)$, $p_{\nu_N}(\nu_N)$ can be conceived as $0.5 H_{0,\sqrt{\nu_N}}\{g(u)\}$, where $H_{0,z}\{g(u)\}$ is the zero$^{\text{th}}$ order Hankel transform (or Fourier-Bessel transform) of $g(u)$ defined as

$$H_{0,z}\{g(u)\} = \int_0^\infty u J_0(zu) g(u) du. \tag{3}$$

Different channel model parameters (a_k, N, P_c) will generate different functions $g(.)$ whose Hankel transforms can be obtained, when feasible, from the Hankel tables published in [21].

In separate papers [22], [32], we examined some advanced stochastic models for partially developed noise power. Particularly in [22], we developed a closed form for the conditional fading power $\nu \mid N$ as a Gram-Charlier series expansion, under the assumption that $A_k = A_0 \ \forall k$ and $V_0 = 0$. Based on these results, we developed the conditional pdf of the fading power $\nu \mid N$ and showed that it obeys a series of Laguerre-weighted exponential law with parameters $\lambda A > 0$ and $A_0 > 0$.

B. Poisson-Modulated PDF of SLAC driven by a Random Process Rate

Since A_k and V_0 are in general random variables, we condition the pdf of the fading power on these variables and on the intensity of the driving Poisson process, and we average the conditional pdf over the Poissonian number of scatterers:

$$\begin{aligned} p_\nu(\nu \mid A_k, V_0, \lambda) &= E_N\left(p_{\nu|N}(\nu \mid N = n, A_k, V_0) \right) \\ &= e^{-\lambda A} \sum_{n=1}^{\infty} \frac{(\lambda A)^n}{n!} p_{\nu|N}(\nu \mid N = n, A_k, V_0). \end{aligned} \tag{4}$$

C. Scatterers' Amplitude and LOS Models

Since the SLAC channels have varying scattering characteristics with respect to the wavelength of the propagating wave, we expect the scatterers' strength of to be governed by different models for different types of channels. The conditional pdf of the fading power becomes after randomizing the scattering amplitudes and the LOS:

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$$p_V(\nu \mid \lambda) = E_{V_0}\left(E_{A_k}\left(E_N\left(p_{\nu|N}(\nu \mid N = n, A_k, V_0)\right)\right)\right)$$
$$= \int_0^{\infty} p_{V_0}(\kappa) \int_0^{\infty} \sum_{n=1}^{\infty} \frac{(\lambda A)^n}{n!} p_{\nu|N}(\nu \mid N = n, A_k = \alpha, V_0 = \kappa) \times \quad (5)$$
$$\times p_{A_k}(\alpha) d\alpha d\kappa e^{-\lambda A}.$$

We consider two models for the amplitudes: (1) the reflectances of the elementary scatterers are Rayleigh distributed with controlling parameter σ, and (2) the reflectances are Ricieаn distributed with controlling parameters s and σ. In the case of Rayleigh distributed amplitudes, we consider the SLAC channel as made up of a small number of scattering centers, with each center made up of a large number of elementary scattering points (Rayleigh center). In the case of Ricieаn distributed amplitudes, we consider the SLAC channel as made up of a small number of scattering centers, with each center made up of a large number of elementary scattering points with a micro LOS (line of sight) component "*s*" present in each center (Ricieаn center).

We also consider two models for the macro LOS parameter V_0: (1) constant model, and (2) V_0 obeys a Nakagami law with controlling parameters μ and w. The latter model is suitable for land mobile station (LMS) applications.

D. Non-conditional Statistics of SLAC under Shadow Fading

The general model shown in (5) assumes that the underlying renewal point process is a doubly stochastic Poisson point process conditioned on the intensity $\lambda > 0$. To cover the most general case when the driving process rate λ is itself random, we consider a lognormally distributed intensity (with controlling parameters μ and σ) suitable for applications with shadow compound fading. The non-conditional pdf of the fading power becomes:

$$p_V(\nu) =$$
$$E_{\Lambda}\left(E_{V_0}\left(E_{A_k}\left(E_N\left(p_{\nu|N}(\nu \mid N = n, A_k, V_0)\right)\right)\right) \Big| \Lambda = \lambda\right), \quad (6)$$

where the nested expectation holds because all the underlying random variables defining the triply stochastic filtered marked Poisson point process are statistically independent.

III. Simulations and Results

Using Monte-Carlo simulations, we illustrate in Figs. 1 – 6 the pdf of partially developed fading power in SLAC channels for different statistical distributions of the underlying model variables and, without loss of generality, for particular controlling parameters.

We also depict the probability density function of the multi-diversity statistic ν_L that is formed by incoherently combining L-independent fading power statistics received over L-independent channel paths, according to the model:

$$\nu_L = \sum_{l=1}^{L} \left| \sum_{k=1}^{N_l} A_{k,l} e^{j\phi_k} \right|^2 . \quad (7)$$

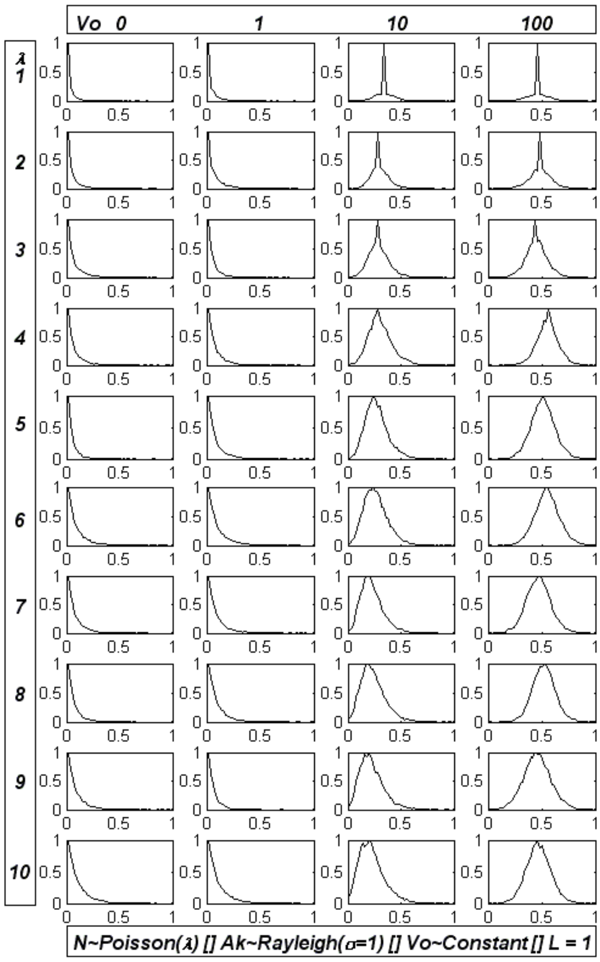

Fig. 1 The power's pdf for different values of mean number of scatterers λ (N is assumed Poisson), different specular components V_0, Rayleigh scattering amplitudes A_k, in a single-input-single-output (SISO) channel (single diversity path L = 1)

We observe in Fig. 1 that for V_0 = 0, the pdf of multipath fading power approaches an exponential distribution as the mean number of scatterers λ increases. In addition, as the specular LOS compoment V_0 and λ increase, the probability density function approaches a Rician law. These results are consistent with the Central Limit Theory for stochastic convergence of random variables in the distribution sense.

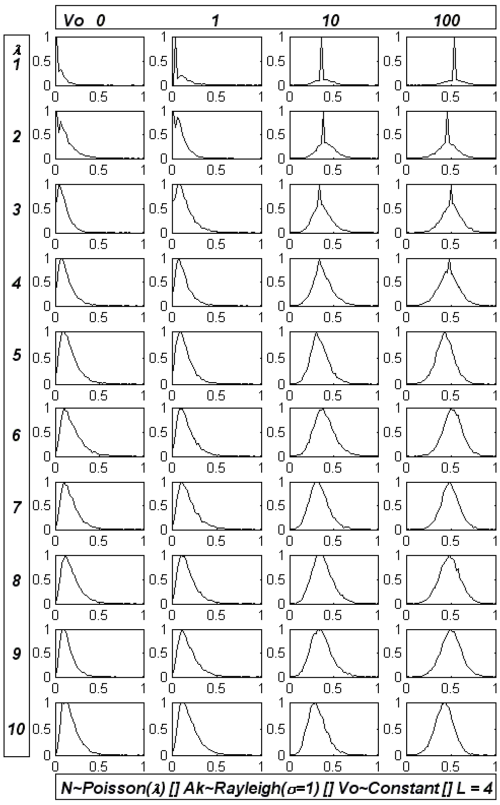

Fig. 2 The power's pdf for different values of mean number of scatterers λ (N is assumed Poisson), different specular components V_0, Rayleigh scattering amplitudes A_k, in a single-input-multiple-output (SIMO) channel ($L = 4$ diversity paths)

We observe in Fig. 2 that for $V_0 = 0$, the pdf approaches a gamma distribution as λ increases. In addition, as V_0 and λ increase, the pdf approaches a modified-Rician law.

We observe in Fig. 3 that for $V_0 = 0$, the pdf approaches an exponential distribution as λ increases. In addition, as V_0 and λ increase, the pdf converges to a Rician law with a faster convergence rate than that of the Rayleigh scattering centers model depicted in Fig. 1.

We observe in Fig. 4 that for $V_0 = 0$, the pdf approaches a gamma distribution as λ increases. In addition, as V_0 and λ increase, the pdf converges to a Rician law with a faster convergence rate than that of the Rayleigh scattering centers model depicted in Fig. 2.

We observe in Fig. 5 that the pdf approaches an exponential distribution for $L = 1$ and a gamma distribution for $L \geq 2$ as λ increases.

We observe in Fig. 6 that for $V_0 = 0$, the pdf is consistent with an exponential distribution for $L = 1$ and with a gamma distribution for $L \geq 2$. As L and V_0 increase, the pdf is consistent with a Rician law.

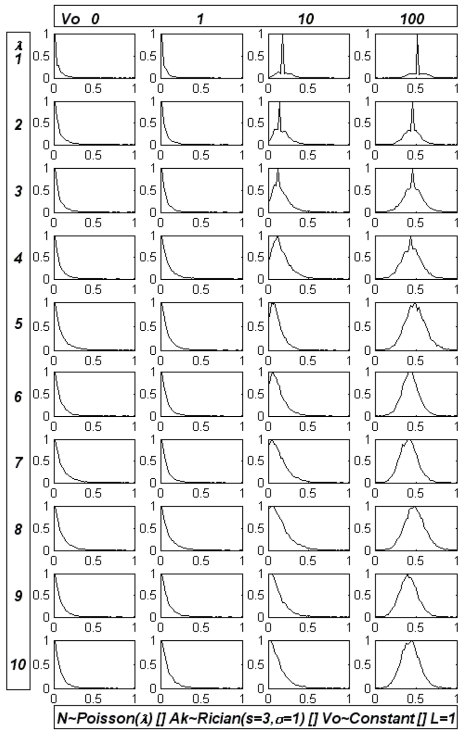

Fig. 3 The power's pdf for different values of mean number of scatterers λ (N is assumed Poisson), different V_0, Rician scattering amplitudes A_k, in a SISO channel (single diversity path $L = 1$)

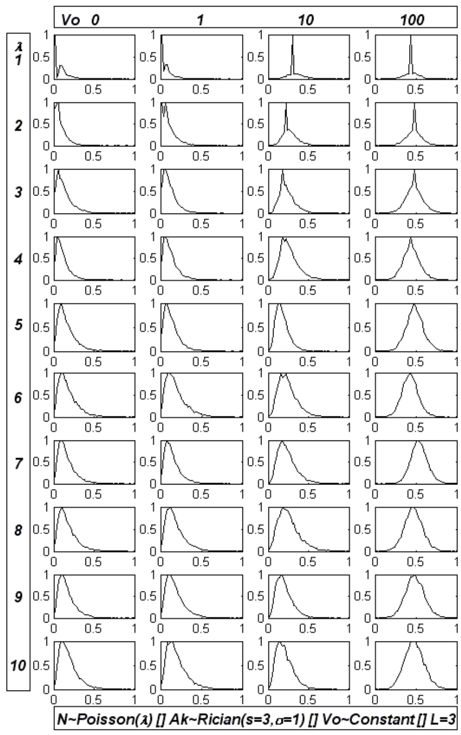

Fig. 4 The power's pdf for different values of mean number of scatterers λ (N is assumed Poisson), different V_0, Rician amplitudes A_k, in a SIMO channel ($L = 3$ diversity paths)

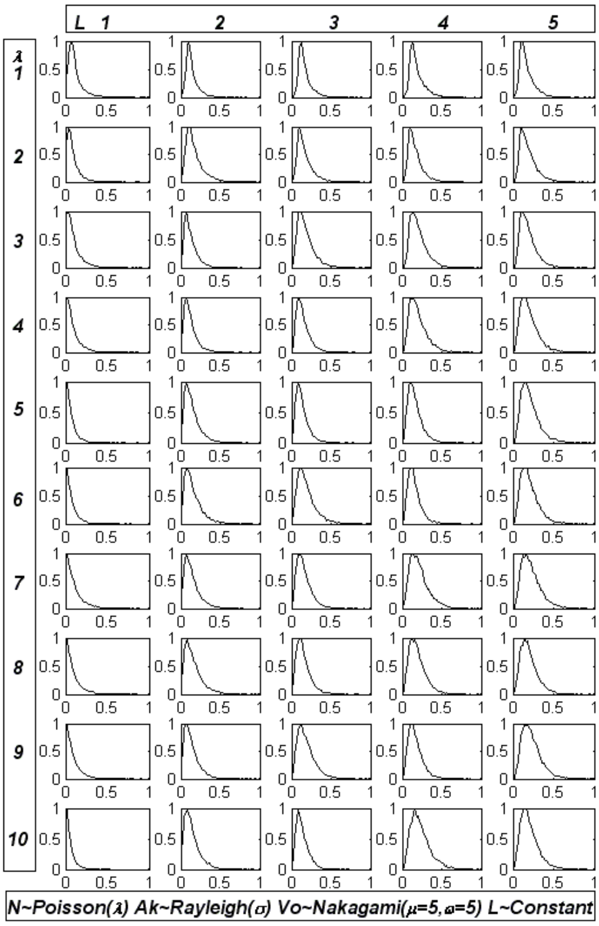

Fig. 5 The power's pdf for different values of mean number of scatterers λ (N is assumed Poisson), different channel diversities L, Rayleigh scattering amplitudes A_k (with controlling parameter $\sigma = 1$), and Nakagami distributed specular component V_0

IV. Conclusions and Future Work

In this paper, we derived the statistics of stochastically local area channels (SLACs) in small cellular structures of the pico- and femto-types. The proposed models extended the stochastic models previously developed in [7], [22], [25], [26] by considering a doubly stochastic Poissonian number of Rayleigh and Rician scattering centers in the presence of a Nakagami-distributed coherent specular line of sight component. Combining these statistics, the proposed model form a unifying triply stochastic filtered marked Poisson point process with a driving underlying lognormally-distributed intensity.

The results were consistent with the Central Limit Theory in the sense that the distribution asymptotically converged to an exponential law with an increasing mean number of scatterers for single diversity, and to a gamma law for multi-receiver diversities.

This work forms a basis for efficient receiver design and channel estimation in cellular systems. Since the statistics of multipath fading are based on the same underlying models as speckle noise in synthetic aperture radars and ultrasound imaging systems, this work can also have an impact on detection and estimation problems in these coherent imaging systems.

As future work, we plan on examining triply stochastic filtered marked point processes driven by an underlying Pascal-Negative Binomial process [33], [34] representing the number of scatterers in the SLAC channel.

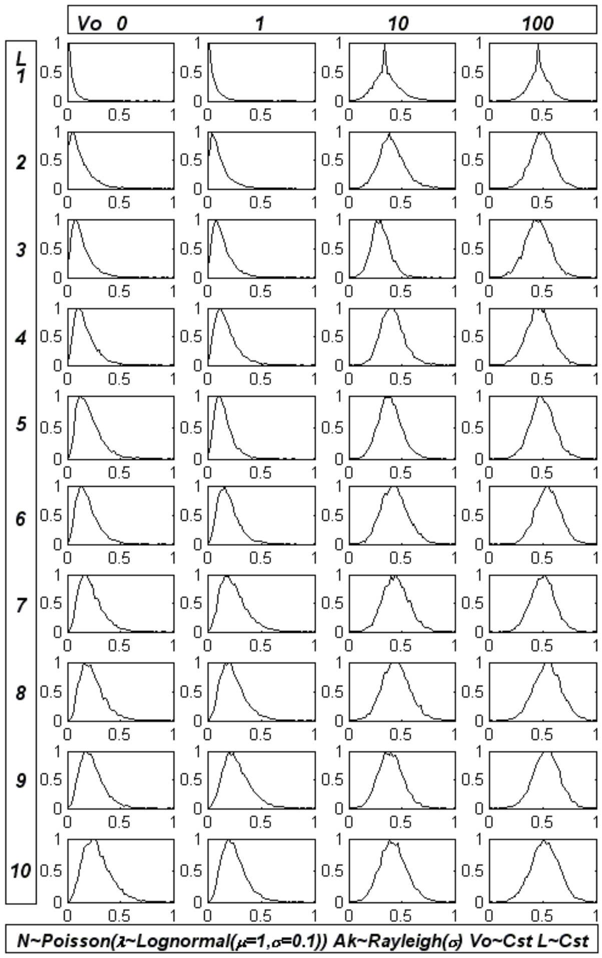

Fig. 6 The power's pdf for different specular components V_0, different channel diversities L, Rayleigh scattering amplitudes A_k (with controlling parameter $\sigma = 1$), and lognormally distributed Poisson intensity λ (mean number of scatterers)

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